Physics [Module 3]

Notes

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Module – 3

Physics

ABHISHEK BOHARE

- Computational Physics
- LASER Physics
- Electronics B
- Electrodynamics
- Nuclear Physics

Computatinal Physics

- Unit 1
  - Roots of Functions
  - Interpolations
  - Extrapolation
  - Integration by trapezoidal
  - Simpson's rule
  - Runge-Kutta Method
  - Least square fitting method
- UNIT 2 MATRIX
- UNIT 3 FLOWCHART, ALGORITHMS
- UNIT 4 C, FORTRON 90/95

• Roots of Functions :

A function has a root when it crosses the x axis i.e. $f(x)=0$. A function can here more than one root, where there are multiple values for x that satisfy the condition.

In mathematics, a zero (also called root) of a real, complex or generally vector valued function f, is a member x of the domain f such that $f(x)$ vanishes at x; that is, the function f attains the value of 0 at x.

or equivalenty, x is the solution of the equation $f(x)=0$. A zero of the function is thus an input value that produces an output of 0.

$$f(x) = x^2 - 5x + 6$$
$$f(x) = 0$$
$$x^2 - 5x + 6 = 0$$
$$x^2 - (2+3)x + 6 = 0$$
$$x^2 - 2x - 3x + 6 = 0$$
$$x(x-2) - 3(x-2) = 0$$
$$(x-2)(x-3) = 0$$
$$x = 2, 3$$

$f(2) = (2)^2 - 5\times2 + 6 = 4 - 10 + 6 = 10 - 10 = 0$
$f(3) = (3)^2 - 5\times3 + 6 = 9 - 15 + 6 = 15 - 15 = 0$

• Trapezoidal Rule-

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[(y_0+y_n) + 2(y_1 + y_2 + \ldots\ldots + y_{n-1})\right]$$

• Simpson 1/3 Rule-

$$\int_a^b f(x)dx = \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+\ldots+y_{n-1}) + 2(y_2+y_4+\ldots+y_{n-2})\right]$$

• Simpson 3/8 Rule -

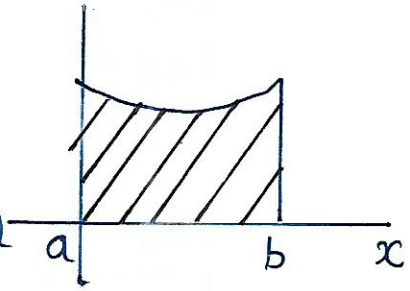
$$\int_a^b f(x)dx = \frac{3h}{8}\left[(y_0+y_n) + 3(y_1+y_2+y_4+y_5+\ldots+y_{n-1}) + 2(y_3+y_6+\ldots+y_{n-3})\right]$$

$$h = \frac{b-a}{n} \quad \text{— Intervals}$$

Numerical Integration -

The area bounded by the curve $f(x)$ & x axis between the limit a and b denoted by

$$I = \int_a^b f(x)dx \quad \ldots\ldots\ldots\ldots (1)$$



devide the interval a and b into n equal interval with the length h (Step Size).

i.e. $(a,b) = (a = x_0, x_1, x_2, \ldots\ldots, x_{n-1}, x_n = b)$

$a = x_0$

$x_1 = x_0 + h$

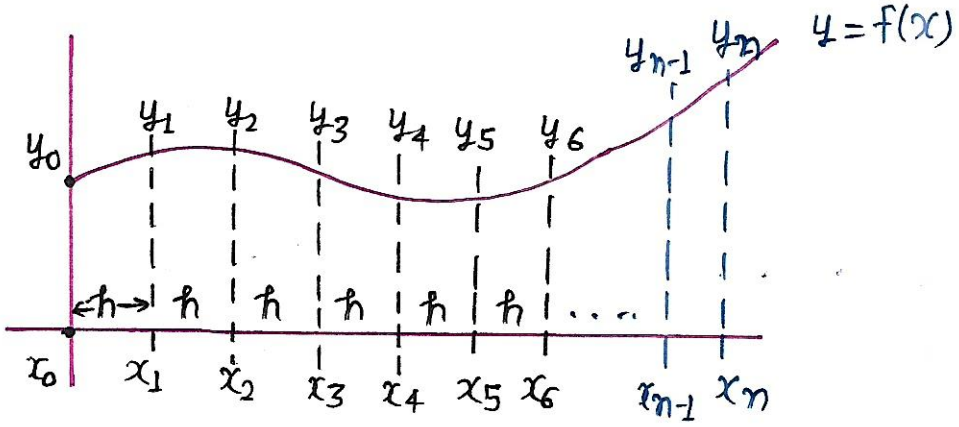
$x_2 = x_1 + h$

. . . . . .

