Physics [Module 4]

Notes

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typeset with LaTeX

Module - 4

Physics

By Abhishek Bohare

P¼C3/D204

- Laplace-Runge-Lenz Vector (Astronomy)
- Time Variable gravity signal during the water impoundment of China Three gorges Reservoirs (Geophysics)
- Classical Mechanics
- Mathematical Physics
- Astrophysics
- Electrodynamics
- Electronics
- How to write an experiment Report?
- How to present your (ppt) seminar?
- Stellar Evolution Applet

(Laplace-Runge-Lenz Vector)

Abhishek Bohare
F-1/C-21

In classical mechanics and astronomy, the Laplace-Runge-Lenz vector is a vector that used to describe and clearify the shape of one astronomical body and orientation of the orbit of one astronomical body around another.

→ It is generally known as LRL vector.

→ For two bodies interacting by Newtonian gravity, the LRL vector is a constant of motion.

change of position

distance dependent

orientation of the orbit one to another.

In 1687, English mathematician sir Issac Newton published principia, which hypothesizes that inverse square law of univeral gravitation.

$$F \propto \frac{1}{r^2}$$

Newton's theory enjoyed its greatest success when it was used to predict the existance of Neptune based on the motion of Uranus could not be accounted for by the actions of the other planets.

→ But newton's Theory has been suspended by einstein's general relativity, most modern non-relativistic gravitational calculations are still made using newton's Theory because it is simpler to work with and it gives sufficient accurate results for more applications involving sufficient small masses, speeds and energies.

In general relativity, the effects of gravitational are ascrible(मढ़ना) to spacetime curvature instedd of a force. Free falling Inertial objects are as being accelerated relative to non-inertial observers on the ground. In newtonian gravity, however, no such acceleration can occur unless at least one of

The LRL vector is named as Laplace-Runge-Lenz vector after Pierre-Simon de Laplace, carl Runge and wilhelm Lenz. It is also known as a Laplace vector, the Runge-Lenz vector. Ironically, none of these scientists discovered it. The LRL vector has been re-disovered several times and is also equivalent to the dimensionless eccentrically vector.

↓

In celestial mechanics, the eccentricity vector of a Kepler orbit is the dimensionless (eccentricity) vector with direction pointing from apoapsis to periapsis and with magnitude equal to the orbit's scalar eccentricity.

↳ explained at next page.

- Kepler's Laws -

Ptolemy (Sun centered theory)

↓

Inaccurate Theory

↓

Then Kepler discovered

↓

At Royal Observatory (Denmark)

↓

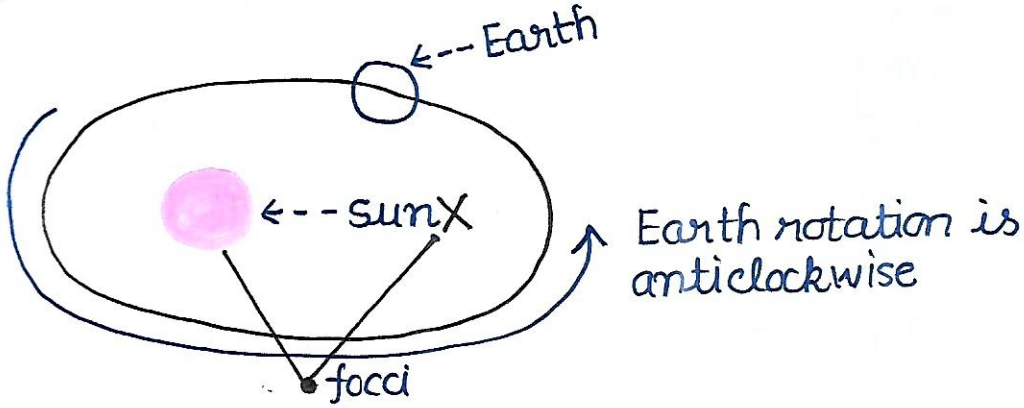
Work on Tycho Brahe Theory

↓

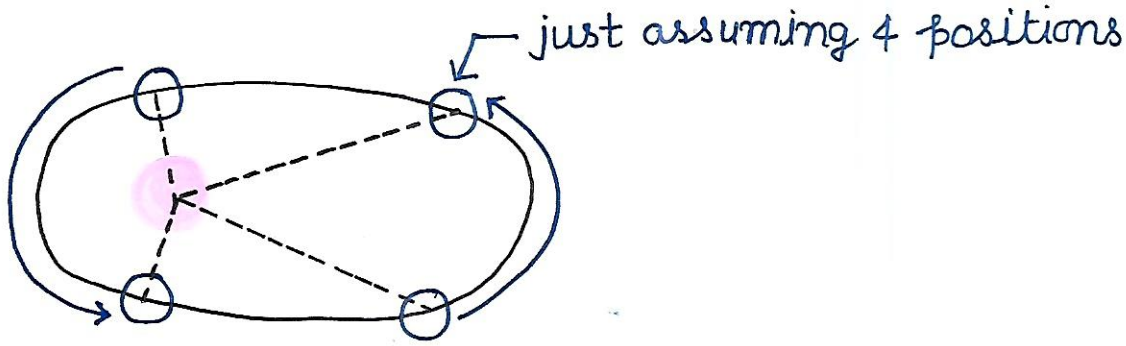
Recorded data and give 3 rules.

Tycho Brahe was a astron-omer and writer known for his comprehensive astronomical and planetery observation.

Law 1- The orbits of the planets are ellipses, with the sun at one focus.

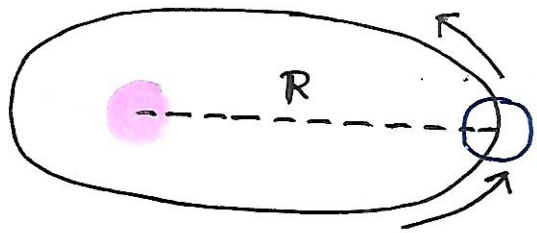


Law 2- The line joining a planet to the sun sweeps out equal areas in equal times as the planet travels around the ellipse.



→ The fastest a planet moves is at perihelion (closest) and the slowest is at aphelion (Farthest).

Law 3- The square of the total time period[T] of the orbit is proportional to the cube of the avarage distance of the planet to the sun.



$$T^2 \propto R^3$$

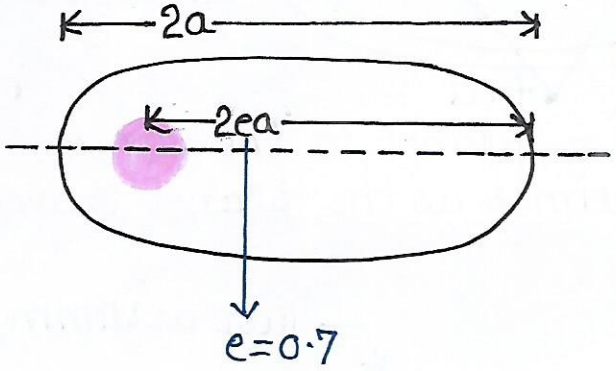
$$T^2 = \frac{4\pi^2}{GM} R^3$$

can be expressed as simply

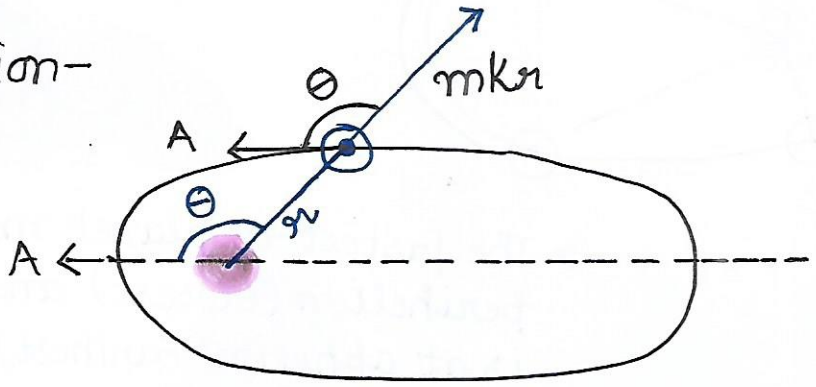
Where T = earth years
a = austronomical unit (AU)
M = solar masses $M_\odot$

Orbit Eccentricity -

The eccentricity of an ellipse can be defined as the ratio of the distance b/w the focci to the major axis of the ellipse. The eccentricity is zero for a circle. Of the planetery orbits, pluto has large eccentricity. But now pluto is no more a planet. so neptune has large eccentricity.



→ Derivation -



The shape and oriantation of the Kepler problem orbit can be determined from the LRL vector.

For a single particle acted on by an inverse square central force described by the equation

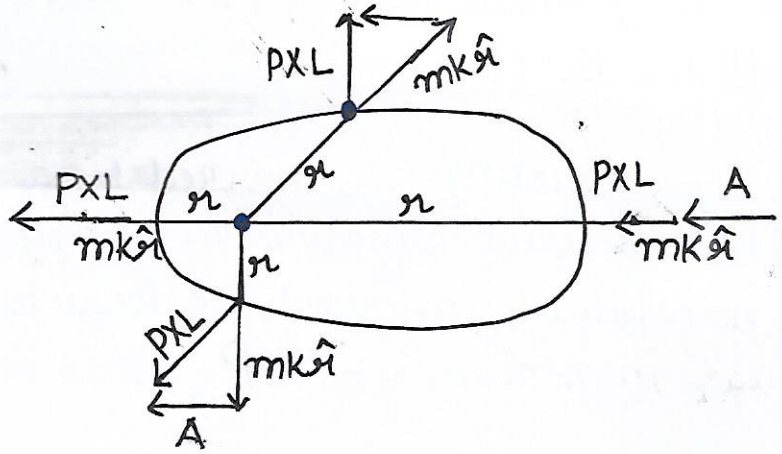
$$F(r) = \frac{-K}{r^2}\hat{r}$$ Why → inverse square

Intensity $\propto \frac{1}{\text{distance}^2}$

central force

In classical mechanics, a central force on an object is a force that is directly along the line joining the object and the origin.

$$\vec{F} = F(r)\hat{r}$$



LRL vector $A = PXL - mk\hat{r}$ ..... (1)

why

where m = mass of a point particle moving under the central force

P = momentum force

L = $r \times P$ angular momentum

K = parameter to describe strength of central force

r = position vector

$\hat{r}$ = corresponding unit vector

$$\hat{r} = \frac{r}{\vec{r}}$$

The angular momentum vector L is perpendicular to the orbit. The coplaner vector PXL and $\left(mK\frac{r}{\vec{r}}\right)$

are shown in diagram/Figure.

Since Force is conservative, total energy E

$$E = \frac{p^2}{2m} - \frac{K}{r} = \frac{1}{2}mv^2 - \frac{K}{r}$$

why

$$E = \frac{p^2}{2m} \Rightarrow P = \sqrt{2mE_K}$$

$$E_K = \frac{1}{2}mv^2$$

$$P = mgh$$

We are interesing in the general Kepler problem, i.e. the Hemiltonian

$$H = \frac{|P^2|}{2m} - \frac{K}{r}$$

This problem is a very intersting one, marked by a great deal of symmetry. Besides the usual conservation energy E, since the Hamiltonian is rotationally invariant, we know to expect conservation of angular momentum $L = r \times p$

In order to find the other relation among E, L, A, we need to look at the direction A points in details. We can take the dot product with the radius vector.

$$A \cdot \vec{r} = Ar\cos\theta = \vec{r} \cdot (P \times L) - mkr \quad [\text{Put value from eqn (1)}] \quad \ldots\ldots\ldots (2)$$

And permuting the triple product we get the form

$$A \cdot \vec{r} = (r \times P) \cdot L = L^2 \qquad (L = r \times P)$$

$$Ar\cos\theta = L^2 - mkr$$

$$\frac{1}{r} = \frac{mk}{L^2}\left(1 + \frac{A}{mk}\cos\theta\right) \quad \ldots\ldots\ldots (3)$$

} I don't know how it has been solved.

$$e = \frac{A}{mk} = \frac{|A|}{mk}$$

$$|2l| = \frac{2L^2}{mk}.$$

$$a(1+e^2) = l = \frac{L^2}{mk}$$

$$[A^2 = m^2k^2 + 2mEL^2]$$

Took dot product of A with itself yield an eqn involving the energy

$$e^2 - 1 = \frac{2L^2}{mk^2}E$$

comparing this eqⁿ (3) with the standard solution of the orbit

$$\frac{1}{r} = \frac{mK}{L^2}\left(1 + \sqrt{1+\frac{2EL^2}{mK^2}}\cos(\theta-\theta')\right)$$

we find that we can write

$$\frac{A^2}{m^2K^2} = 1+\frac{2EL^2}{mK^2} = \frac{mK^2+2EL^2}{mK^2}$$

which gives me

$$A^2 = m^2K^2 + 2mEL^2$$

which is precisely the other relation we were seeking.

Hamiltonian may have been the first to utilize the LRL conservation. He referred to it as the eccentricity vector, which if we rearrange our expression as

$$mK\frac{r}{\vec{r}} = PXL - A$$

And take the dot product to it with itself, we obtain

$$(mK)^2 = A^2 + P^2L^2 + 2L(PXA)$$

And now choosing L to be along the Z axis and the major semiaxis along the X axis, we obtain

$$\boxed{P_x^2 + \left(P_y^2 - \frac{A}{L}\right)^2 = \left(\frac{mK}{L}\right)^2}$$

In other words, the momentum P is confirmed to a circle of radius $mK/L = L/\ell$ centered on $(0, A/L)$. The essentricity e corresponds to the cosine of the angle η.

- **Hydrogen Atom–**

Another interesting problem that fits the bill for the above description central force and inverse square force,

∞ is the Hydrogen Atom, Just as before we have the Hamiltonian.

$$H = \frac{p^2}{2m} - \frac{e^2}{r}$$

• Time Variable gravity signal during the water impoundment of china Three-gorges Resevoir

→ Beginning in 2003

→ Jean Paul Boy and Benjamin F. Chao

- By 2009, it will be holding 400 Km^3 of water, flooding a stretch of middle Yangtze River about 600 Km in length.

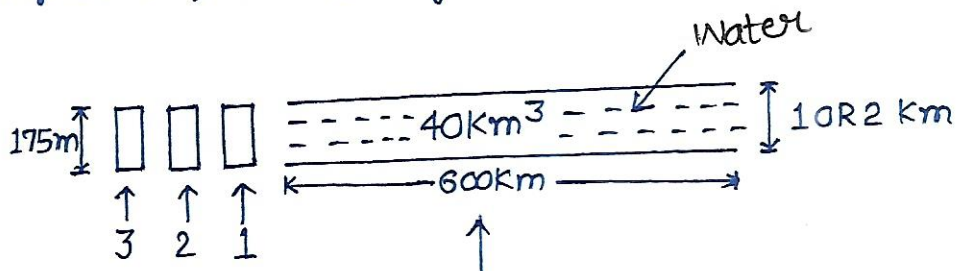
↓ used geophysical controlled experiment

Classical forward/inverse modeling problem of surface Loading

↓ GRACE

Time variable gravity signal

→ Result using green function model adopting the PREM elastic Earth model indicates that the per year geoid height increase is above the GRACE observational sensitivity out to harmonic degree 20, and to degree 50.



Water mass re-distribution (Resulting)

Loading on Land

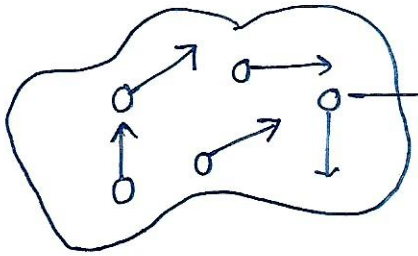
unloading on ocean

Monitoring — Lithospheric Loading response

- area gravity

Geophysical monitoring programme at the surface are indeed taking place. This is mainly for earthquake risk assessment. Wang has studied that possible loading effects on a local scale within about 100 Km upstream from the dam. He calculated that, depending on the local rheology, the local peak changes are; 3.41 mGal in free-air gravity, 7.5 mm in surface geoid, up to 4.8 cm in surface subsidence, and 7.8 arcseconds east-west and 17.5 arcseconds north-south in the tilt.

#Conservation Law-



How these objects are moving

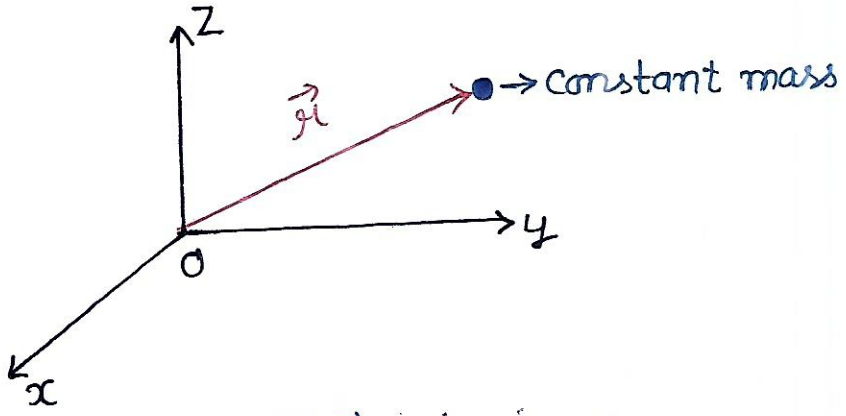
↓

Study of moving objects

- Mechanics
- dynamics
- Analytical Mechanics

- classical Mechanics
- Quantum Mechanics (New prespective)

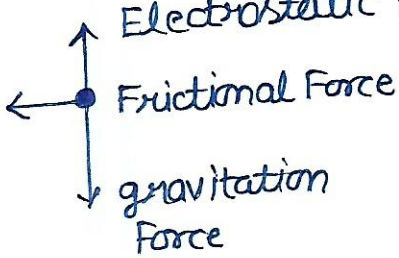
- mass
- velocity
- Space
- time

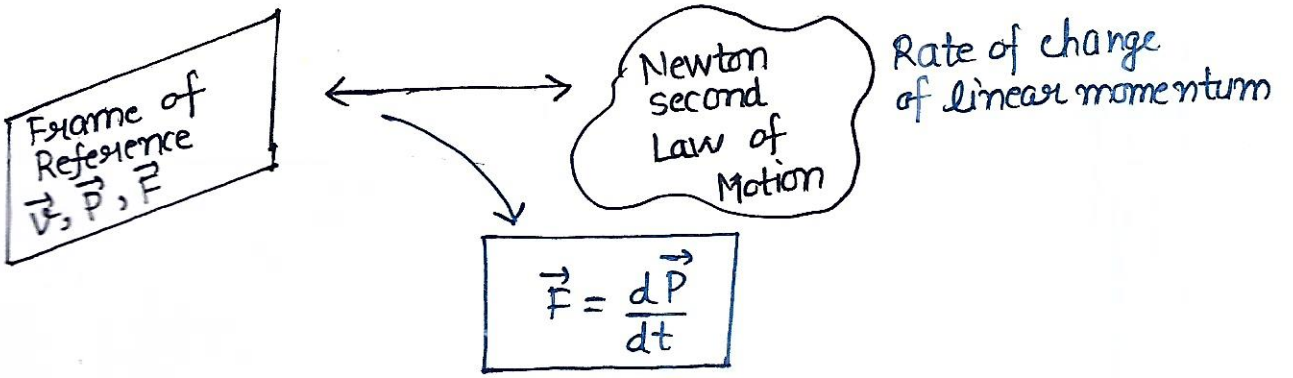


velocity vector = $\frac{d\vec{r}}{dt}$

Linear momentum = $m \cdot \vec{v}$

Force vector = $\vec{F}$ ← external force acting on the particle





$$\vec{F} = \frac{d}{dt}(m\vec{v})$$

$$\vec{F} = m\frac{d\vec{v}}{dt} \qquad \left[\because \vec{a} = \frac{d\vec{v}}{dt}\right]$$

$$\boxed{\vec{F} = ma}$$

Mechanics

↓

Many conservation Laws

Linear momentum conservation -

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{F} = 0$$

$$\frac{d\vec{p}}{dt} = 0$$

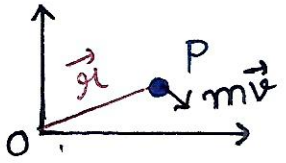
$$\vec{p} = \text{Constant}$$

If the external force acting on the particle are zero then the linear momentum of the particle remain constant.

Angular Momentum Conservation -

⇓

Amount of motion contained in an object.

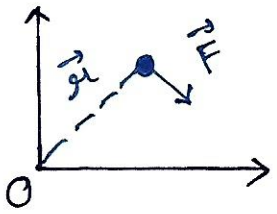


$\vec{L} = \vec{r} \times m\vec{v}$

$\vec{L} = \vec{r} \times \vec{P}$

↑ ~~Linear~~ Angular momentum of the particle

moment of the Force / Torque-



$\vec{N}\ (\vec{\tau}) = \vec{r} \times \vec{F}$

Law of conservation of ~~Angular~~ Momentum-

$$\frac{d}{dt}(m\vec{v}) = \vec{F}$$

$$\vec{r}\frac{d}{dt}(m\vec{v}) = \vec{r} \times \vec{F} = (\vec{\tau})\vec{N}$$

$$\frac{d\vec{r}}{dt}(m\vec{v}) + \vec{r}\frac{d}{dt}(m\vec{v}) = \vec{N}$$

$$\vec{v}(m\vec{v}) + \vec{r}\frac{d}{dt}(m\vec{v}) = \vec{N}$$

$\Rightarrow |\vec{a} \times \vec{b}|$

$\Rightarrow |\vec{a}||\vec{b}| \sin\theta$

$\Rightarrow m|\vec{v}||\vec{v}| \underbrace{\sin 0}_{0}$

$\Rightarrow 0$

$$0 + \vec{r}\frac{d}{dt}(m\vec{v}) = \vec{N}$$

$$\frac{d}{dt}(\vec{r} \times m\vec{v}) = \vec{N}$$

↓

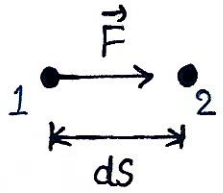
Angular momentum

$$\boxed{\frac{d\vec{L}}{dt} = \vec{N}}$$

If $\vec{N} = 0 \Rightarrow \frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L}$ = Constant

If the external Torque about point O on a particle is zero then Angular momentum about that point is constant.