. . . . . .

$x_n = x_{n-1} + h$

$$n = \frac{b-a}{h} \quad \text{or} \quad h = \frac{b-a}{n}$$



eq^n(1) can be solved –

(1) Trapezoidal Rule

$$TR = \int_a^b f(x)\,dx = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + \ldots\ldots y_{n-1})\right]$$

$$\int_a^b f(x)dx = h\left[\frac{y_0 + y_n}{2} + y_1 + y_2 + \ldots\ldots y_{n-1}\right]$$

It is used for any no. of Interval.

(ii) Simpson 1/3 Rule-

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+\dots)+2(y_2+y_4+\dots)\right]$$

It is applicable only when the total no. of Interval is even.

(iii) Simpson 3/8 Rule-

$$\int_a^b f(x)\,dx = \frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2+y_4+\dots)+2(y_3+y_6+\dots)\right]$$

It is applicable only when the total no of Interval is multiple of three.

General Quadratic Formula-

→ Quardratic Formula solve quardratic equations.

Let $ax^2+bx+c=0$ be the quardratic equations.

$$ax^2+bx+c=0$$

devide by a

$$\frac{a}{a}x^2+\frac{b}{a}x+\frac{c}{a}=0$$

$$x^2+\frac{b}{a}x+\frac{c}{a}=0$$

$$x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2-\left(\frac{b}{2a}\right)^2+\frac{c}{a}=0$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

$$x+\frac{b}{2a}=\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2-4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$

Newton Backward and Forward Interpolation-

Let $y = f(x)$ and $y_0, y_1, y_2, y_3 \ldots y_n$ are values corresponding to points $x_0; x_0+h, x_0+2h, x_0+3h, \ldots x_0+nh$. Suppose we want to find out $f(x) = y$ at point $x = x_0 + ph$ $\left(\text{or } p = \frac{x-x_0}{h}\right)$

- We know by definition of E

$$E^p f(x) = f(x+ph)$$

$$\Rightarrow E^p f(x_0) = f(x_0+ph)$$

$$\Rightarrow f(x_0+ph) = y(x_0+ph) = y(x) = E^p f(x_0)$$

$$\Rightarrow y(x) = E^p y_0$$

$$\Rightarrow y(x) = (1+\Delta)^p y_0 \qquad [\because E = 1+\Delta]$$

$$\Rightarrow y(x) = \left(1 + p\Delta + \frac{p(p-1)}{\lfloor 2}\Delta^2 + \frac{p(p-1)(p-2)}{\lfloor 3}\Delta^3\right) y_0 \qquad \left[(1+x)^n = 1 + nx + \frac{n(n-1)}{\lfloor 2}x^2 + \ldots\right]$$

$$\boxed{\Rightarrow y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{\lfloor 2}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{\lfloor 3}\Delta^3 y_0 + \ldots}$$

This is newton's Forward Interpolation Formula.

- We know that

$f(x) = y$ at point $x = x_n + ph$ or $\left(p = \frac{x-x_n}{h}\right)$

We know by definition of E

$$E^p f(x) = f(x+ph)$$

$$E^p f(x_n) = f(x_n+ph)$$

$$f(x) = E^p y_n \quad (\because y_n = f(x_n))$$

$$f(x) = (E^{-1})^{-p} y_n \quad [\because E^{-1} = 1-\Delta]$$

$$f(x) = (1-\nabla)^{-p} y_n$$

$$f(x) = \left[1 + p\nabla + \frac{p(p+1)}{L^2}\nabla^2 + \frac{p(p+1)(p+2)}{L^3}\nabla^3\right] y_n$$

$$\boxed{f(x) = y_n + p\nabla y_n + \frac{p(p+1)\nabla^2 y_n}{L^2} + \frac{p(p+1)(p+2)}{L^3}\nabla^3 y_n + \dots}$$