#Law of conservation of Energy-



$$dW = \vec{F}\cdot d\vec{s}$$

$$W_{12} = \int_1^2 \vec{F}\cdot d\vec{s} \qquad \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

$$W_{12} = \int_1^2 m\frac{d\vec{v}}{dt}\cdot d\vec{s} \qquad d\vec{s} = \vec{v}\,dt$$

$$W_{12} = \frac{m}{2}\int_1^2 2\frac{d\vec{v}}{dt}(\vec{v}\,dt)$$

$$W_{12} = \frac{m}{2}\int_1^2 \frac{d}{dt}(\vec{v}\cdot\vec{v})dt$$

$$W_{12} = \frac{1}{2}m\int_1^2 \frac{d}{dt}(v^2)dt = \frac{1}{2}m\int_1^2 dv^2$$

$$W_{12} = \frac{m}{2}\left[v_2^2 - v_1^2\right]$$

$$W_{12} = \frac{mv_2^2}{2} - \frac{mv_1^2}{2} \qquad E_K = \frac{1}{2}mv^2$$

$$\boxed{W_{12} = K_2 - K_1}$$

Force is such that, the work done by it is independent of the path. The work depends on the initial and Final position. Such Force is called Conservative Force.

Frictional Force is non conservative Force.

Let us assume that the Force is conservative.

$$\vec{F} = -\vec{\nabla}V$$

(V: Potential of the Force)

$$\vec{F}\cdot d\vec{s} = -\vec{\nabla}V\cdot d\vec{s}$$

$$= -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)\cdot\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$$

$$= -\frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz = -dV$$

$$\vec{F}\cdot d\vec{s} = \int_1^2 \vec{F}\cdot d\vec{s} = \int_1^2 -dV = -[V]_1^2$$

$$\vec{F}\cdot d\vec{s} = V_1 - V_2$$

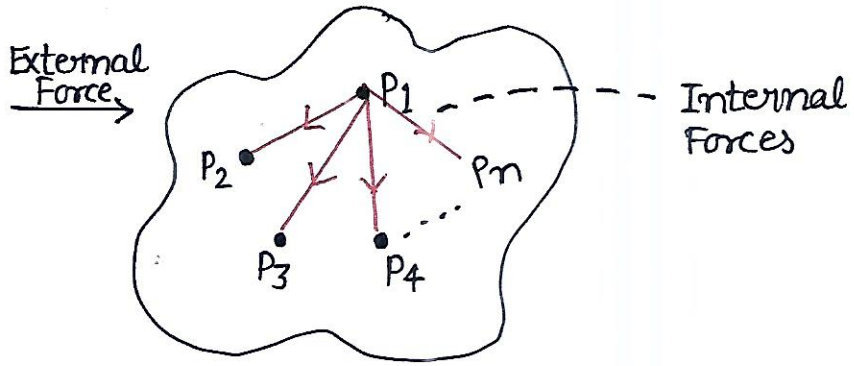
$$W_{12} = K_2 - K_1$$

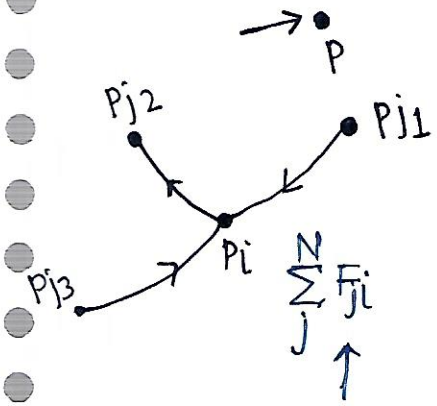
$$K_2 + V_2 = K_1 + V_1$$

(K: Kinetic Energy, V: Potential Energy)

$\Rightarrow P_E + K_E \Rightarrow$ Conservation of Energy

System of Particle-





$\sum_j^N F_{ji}$ ← Total Internal Force

F_i^e ← External Force acting on the ith Particle

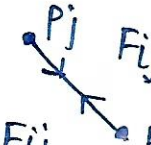
$\vec{r_i}$ = position vector

$$m_i \times \frac{d^2\vec{r_i}}{dt^2} = F_i^e + \sum_j^N F_{ji}$$

$$\sum_{i=1}^N m_i \frac{d^2\vec{r_i}}{dt^2} = \sum_{i=1}^n F_i^e + \underbrace{\sum_i \sum_j F_{ij}}_{=0}$$

F_{ij} ↑ Internal Force exerted by j^{th} Particle on i^{th}.

Newton's third Law of Motion-



$$\sum_i m_i \frac{d^2\vec{r}_i}{dt^2} = \sum_{i=1}^{n} F_i^e \quad \leftarrow \text{--- Total external force Acting on the system}$$

↑ It obeys newton's third Law of Motion

$$\frac{d^2}{dt^2}\left(\sum_i^n m_i r_i\right) = F_i^e$$

system of N Particle

$P_1, P_2, \ldots P_n$ } Center of Mass
$r_1, r_2, \ldots r_n$ }

$$\vec{R} = \frac{\sum m_i r_i}{\sum m_i}$$

$\vec{R}$ $\vec{M}$

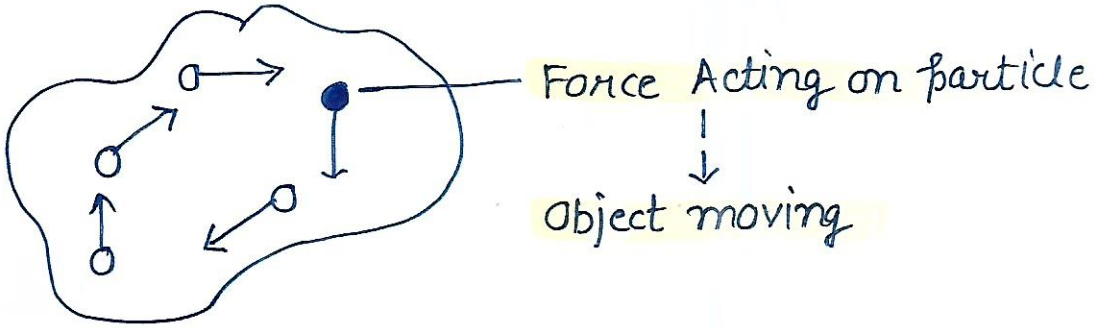
$$\frac{d^2}{dt^2}\left(\sum \frac{m_i r_i}{m_i} \times \sum m\right) = \vec{F}_i^e$$

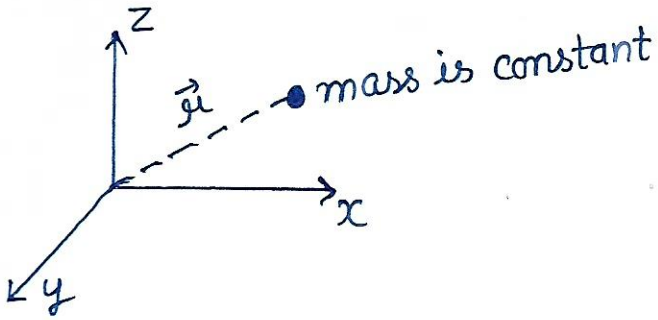
$$M\frac{d^2\vec{R}}{dt^2} = \vec{F}_i^e$$

The Internal Force which are following newton's third Law of Motion in the weak form, don't have any effect on the motion of the system.

MECHANICS OF A PARTICLE; CONSERVATION LAWS-

If a force $\vec{F}$ is acting on a particle of mass m, then according to Newton's second Law of motion, we have





$$\vec{F} = \frac{d\vec{P}}{dt}$$

Where $P = m\vec{v}$

$$\vec{F} = \frac{d}{dt}(m\vec{v})$$

$$\vec{F} = m\frac{d\vec{v}}{dt} \qquad \left[\because \vec{a} = \frac{d\vec{v}}{dt}\right]$$

$$\boxed{\vec{F} = ma}$$

If a external force acting on the particle is zero then,

$$\frac{d\vec{P}}{dt} = 0$$

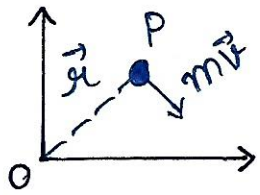
$$\frac{d}{dt}(mv) = 0$$

$$\vec{P} = \text{Constant}$$

If the external force acting on the particle is zero then

CONSERVATION OF ANGULAR MOMENTUM-

Angular momentum refers to amount of motion contained in an object.



The particle P has tendency to rotate about the fix axis O. Then Angular momentum of Particle P of a mass m about a fix point O is defined as

$$\vec{L} = \vec{r} \times m\vec{v}$$

$$\vec{L} = \vec{r} \times \vec{P} \quad \ldots\ldots\ldots (1)$$

If the force on the particle is $\vec{F}$, then the moment of the force or torque is

$$\vec{N} = \vec{r} \times \vec{F} \quad \ldots\ldots\ldots (2)$$

According to the law of Angular momentum-

$$\frac{d}{dt}(m\vec{v}) = \vec{F}$$

multiply by $\vec{r}$ on the both sides

$$\vec{r}\,\frac{d}{dt}(m\vec{v}) = \vec{r} \times \vec{F} \quad [\text{From eqn (2)}]$$

$$\vec{r}\,\frac{d}{dt}(m\vec{v}) + \underbrace{\frac{d\vec{r}}{dt}(m\vec{v})}_{v = \frac{d\vec{r}}{dt}} = \vec{N}$$

$$\vec{r}\,\frac{d}{dt}(m\vec{v}) + \vec{v}(m\vec{v}) = \vec{N}$$

$$\vec{r}\,\frac{d}{dt}(m\vec{v}) + \underbrace{\vec{v}(mv)}_{0} = \vec{N}$$

because $|\vec{a} \times \vec{b}|$
$\Rightarrow |\vec{a}||\vec{b}|\sin\theta$
$\Rightarrow |\vec{a}||\vec{b}|\sin 0$

$$0 + \vec{r}\,\frac{d}{dt}(m\vec{v}) = \vec{N}$$

$$\frac{d}{dt}(\underbrace{\vec{r} \times m\vec{v}}) = \vec{N}$$

From eqⁿ (1)

$$\frac{d\vec{L}}{dt} = \vec{N}$$

If $\vec{N} = 0 \Rightarrow \frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L}$ = Constant

If the external torque about point O on a particle is zero then Angular momentum about that point is constant.

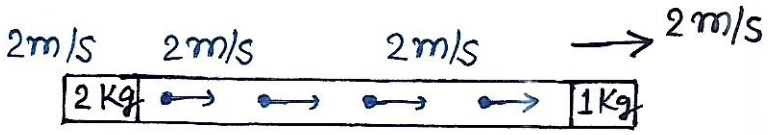
MECHANICS OF A PARTICLE–

We shall study the conservation laws for a particle in motion using Newtonian Mechanics.

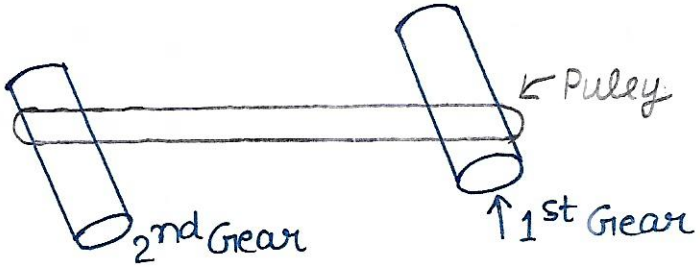
→ Newtonian's Mechanics–

- The total Linear momentum 'P' of a particle is conserved when the total Force on it is 0.
- The Angular momentum of a particle is conserved when the torque is 0.
- The total energy of a particle in a conservative field is constantly time.

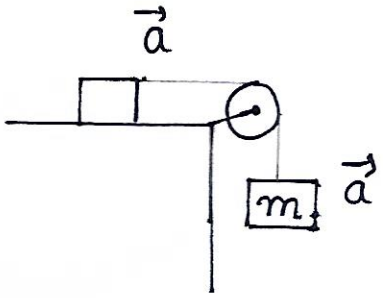
CONSTRAINTS–



A constrainted motion is a motion which can't proceed arbitrary in any manner. Particle motion is restricted only.



If 1st gear moves forward, then 2nd gear will also move



↔ CLASSIFICATION OF THE CONSTRAINTS-

(1) HOLONOMIC CONSTRAINTS-

The constraints are present in the system of N particles. If the constraints are expressed in the form of eqn

$$f(r_1, r_2, \ldots, t) = 0$$

Then they are called Holonomic constraints. It is independent to v.
e.g. Rolling of cylinder

(2) NON-HOLONOMIC CONSTRAINTS-

The constraints which are not holonomic, can't expressed in the form of eqn $f(r_1, r_2, \ldots t) = 0$, are called non holonomic constraints.

$$|r| \geq a \text{ or } r - a \geq 0$$

In the gravitational field, where r is the position vector of the particle relative to the center of the sphere. It is dependent to velocity.

(3) Scleronomic Constraints- It doesnot depend on time.

(4) Rheonomic Constraints- It depends on time.

(5) Bilateral Constraints- It can be in the form of equation.

(6) Unilateral Constraints- It will be expressed in the form of inequalities.

(7) Conservative Constraints- If force of constraints do not do any work and the mechanical energy of the system is conserved.

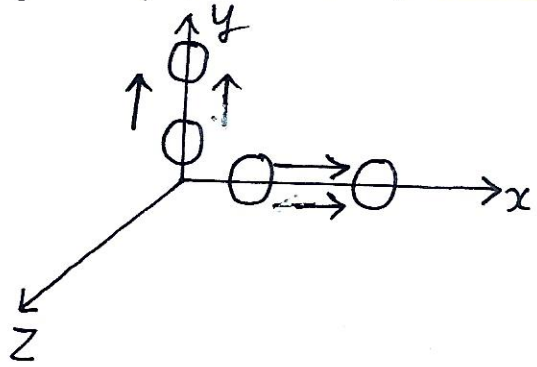
(8) Dissipative Constraints- If the force of constraints do work and the mechanical energy of the system is not conserved.

DEGREE OF FREEDOM-

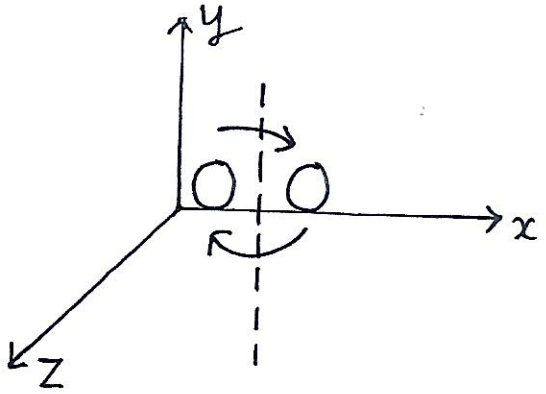
The Number of independent ways in which mechanical system can move without voilating any constraints which may be imposed is called the number of degree of freedom of the system.

Types of degree of freedom⇒

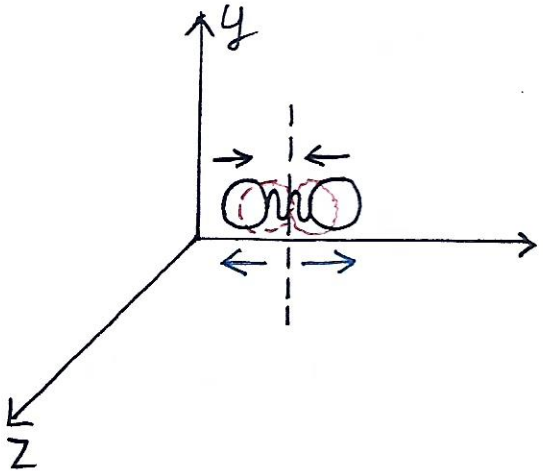
- Degree of freedom of Translational motion-



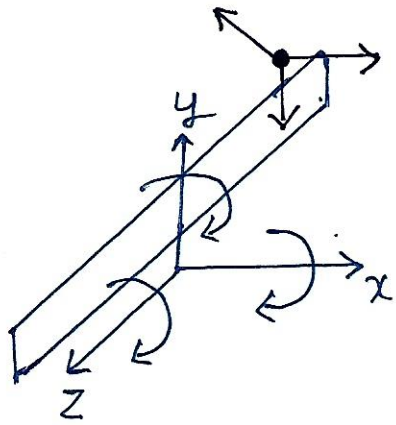
- Degree of freedom of Rotational Motion-



- Degree of freedom of viberation Motion-



eg →



Translational
x y z
Rotational
θ_1 θ_2 θ_3

$f_T = 3$

$f_R = 3$

$f_V = 0$

$f = f_T + f_R + f_V = 3+3+0$

$\boxed{f = 6}$ N

#GENERALISED (MECHANICS) CO-ORDINATIONS-

The name generalised co-ordinates is given to a set of independent co-ordinates sufficient in number to describe the state of configuration of a dynamical system.

- Without constraints, a system of N particle has 3N dof
- With K constraints, a system has (3N-K) dof
- With holonomic constraints, one can introduce (3N-K) dof generalized coordinates $(q_1, q_2, \ldots q_{3N-K})$ s.t.

$$\left.\begin{aligned} r_1 &= r_1(q_1, q_2, \ldots, q_{3N-K}, t) \\ r_2 &= r_2(q_1, q_2, \ldots, q_{3N-K}, t) \\ &\ldots\ldots\ldots\ldots \\ &\ldots\ldots\ldots\ldots, \\ r_N &= r_N(q_1, q_2, \ldots q_{3N-K}) \end{aligned}\right\} \text{Transformation}$$

- Generalized coordinates can be anything; Angle, Energy, Unit, momentum, units, or even amplitude

#EULAR-LANGRANGE DIFFERENTIAL EQ^{n}

Let f be a function of many independent variables of y_j and their derivatives $\dot{y}_j$, then the Integral I representing a path between 1 & 2 will be written as

$$I = \int_1^2 f\left[y_1(x)\, y_2(x) \ldots\ldots \dot{y}_1(x)\, \dot{y}_2(x), x\right] dx$$

Now, for all possible curves between the two points 1 & 2, we merely produce assign different values of a parameter α to these curves show that y_j will also be a function of α. The Integral I will be a function of α and hence the variation can be represented as $\frac{\partial I(\alpha)}{\partial \alpha} d\alpha$

$$\frac{\partial I(\alpha)}{\partial \alpha} d\alpha = \int_1^2 \sum \left(\frac{\partial f}{\partial y_j} \cdot \frac{\partial y_j}{\partial \alpha} + \frac{\partial f}{\partial \dot{y}_j} \cdot \frac{\partial \dot{y}_j}{\partial \alpha} \partial \alpha \right) dx$$

Integrating by parts, we get

$$\frac{\partial I(\alpha)}{\partial \alpha} d\alpha = \int_1^2 \sum_j \left(\frac{\partial f}{\partial y_j} \cdot \frac{\partial y_j}{\partial \alpha} d\alpha\, dx \right) + \left(\sum_j \frac{\partial f}{\partial \dot{y}_j} \cdot \frac{\partial y_j}{\partial \alpha} \right)_1^2 -$$

$$\int_1^2 \sum_j \frac{d}{dx} \left(\frac{\partial f}{\partial \dot{y}_j} \right) \frac{\partial y_j}{\partial \alpha} d\alpha\, dx$$

At n points which are held fixed all path need $\left.\frac{\partial y_j}{\partial \alpha}\right|_1^2 = 0$

$$\frac{\partial I(\alpha)}{\partial \alpha} = \int_1^2 \sum_j \left(\frac{\partial f}{\partial y_j} \cdot \frac{\partial y_j}{\partial \alpha} \partial \alpha\, \partial x \right) - \int_1^2 \left(\sum_j \frac{d}{dx} \left(\frac{\partial f}{\partial \dot{y}_j} \right) \frac{\partial y_j}{\partial \alpha} d\alpha\, dx \right)$$

$$\frac{\partial I(\alpha)}{\partial \alpha} = \int_1^2 \sum_j \left[\left(\frac{\partial f}{\partial y_j} \right) - \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial \dot{y}_j} \right) \right] \frac{\partial y_j}{\partial \alpha} \partial \alpha\, \partial x$$

Put $\frac{\partial I(\alpha)}{\partial \alpha} d\alpha = \delta I$

Put $\frac{\partial y_j}{\partial \alpha} \partial \alpha = \delta y_j$

$$\delta I = \int_1^2 \sum_j \left[\left(\frac{\partial f}{\partial y_j} \right) - \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial \dot{y}_j} \right) \right] \delta y_j \, dx$$

for the Integral I to be extramum then $\delta y_j \, dx = 0$

$$\int_1^2 \sum_j \left[\left(\frac{\partial f}{\partial y_j} \right) - \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial \dot{y}_j} \right) \right] \delta y_j \, dx = 0$$

Since, δy_j are independent of each other, cofficient of each δy_j, should seprately vanish it above eq^n is to be satisfied, thus

$$\left[\frac{\partial f}{\partial y_j} - \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial \dot{y}_j} \right) \right] = 0 \quad \ldots\ldots\ldots\ldots (1)$$

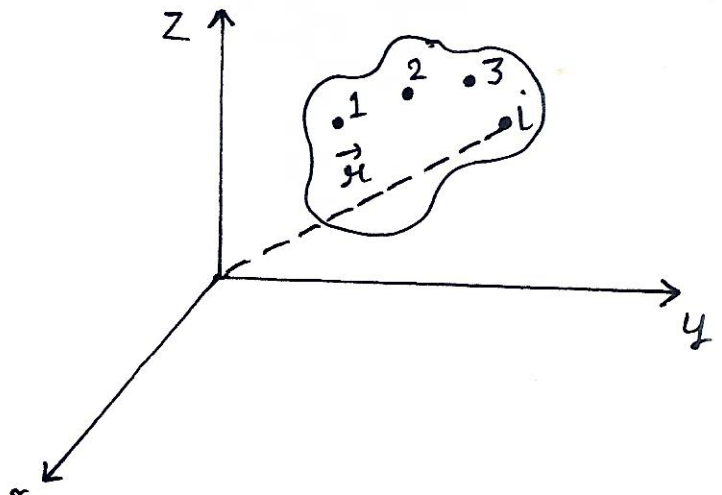
The set of differential eq^n is represented by eq^n (1) known as eular Langrange differential equation. The solution of Eular Langrange differential equation represent those curve for which the Integral

$$I = \int_1^2 f(y_j, \dot{y}_j, x) \, dx$$

assumes an extramum value.