- Forward $\Delta f(x) = f(x+h) - f(x)$

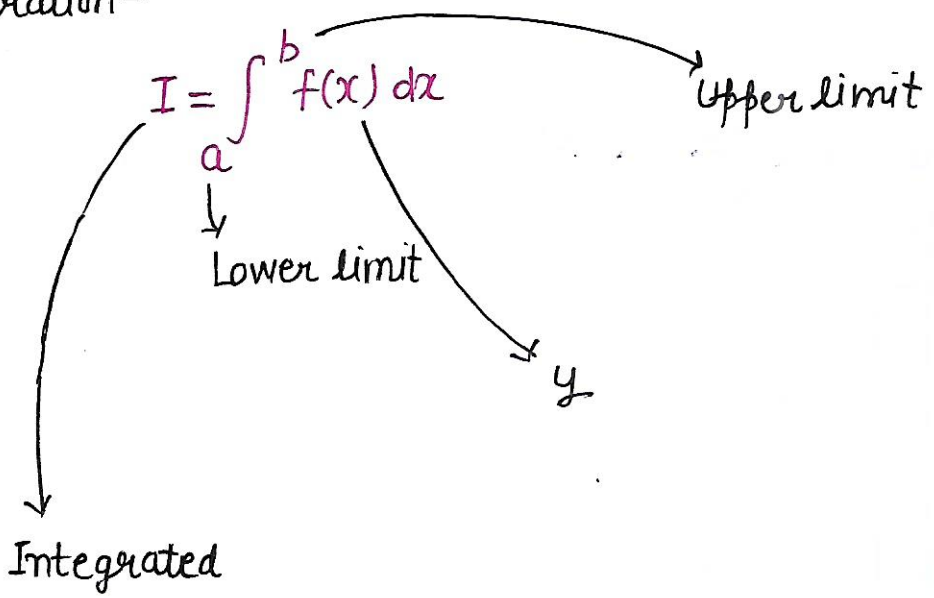
$$f(a+hu) = f(a) + u\Delta f(a) + \frac{u(u-1)}{L^2}\Delta^2 f(a) + \frac{u(u-1)(u-2)}{L^3}\Delta^3 f(a) + \dots$$

- Backward $\nabla f(x) = f(x) - f(x-h)$

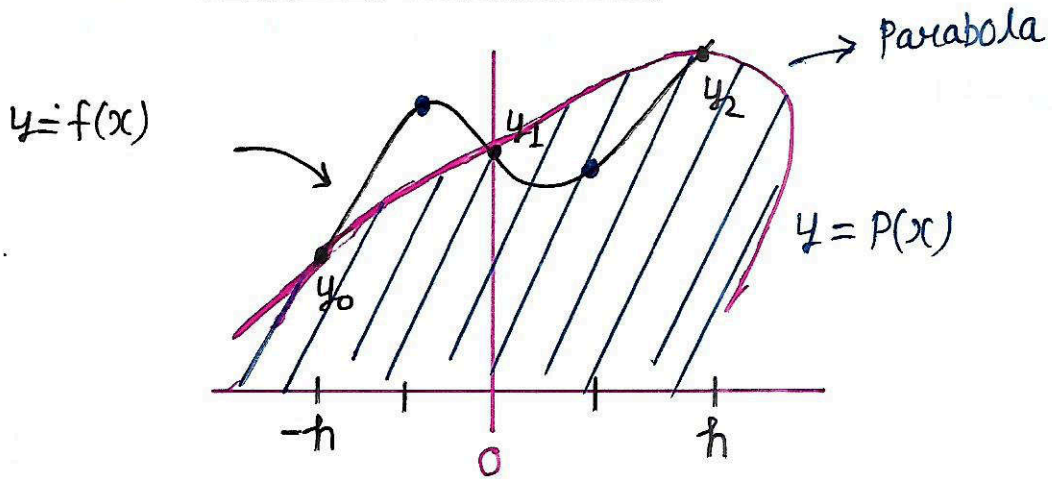
$$f(a+hu) = f(a) + u\nabla f(a) + \frac{u(u+1)}{L^2}\nabla^2 f(a) + \frac{u(u+1)(u+2)}{L^3}\nabla^3 f(a) + \dots$$