De'Almbert Principle-

Suppose that there is a system of n particles.



According to the Newton's second Law of Motion, the force on i^{th} particle is given by

$$\vec{F}_i = \frac{d\vec{P}_i}{dt} = \dot{\vec{P}}_i$$

$$\vec{F}_i = \dot{\vec{P}}_i$$

$$\vec{F}_i - \dot{\vec{P}}_i = 0$$

↑ Actual Force Acting on the ith particle

↖ Reserved effective force

$$F = ma$$

$$F = \frac{dV}{dt}(m)$$

$$F = \frac{d}{dt}(mV)$$

$$F = \frac{dP}{dt}$$

$$F = \dot{P}$$

This means

$$\left[\vec{F}_{net}\right]_i = \vec{F}_i + (-\dot{\vec{P}}_i) = 0 \quad \ldots\ldots\ldots (1)$$

Which is the condition of equilibrium. This means that any particle in the equilibrium under force which is equal to the actual force f_i with the reversed effective force.

For Virtual displacement $\delta\vec{r}_i$ of ith particle because displacement is imaginary.

Then by the principle of the virtual work

$$\sum_{i=1}^{n} (\vec{F}_{net})_i \cdot \delta\vec{r}_i = 0$$

$$\sum_{i=1}^{n} (\vec{F}_i - \dot{\vec{P}}_i) \cdot \delta\vec{r} = 0 \quad [\text{By eq}^n(1)]$$

But $\vec{F}_i = \vec{F}_i^a + \vec{f}_i \quad \ldots\ldots\ldots (2)$

↑ force on i^{th} particle

↑ Applied force

↑ force due to constraints

$$\sum_{i=1}^{N}\left(\vec{F}_i^{a}+\vec{f}_i-\dot{\vec{P}}_i\right)\cdot\delta\vec{r}=0$$

$$\sum_{i=1}^{N}\left(\vec{F}_i^{a}\,\delta\vec{r}+\vec{f}_i\cdot\delta\vec{r}-\dot{\vec{P}}_i\cdot\delta\vec{r}\right)=0$$

$$\sum_{i=1}^{N}\vec{F}_i^{a}\,\delta\vec{r}+\sum_{i=1}^{N}\vec{f}_i\cdot\delta\vec{r}-\sum_{i=1}^{N}\dot{\vec{P}}_i\cdot\delta\vec{r}=0 \quad \ldots\ldots\ldots (3)$$

Now we restrict ourself to the systems for which virtual work of the constraints are zero.

$$\sum_{i=1}^{N}\vec{f}_i\cdot\delta\vec{r}=0 \quad \ldots\ldots\ldots (4)$$

By Eq^{n} (3) & (4)

$$\sum_{i=1}^{N}\vec{F}_i^{a}\,\delta\vec{r}+\sum_{i=1}^{N}\dot{\vec{P}}_i\,\delta\vec{r}=0$$

In usually notation we denote $\vec{F}_i^{a}$ by simply $\vec{F}_i$

$$\boxed{\sum_{i=1}^{N}\left(\vec{F}_i-\dot{\vec{P}}_i\right)\cdot\delta\vec{r}=0}$$

Answer Proved

This is known as D'Alembert's Principle.

Lagrange's Equation From D'Alembert's Principle-

Consider a system of Particles N. The transformation equations for the position vectors of the particles are

$$r_i=r_i(q_1,q_2,\ldots q_K,\ldots q_n,t) \quad \ldots\ldots(1)$$

Where t is time

$q_K=(K=1,2,\ldots n)$

differenciating eq^n (1) with respect to t, we get the velocity of the ith particle.

$$\frac{d\vec{r}_i}{dt} = \frac{\partial \vec{r}_i}{\partial q_1}\cdot\frac{\partial q_1}{\partial t} + \frac{\partial \vec{r}_i}{\partial q_2}\cdot\frac{\partial q_2}{\partial t} + \dots + \frac{\partial r_i}{\partial t}\cdot\frac{\partial t}{\partial t}$$

$$v_i = \frac{dr_i}{dt} = \frac{\partial r_i}{\partial q_1} q_j + \frac{\partial r_i}{\partial t}\partial t$$

$$v_i = \dot{r}_i = \frac{\partial r_i}{\partial q_1} q_j + \frac{\partial r_i}{\partial t}\partial t \quad \text{——— (2)}$$

But the last term is zero, since invirtual displacement only co-ordinate displacement is considered Φ not on that time.

Therefore

$$\delta r_i = \sum_j \frac{\partial r_i}{\partial q_1}\delta q_j \quad \text{——— (3)}$$

We know that the D'Almbert Principle

$$\sum_i (\vec{F}_i - \dot{\vec{P}}_i)\,\delta r_i = 0$$

$$\sum_{ij} F_i \frac{\partial r_i}{\partial q_j}\delta q_j - \sum_{ij} P_i \frac{\partial r_i}{\partial q_j}\delta q_j = 0$$

we define that term

$$\sum_i F_i \frac{\partial r_i}{\partial q_j} = Q_j$$

Then

$$\sum_j Q_j \delta q_j - \sum_j \dot{P}_i \frac{\partial r_i}{\partial q_j}\cdot\delta q_j = 0 \quad \text{——— (4)}$$

Let us evaluate the 2nd term of above equation

$$\sum_{ij} P_i \frac{\delta r_i}{\partial q_j}\delta q_j = \sum_{ij} m_i \dot{r}_i \frac{\partial r_i}{\partial q_j}\delta q_j$$

$$\sum \left[\dots \frac{\partial r_i}{\dots} + m_i \dot{r}_i \frac{d}{dt}\left(\frac{\partial r_i}{\dots}\right) - m_i \dot{r}_i \frac{d}{dt}\left(\frac{\partial r_i}{\dots}\right)\right]\delta q_j$$

$$= \sum_{ij} \left[\frac{d}{dt}\left(m_i r_i \frac{\partial r_i}{\partial q_j} \right) - m_i v_i \frac{d}{dt}\left(\frac{\partial r_i}{\partial q_j} \right) \right] \delta q_j \quad \ldots\ldots (5)$$

Further

$$\frac{d}{dt}\left(\frac{\partial r_i}{\partial q_j} \right) = \frac{\partial}{\partial q_k}\left(\frac{\partial r_i}{\partial q_j} \right) \frac{\partial q_k}{\partial t} + \frac{\partial}{\partial t}\left(\frac{\partial r_i}{\partial q_j} \right) \frac{dt}{dt}$$

$$= \sum_k \frac{\partial^2 r_i}{\partial q_k \partial q_j} \cdot q_k + \frac{\partial^2 r_i}{\partial t \partial q_j}$$

$$= \sum_k \frac{\partial^2 r_i}{\partial q_j \partial q_k} q_k + \frac{\partial^2 r_i}{\partial q_j \partial t}$$

$$= \sum_k \frac{\partial}{\partial q_j}\left[\frac{\partial r_i}{\partial q_k} q_k + \frac{\partial r_i}{\partial t} \right]$$

$$\frac{d}{dt}\left(\frac{\partial r_i}{\partial q_j} \right) = \frac{\partial}{\partial q_j}\left[\frac{\partial r_i}{\partial t} \right] = \frac{\partial}{\partial q_j}\left[v_i \right]$$

$$\frac{d}{dt}\left(\frac{\partial r_i}{\partial q_j} \right) = \frac{\partial v_i}{\partial q_j} \quad \ldots\ldots (6)$$

differential eq^n (1) with respect to q_j

$$v_i = \sum_j \frac{\partial r_i}{\partial q_j} q_j + \frac{\partial r_i}{\partial t}$$

$$\frac{\partial v_i}{\partial q_j} = \frac{\partial r_i}{\partial q_j} \quad \ldots\ldots (7)$$

Put eq^n ⑥ & ⑦ in the eq^n ⑤

$$\sum_{ij} \dot{p} \frac{\partial r_i}{\partial q_j} \delta q_j = \sum_{ij} \left[\frac{d}{dt}\left(m_i v_i \frac{\partial v_i}{\partial q_j} \right) - m_i v_i \frac{\partial v_i}{\partial q_j} \right] \delta q_j$$

$$\sum_{ij} \dot{p} \frac{\partial r_i}{\partial q_j} \delta q_j = \sum_{ij} \left[\frac{d}{dt}\left\{ \frac{\partial}{\partial \dot{q}_j}\left(\sum_i \frac{1}{2} m_i v_i^2 \right) \right\} - \frac{\partial}{\partial q_j}\left(\sum_i \frac{1}{2} m_i v_i^2 \right) \right]$$

Putting this in eq^n ④

$$\sum_j Q_j \delta q_j - \sum_j \left[\frac{d}{dt}\left\{ \frac{\partial}{\partial \dot{q}_j}\left(\sum_i \frac{1}{2} m_i v_i^2 \right) \right\} - \frac{\partial}{\partial q_j}\left(\sum_i \frac{1}{2} m_i v_i^2 \right) \right] = 0$$

$$\sum_j Q_j \delta q_j - \sum_j \left[\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j}\right] \delta q_j$$

$$\sum_j \left[\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j}\right] \delta q_j = \sum_j Q_j \delta q_j$$

$$\boxed{\left[\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j}\right] = Q_j} \quad \ldots\ldots (8) \quad j = 1, 2, \ldots\ldots n$$

CASE I-

FOR CONSERVATION SYSTEM-

We consider a conservative system & use above set of eqⁿ to obtain Lagrange eqⁿ of motion for a conservative system.

$$F_i = -\nabla_i V$$

$$F_i = -\left(-\frac{\partial V}{\partial r_i}\right)$$

Thus generalized form can be expressed as

$$Q_j = \sum_i F_i \frac{\partial r_i}{\partial q_j} = -\sum_i \nabla_i V \frac{\partial r_i}{\partial q_j}$$

$$= -\sum_i \frac{\partial V}{\partial r_i} \cdot \frac{\partial r_i}{\partial q_j}$$

$$Q_j = -\frac{\partial V}{\partial q_j}$$

From eqⁿ (8), we get

$$\left[\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j}\right] = -\frac{\partial V}{\partial q_j}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial V}{\partial q_j} = 0$$

$$\left[\frac{d}{dt}\left(\frac{\partial (T-V)}{\partial \dot{q}_j}\right) - \frac{\partial}{\partial q_j}(T-V)\right] = 0$$

Since V is not a function of $\dot{q}_j$ and $L = T - V$

The Langrange function of the conservative system eqⁿ becomes

$$\left[\frac{d}{dt}\left(\frac{\partial t}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j}\right] = 0 \quad \ldots\ldots\ldots\ldots (9)$$

Which is known as Lagrange eqⁿ of motion for a conservative system.

CASE II–

FOR NON- CONSERVATIVE SYSTEM-

If Potential are velocity dependant called generalised potential then through the system is not conservative.

The compounds of generalized force are obtained from a funⁿ $V(q_j, \dot{q}_j)$ called generalized Potential s.t.

$$Q_j = \frac{d}{dt}\left(\frac{\partial V}{\partial \dot{q}_j}\right) - \frac{\partial V}{\partial q_j}$$

Putting the value of Q_j in eqⁿ (8), we get

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = \frac{d}{dt}\left(\frac{\partial V}{\partial \dot{q}_j}\right) - \frac{\partial V}{\partial q_j}$$

$$\frac{d}{dt}\left(\frac{\partial (T-V)}{\partial \dot{q}_j}\right) - \frac{\partial}{\partial q_j}(T-V) = 0$$

Put $L = T - V$

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0} \qquad j = 01, 02, \ldots\ldots n$$

Which is same as the eqⁿ (9) & this eqⁿ is known as Lagrangian eqⁿ of motion for non-conservative system.

HAMILTONIAN FUNCTION H-

In the Lagrangian Formulation one may expect the deduction of the theorem of conservation of the total energy for the system where the potential energy is a function of position only. In fact we shall see as discussed below, the theorem of conservation of total energy in a special case of a more general conservation theorem.

Consider a general Lagrangian L of a system given by

$$L = L(q_1, q_2, \ldots q_K, \dot{q}_1, \dot{q}_2, \ldots \dot{q}_K, t)$$

We denote for our convience by

$$L = L(q_K, \dot{q}_K, t)$$

The total time derivation of L is

$$\frac{dL}{dt} = \sum_K \frac{\partial L}{\partial q_K} \cdot \frac{dq_K}{dt} + \sum_K \frac{\partial L}{\partial \dot{q}_K} \frac{d\dot{q}_K}{dt} + \frac{\partial L}{\partial t} \quad \ldots\ldots (1)$$

From Langrangian eq^n; we have

$$\frac{\partial L}{\partial q_K} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_K}\right)$$

Substituting the value of $\frac{\partial L}{\partial q_K}$ in eq^n(1), We get

$$\frac{dL}{dt} = \sum_K \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_K}\right) \dot{q}_K + \sum_K \frac{\partial L}{\partial \dot{q}_K} \cdot \frac{d\dot{q}_K}{dt} + \frac{\partial L}{\partial t}$$

$$\frac{dL}{dt} = \sum_K \frac{d}{dt}\left(\dot{q}_K \frac{\partial L}{\partial \dot{q}_K}\right) + \frac{\partial L}{\partial t}$$

$$\frac{d}{dt}\left(\sum_K \dot{q}_K \frac{\partial L}{\partial \dot{q}_K} - L\right) = -\frac{\partial L}{\partial t}$$

Then

$$\frac{dh}{dt} = -\frac{\partial L}{\partial t}$$

If the Lagrangian L does not depend on time t explicity, then $\frac{\partial L}{\partial t} = 0$

$\frac{dh}{dt} = 0$ i.e. $h = \text{Constant}$

Thus when the Lagrangian is not explicit function of time, the energy function is the constant of motion.

$$\frac{d}{dt}\left(\sum_K P_K \dot{q}_K - L\right) = \frac{-\partial L}{\partial t}$$

The quantity in the bracket is called as Hamiltonian Function.

$$H = \sum_K (P_K \dot{q}_K - L) = \text{Constant}$$

Thus if the time t does not appear in the Lagrangian L explicitly, we see that the Hamiltonian H is a constant of time.

Physical Significance of H-

The Hamiltonian takes a special form, if the system is conserved.

$$r_i = r_i(q_1, q_2, q_3, \ldots q_n)$$

For the conservation system $\frac{\partial V}{\partial q_K} = 0$ Then

$$P_K = \frac{\partial L}{\partial \dot{q}_K} = \frac{\partial}{\partial \dot{q}_K}(T-V) = \frac{\partial T}{\partial \dot{q}_K}$$

$$H = \sum_K P_K \dot{q}_K - L = \sum_K \frac{\partial T}{\partial \dot{q}_K} - L$$

$$\sum_K \frac{\partial T}{\partial \dot{q}_K} \dot{q}_K = 2T$$

$$H = 2T - L = 2T - (T-V)$$

$$\boxed{H = T+V = E \text{ (constant)}}$$

Matrix- A matrix is a collection of numbers arranged into a fixed numbers of row and column.

Types of Matrix-

(1) Square matrix- A matrix having the same no. of row of column is called as square matrix.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3}$$

(2) Diagonal Matrix-

$$\begin{bmatrix} a_{11} & 0 & 0 & \cdots & 0 \\ 0 & a_{22} & 0 & \cdots & 0 \\ 0 & 0 & a_{33} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & a_{nn} \end{bmatrix}$$

(3) Scalar Matrix-

$$\begin{bmatrix} \lambda & 0 & 0 & \cdots & 0 \\ 0 & \lambda & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \lambda & \cdots & \lambda \end{bmatrix}$$

(4) Identity or Unitary Matrix-

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(5) Row and Column Matrix-

$$\begin{bmatrix} a_{11}, a_{12}, a_{13} \cdots \end{bmatrix}$$

$$\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix}$$

(6) Null Matrix-

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(7) Upper triangle Matrix- $i > j$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ 0 & a_{22} & a_{23} & \dots & a_{2n} \\ 0 & 0 & a_{33} & \dots & a_{3n} \\ 0 & 0 & 0 & \dots & a_{mn} \end{bmatrix}$$

(8) Lower Δ Matrix- $j > i$

$$\begin{bmatrix} a_{11} & 0 & 0 & \dots & 0 \\ a_{21} & a_{22} & 0 & \dots & 0 \\ a_{31} & a_{32} & a_{33} & \dots & 0 \\ a_{n1} & a_{n2} & a_{n3} & \dots & 0 \end{bmatrix}$$

(9) Periodic Independent & Nilopotent Matrix-

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is an idempotent matrix

$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ is a nilopotent matrix

Transpose of a matrix-

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{i \times j = 2 \times 3}$$

$$A' = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}_{i \times j = 3 \times 2}$$

Conjugate Matrix -

$$A = \begin{bmatrix} 1+2i & 2 & i \\ 3 & 5i & 2+3i \end{bmatrix}$$

$$A^*, \bar{A} = \begin{bmatrix} 1-2i & 2 & -i \\ 3 & -5i & 2+3i \end{bmatrix}$$ ← Replaced complex number sign

Symmatric & Antisymmatric Matrix-

$$A = \begin{bmatrix} a & b & c \\ b & e & f \\ c & f & d \end{bmatrix}_{3\times3}$$

$$A^T = \begin{bmatrix} a & b & c \\ b & e & f \\ c & f & d \end{bmatrix}_{3\times3} \Leftarrow \text{Symmatrix}$$

$A^T = A$

Inverse of a matrix-

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}_{2\times2}$$

$A^{-1} = ?$

$$|A| = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}$$

$|A| = 1-(-1)$

$|A| = 1+1$

$|A| = 2 \neq 0$

Find A^{-1}

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5 \\ 5 & 6 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5 \\ 5 & 6 & 0 \end{bmatrix} \begin{matrix} 15 & 30 & 0 \\ \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 5 & 6 \end{bmatrix} \\ 0 & 50 & 0 \end{matrix}$$

$|A| = (0+50+0)-(15+30+0)$

$|A| = 50-45$

$\boxed{|A| = 5}$

$$N = \begin{bmatrix} \begin{vmatrix} 1 & 5 \\ 6 & 0 \end{vmatrix} & \begin{vmatrix} 0 & 5 \\ 5 & 0 \end{vmatrix} & \begin{vmatrix} 0 & 1 \\ 5 & 6 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ 6 & 0 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 5 & 0 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 5 & 6 \end{vmatrix} \\ \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 0 & 5 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix} \end{bmatrix}$$

$$N = \begin{bmatrix} +-30 & --25 & +-5 \\ +-18 & +-15 & +-4 \\ +7 & -5 & +1 \end{bmatrix}$$

$$N = \begin{bmatrix} -30 & 25 & -5 \\ -18 & -15 & +4 \\ 7 & -5 & 1 \end{bmatrix}$$

$$N = \begin{bmatrix} -30 & 18 & 7 \\ 25 & -15 & -5 \\ -5 & 4 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj}A$$

$$A^{-1} = \frac{1}{5} \begin{bmatrix} -30 & 18 & 7 \\ 25 & -15 & -5 \\ -5 & 4 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -6 & 18/5 & 7/5 \\ 5 & -3 & -1 \\ -1 & 4/5 & 1/5 \end{bmatrix}$$

Eigen Value-

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = 0$$

$$\begin{bmatrix} 2-\lambda & 1 & 0 \\ 1 & 2-\lambda & 1 \\ 0 & 1 & 2-\lambda \end{bmatrix} = 0$$

$$\begin{vmatrix} 2-\lambda & 1 & 0 \\ 1 & 2-\lambda & 1 \\ 0 & 1 & 2-\lambda \end{vmatrix} = 0$$

$$2-\lambda \begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ 0 & 2-\lambda \end{vmatrix} + 0 = 0$$

$$(2-\lambda)[(2-\lambda)^2 - 1] - 1[2-\lambda] = 0$$

$$(2-\lambda)(\lambda^2 - 4\lambda + 2) = 0$$

$\lambda = 2$

$$\lambda^2 - 4\lambda + 2 = 0$$

$$\lambda = \frac{4 \pm \sqrt{16 - 4 \times 1 \times 2}}{2}$$

$$\lambda = \frac{4 \pm \sqrt{8}}{2}$$

$$\lambda = 2 \pm \sqrt{2}$$

$\lambda_1 = 2$

$\lambda_2 = 2 + \sqrt{2}$

$\lambda_3 = 2 - \sqrt{2}$

Find Eigen Vectors-

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$\lambda_1 = 2$ $\quad |A - \lambda I| X = 0$

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$y = 0$$
$$x + z = 0 \Rightarrow 1 - 1 = 0$$
$$y = 0$$

$$x_i = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$|x_i| = \sqrt{1^2 + 0^2 + (-1)^2} = \sqrt{2}$$

$$|\hat{x}_i| = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$$

same process with λ_2 and λ_3

- Now One can answer and solve how to solve eigen value and eigen vector problems?

Inverse of a matrix-

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & 2 \\ 1 & 1 & 0 \end{bmatrix} \qquad A^T = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 2 & 0 \end{bmatrix}$$

$A^{-1} = ?$

$$A^{-1} = \frac{1}{|A|} adjA$$

$|A| = 2(0-2)-(-1)(0-2)+0(0-1)$

$|A| = -4+1(-2)+0$

$|A| = -6 \neq 0$

$$adjA = \begin{bmatrix} +(-2) & -0 & +(-2) \\ -(-2) & +0 & -4 \\ +(-1) & -3 & +2 \end{bmatrix}$$

$$adjA = \begin{bmatrix} -2 & 0 & -2 \\ +4 & 0 & -4 \\ -1 & -3 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adjA$$

$$A^{-1} = \frac{1}{-6} \begin{bmatrix} -2 & 0 & -2 \\ 4 & 0 & -4 \\ -1 & -3 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1/3 & 0 & 1/3 \\ -2/3 & 0 & 2/3 \\ 1/6 & 1/2 & -1/3 \end{bmatrix}$$

#CAYLEY-HAMILTON THEOREM-

Statement- Every square matrix have its own characteristic eq^n i.e. if for a square matrix $(n \times n)$ of order n.

The characteristic Polynomial is

$|A-\lambda I| = a_0 + a_1\lambda + a_2\lambda^2 + a_3\lambda^3 + \ldots\ldots + a_n\lambda^n$

Then the matrix eq^n

$$a_0 I + a_1 X + a_2 X^2 + a_3 X^3 + \ldots\ldots + a_n X^n = 0$$

Proof;

The characteristic Polynomial is

$|A-\lambda I| = a_0 + a_1\lambda^1 + a_2\lambda^2 + a_3\lambda^3 + \ldots\ldots + a_n\lambda^n \ldots\ldots\ldots (1)$

The characteristic of A is

$|A-\lambda I| = a_0 + a_1\lambda^1 + a_2\lambda^2 + \ldots\ldots\ldots + a_n\lambda^n = 0 \ldots\ldots\ldots (2)$

The matrix eq^n is

$a_0 I + a_1 X + a_2 X^2 + \ldots\ldots a_n X^n = 0 \ldots\ldots\ldots (3)$

If this eq^n is satisfied by A then we have to show that

$a_0 I + a_1 A + a_2 A^2 + \ldots\ldots a_n A^n = 0 \ldots\ldots\ldots (4)$

Since each element of characteristic matrix $|A-\lambda I|$ is an ordinary Polynomial of degree n.

The co-factor of every element of $|A-\lambda I|$ is an ordinary Polynomial of degree $n-1$; then,

$B = \text{adj}\,|A-\lambda I|$ is an ordinary Polynomial, we can write

$B = \text{adj}\,|A-\lambda I|$

$B = \text{adj}\,|A-\lambda I| = B_0 + B_1\lambda + B_2\lambda^2 + B_3\lambda^3 + \ldots\ldots + B_{n-1}\lambda^{n-1} \ldots\ldots (5)$

Where $B_0, B_1 \ldots B_{n-1}$ are all square matrix of same order n.

using eqn (5) & (1)

$$|A-\lambda I|(B_0+B_1\lambda+B_2\lambda^2+\ldots+B_{n-1}\lambda^{n-1}=(a_0+a_1\lambda+a_2\lambda^2+\ldots+a_n\lambda^n)I$$

$$(a_0+a_1\lambda+a_2\lambda^2+\ldots a_n\lambda^n)(B_0+B_1\lambda+B_2\lambda^2+\ldots+B_{n-1}\lambda^{n-1})$$

$$=(a_0+a_1\lambda+a_2\lambda^2+\ldots.\ a_n\lambda^n)I$$

Compare the coffiients

$$AB_0 = a_0 I$$

$$AB_1-B_0 = a_1 I$$

$$AB_2-B_1 = a_2 I$$

. . . . . .

. . . . . .,

$$AB_{n-1}-B_{n-2} = a_{n-1} I$$

Now Pre multiply these equations

$$\boxed{a_0 I+a_1 A+a_2 A^2+\ldots.+a_n A^n=0}$$

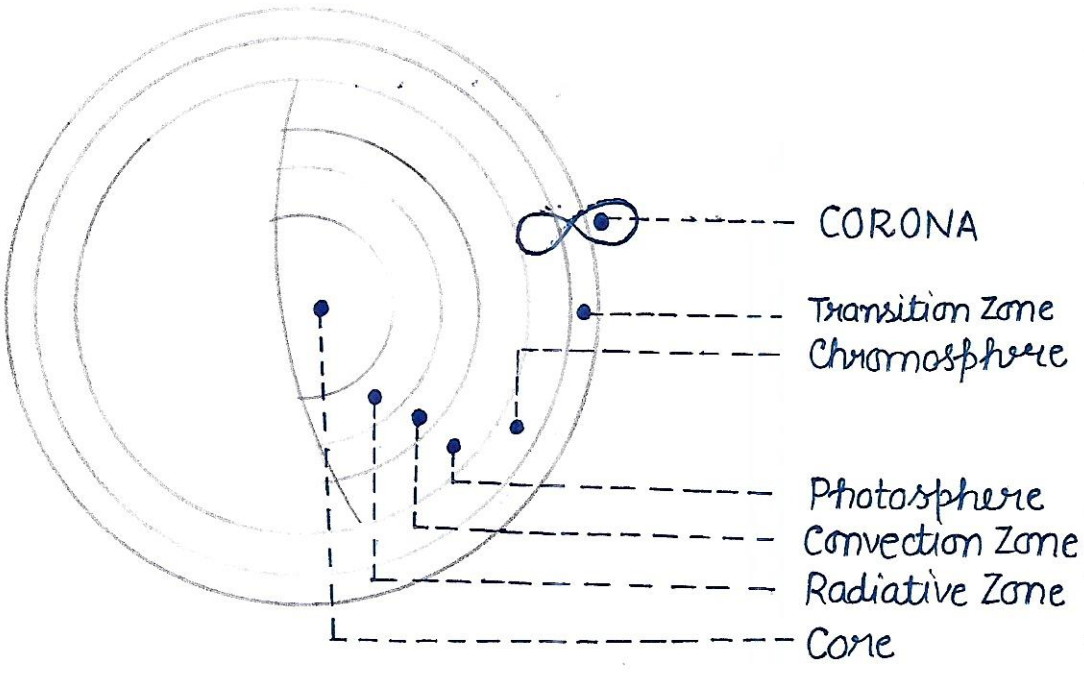
[ASTROPHYSICS]

Important Topics

- SUN and Histoscy of Sun
- Big Bang Theory [5]
- Astronomical Unit
- Butterfly Diagram
- HR diagram [10]
- Black Hole
- Stellar Evolution [3]
- Neutron Star
- Hubble's classification of galaxies
- Solar Flame
- Sun Spot [5]
- Binary Star [6]
- Solar Flame
- Saha Eq^{n} of Ionisation
- Inverse Square Law of Astronomy
- Variable Star
- Butterfly diagram
- Nuclear Fission/Nuclear Fusion

SUN -

Our Sun had formed 4.5 Billion years ago in the Milky way galaxy (ORION SPUR). It was born on clouds of dust known as Solar Nebula. It have tremendous Gravitational Pull and extensive Magnetic Field.

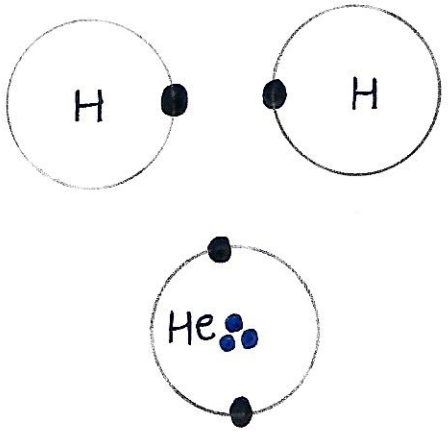


→ ATom Counts

91% Hydrogens
8.9% Helium
0.1% Hevier Elements

→ Due to the sun extreme temperature, the elements stay in the form of Plasma.

→ CORE temperature 15,000,000 °C / 27,000,000 °F



combined with the powerful gravity, Hydrogen Atoms combined and gravity create Helium Atom

→ Thermonuclear Fusion

The Sun release energy in the form of

→ Radiation
→ Electricity
→ Solar Wind

It is nearly perfect sphere of hot Plasma. Its diameter is about 1.39 million kilometers i.e. 109 times of earth.
For most of human history the Sun has been a object of ~~mistery~~ mystery, of reverence and sometimes, of fear. Early civilizations learned about seasonal changes in the sky very early on. Why It is high up in the sky, it is summer and it is warm. When it is low in the sky, it is winter and it is cold.

#Sunspots - When Galileo first observed the sun in detail, he saw dark spots on its surface, carried around by the sun's rotation. For centuries astronomers wondered whether they were clouds floating in the Sun's atmosphere, but we now know that they are really very huge region of magnetic field, about as intense as the field at the poles of an iron magnet.

Important Topics
- Maxwell's Equation and Physical Significance[6]
- Poynthing Theorem[6]
- Lienard Wiechard Theorem[4]

Important
• Maxwell Eq^n –

(1) $\text{div}\, D = \nabla D = \rho$

$$\int_S E\, dS = \frac{1}{\epsilon_0}\int_V (\rho + \rho_p)\, dV$$

$\because \rho_p = -\text{div}\, P$

$$\int_S \epsilon_0 E\, dS = \int_V (\rho - \text{div} P)\, dV$$

$$\int_V \text{div}\, \epsilon_0 E\, dV = \int_V \rho\, dV - \int_V \text{div}\, P\, dV$$

$$\int_V \text{div}(\epsilon_0 E + P)\, dV = \int_V \rho\, dV$$

$$\int_V \text{div}(D)\, dV = \int_V \rho\, dV$$

$$\boxed{\nabla D = \rho}$$

(2) $\nabla B = 0$

$$\int_S B\, dS = 0$$

$$\int_V \text{div}\, B\, dV = 0$$

$$\text{div}\, B = 0$$

$$\boxed{\nabla B = 0}$$

(3) $\text{curl}\, E = -\frac{\partial B}{\partial t}$

$$\int_S B \cdot dS = 0$$

46 $e = -\frac{d\phi}{dt}$

$e = -\frac{d}{dt}\int_S B \cdot dS$

$e = -\int_S \frac{\partial B}{\partial t}\, dS$

$e = \int_C E \cdot dl$

$\int_C E \cdot dl = -\int_S \frac{\partial B}{\partial t}\, dS$

$\int_S \text{curl}\, E\, dS = -\int_S \frac{\partial B}{\partial t}\, dS$

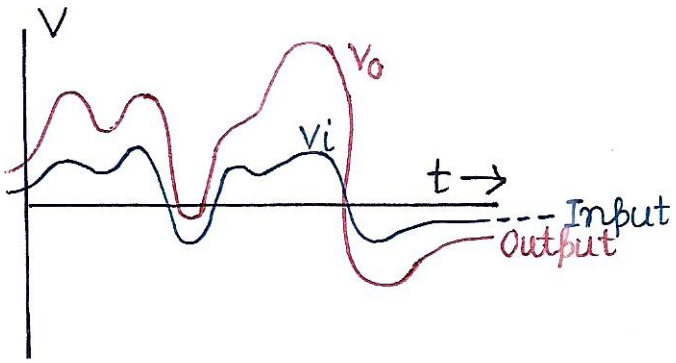
$$\text{curl}\, E = -\frac{\partial B}{\partial t}$$

[Electronics]

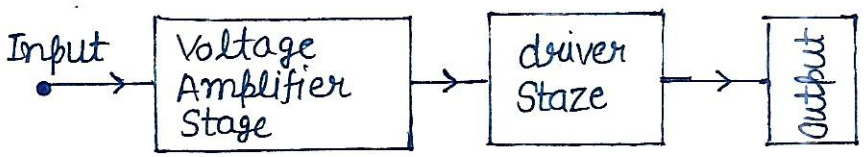
- Feedback in Amplifier[4]
- Wein bridge Oscillator[3]
- Multiviberator[4]
- Push Pull Amplifier[3] ✓
- Mean of Amplification on a Signal ✓
- Class B Push Pull Amplifier[1] ✓
- difference b/w Amplifier and oscillator ✓
- CLASS A, B, C, D, AB Amplifier

Mean of Amplification on a signal-

An amplifier, electronic amp is an electronic device that can increase the power of the signal(a time varying voltage or current). It is a two-port electronic circuit that uses electric power from a power supply to increase the amplitude of a signal applied to its input terminal, producing a proportional greater amplitude signal to its output.

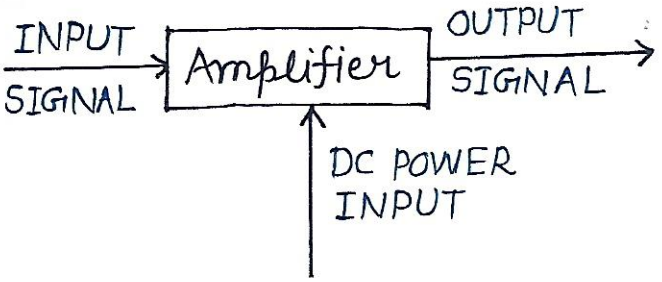


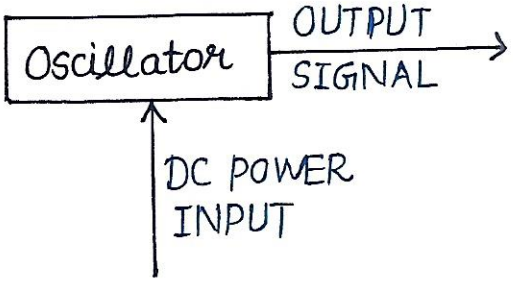
Amplification is the set of techniques used to boost a signal's strength.



Difference between Amplifier and Oscillators-

Amplifier	Oscillators
Amplifier is an electronic circuit which gives output as amplified form of Input.	Oscillator is an electronic circuit which gives output without application of Input
Amplifier uses negative feedback.	Osallator uses positive feedback.
Amplifier operates as a multiplier	Oscillator operates as a source.
The Amplifier provides amplified signal.	Oscillator is given oscillatory signal.
Amplifier do nothing till input signal is fed to the Input.	Oscillators produce signals from the moment of Powered.





Push-Pull Amplifier-

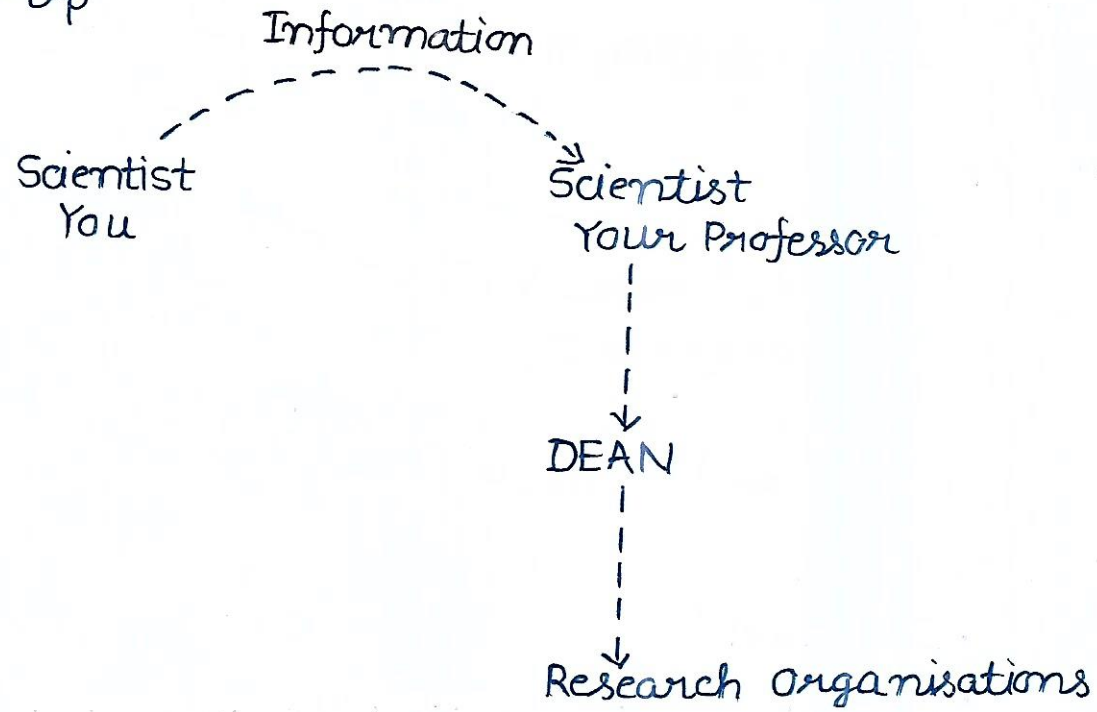
A push-Pull amplifier is a type of electronic circuit that uses a pair of active devices that alternately supply current to, or absorb current from, a connected load.

[How to write an experiment Report]

#Sherri Seligson

- What you have done in an experiment.
- Describe Result

→ Write-Up



- Concise
- Clear
- just get Points
- Don't Use pronouns

} Most Important

- Grammar
- Spellings
- Punctuations

} Important

- don't use slang

1. Title
2. Abstract
3. Introduction
4. Procedure
5. Results
6. Discussion
7. Conclusion
8. References

} Contents

Title-

- Title
- Names
- Date

Experiment Title -

- Don't use A, An, The as first word

Abstract -

- Abstracts are a completly self contained way to briefly summarize the procedure, results, and conclusion of the experiment.
- Abstract is usually a single short paragraph and is written in the (short) present tense.

Introduction-

- Background
- Why are you performing this experiment?
- What is experiment.
- Purpose of the experiment.
- Hypothesis.

Procedure-

- Exactly what you did.
- What you want to do? (For Proposal).
- describe the steps.
- Use past tense.
- Diagram

Results -

- Graph
- Table

→ In graph, Independent Variable should be on x-axis

→ Show mathematical Calculations.
→ Do not give any interpretation of the data;
→ Use past tense.

Discussion-
→ Consider and Support what you did.
→ Avoid Emotions here.
→ No ~~opetition~~ openion.
→ Third Person

Conclusion-
→ Show error
→ Final data you get
→ expected Results
} Show

Reference-
→ Write all References.

What to do?
- Write neatly
- No grammatical errors
- Pay attention to significant figures
- (perpose) Proper accuracy.
- Use third person / Past tense.
- Check Plagiarism.

Experiment

1. AIM/OBJECTIVE
2. Equipment/Apparatus
3. Theory
4. Procedure
5. Observations
6. Results
7. Conclusion

} #1

- Record data and Analysis.

[Stellar Evolution Applet]

Abhishek Bohare

What is astrophysics?

1. Particle Physics- Particle Physics (or High energy Physics) is a branch of Physics that studies the nature of the particles that consitute matter and radiation.

2.(Astrology) Atomic Physics- Atomic Physics is the field that studies atoms as an isolated system of e^- and atomic nucleus.

3. Gravitational Physics- Gravitational Physics is the field that studies general relativity, space time, block hole, motion of the object, gravitational collapse.

4. Cosmology- Cosmology is the study of the origin, evolution and evantual fate of the universe.

5. Neutrino Physics- It is physics of weak subatomic force.

6. Nuclear Physics- Nuclear Physics is the field of Physics that studies atomic nuclei and their constituents and interactions.

Staller Evolution | Applet Introduction-

Applet is a simulator that is used for the evalution of stars. The followings are displayed as the star progresses through its life.

- Age luminosity
- Radius
- Effective temperature

Story of Stars-

The ancients believe that the star at night turn on and changing. Today we know this is not true. Stars are born, live their life and die. The way of stars dieing depends largly on its mass. A low mass star called White dwarf, a high mass star becomes black hole but in between, a star becomes neutron star.

- Stars spends their life using matter together. This process begins with simplest Atom ${}_1H$.

$$ {}_1H^1 + {}_1H^1 \longrightarrow {}_2He^4 + \text{energy}(\gamma) $$

↓

which cause stars to shine

- Mass reduce of white dwarf ⟶ Slowly cool down over billions of years

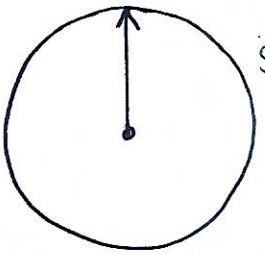
↓

After fusion, they still shine.

↓

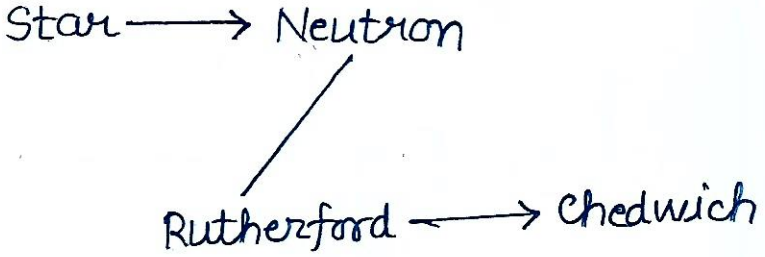
Death of Our Sun { Bacause astronomically high temperature.

- For the really big stars, the end of fusion enables gravity to do some real damage. Unconstrainded by the fusion, the gravity of the stars breaks down particles and squized everything together. The result is here i.e. Black hole.



Schwarzschild Radius

• Neutron Star-



Here is a (press) e^- capture -

[How to present your Seminar]

#ABHISHEK BOHARE

- How to give presentation?

- Focus on the Audiance
- Delete all excess text from your slides
- Keep it simple.
- Rehearse 3.
- Make friend
- Dress to impress
  - → Smart hair
  - → Good/Smart Shoes
  - → Smart dress.
- Be comfortable
- Stand Still
- Tell a story
- Be enthusiastic!
- Know your audience.
- Use structures to build on Ideas.
- Use Visuals

  65% of people are visual learner
- Repetition is your friend.
- Give a brief rundown
- Have a story to tell
- Be Relatable
- Built your confidence.
- Practice a lot!
- Read whatever you wrote
- Be Natural
- Impress your Audiance.

- How can we start?

#Show you emotion today what have you prepared. If you are in fear, express fear in public. Everyone will take you positively.

#Can Humor situation-

1. Are you ready?

Yes!

But I am not ready.

2. predefined things don't work.

Un-predicted work is enough to impress audience:

Be energetic. (Mirror effect)

Demonstrate you message or work.

You can use magic tricks.

Don't left the staze.

You can go to imagination with your audience.

I have a question for you. ← you can start like this

Take them to the future.

Say 'sayonara' at last.