Inegration-

$$I = \int_a^b f(x)\,dx$$



#Simpson 1/3 Rule and 3/8 Rule-



$$\int_{-h}^{h} P(x)\,dx = \frac{h}{3}(y_0+4y_1+y_2)$$

$\Downarrow$

Goal

$$P(x) = Ax^2+Bx+C$$

$$\int_{-h}^{h} Ax^2+Bx+C = \left[\frac{Ax^3}{3}+\frac{Bx^2}{2}+Cx\right]_{-h}^{h}$$

$$= \frac{2Ah^3}{3}+2Ch$$

$$= \frac{h}{3}(2Ah^2+6C)$$

$$P(h) = Ah^2+Bh+C = y_2$$
$$P(-h) = Ah^2-Bh+C = y_0$$

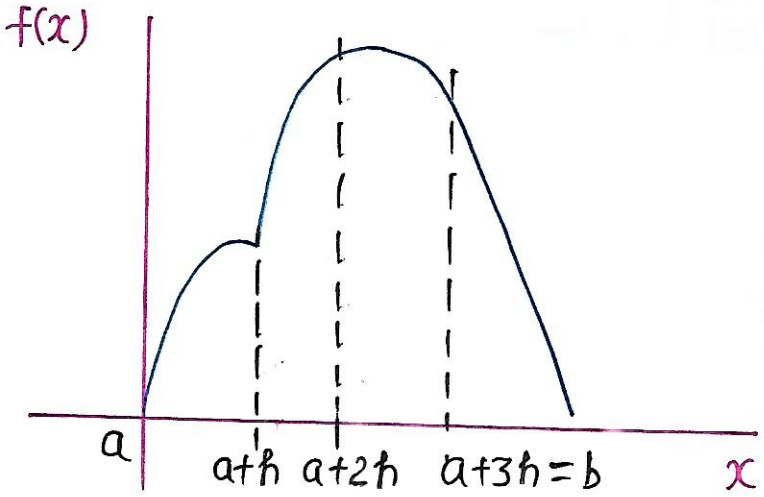
$$2Ah^2+2C = y_0+y_2$$
$$2Ah^2 = y_0+y_2-2y_1$$

$$\Rightarrow \frac{h}{3}(y_0+y_2-2y_1+6y_1)$$

$$\Rightarrow \frac{h}{3}(y_0+4y_1+y_2)$$

• 3/8 Rule -

It is used to approximate given function $f(x)$ with cubic polynomial.



Trapazoidal Rule ⟶ 1st Order ⟶ 2 points
1/3 Rule of Simpson ⟶ 2nd order ⟶ 3 points
3/8 Rule of Simpson ⟶ 3rd order ⟶ 4 points

$$\int_a^b f(x)dx = \frac{3}{8}h\left[f(a)+3(f(a+h))+f(a+2h)+f(b)\right]$$

$$\int_a^b f(x)dx = \frac{3}{8}h\Big[f(a)+3\Big(f(a+h)+f(2h+a)+f(a+3h)+\dots\Big) + 2\big(f(a+3h)+f(a+6h)+f(a+9h)+\dots + f(b)\big)\Big]$$

$$\int_a^b f(x)dx = \frac{3}{8}h\left[f(a)+f(b)+3\sum_{i=1}^{n-1}(a+ih)+2\sum_{i=1}^{n-1}f(a+ih)\right]$$

Non Multiple of 3

Runge-Kutta Method -

- Runge Kutta method of 4th order.

Consider Initial value problem $\frac{dy}{dx} = f(x,y)$

When $y(x_0) = y_0$

$$K_1 = hf(x_n, y_n)$$

$$K_2 = hf\left(x_n + \frac{h}{2}, y_n + K_1/2\right)$$

$$K_3 = hf\left(x_n + \frac{h}{2}, y_n + K_2/2\right)$$

$$K_4 = hf\left(x_n + h, y_n + K_3\right)$$

$$K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

$$\boxed{y_{n+1} = y_n + K}$$

Least Square fitting Method -

(1) Curve Fitting and optimization

x	x_1 x_2 ..... x_n → no. of terms
y	y_1, y_2 ..... y_n

(i) Fitting straight line

$$y = a + bx$$

$$\Sigma y = na + b\Sigma x$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2$$

(ii) Fitting second degree parabola

$$y = a + bx + cx^2$$

$$\Sigma y = na + b\Sigma x + c\Sigma x^2$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$$

$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$$

Eigenvalues and Eigenvectors of Matrices–

Let $A = (a_{ij})_{m \times n}$

(i) Characteristic Matrix of A–

The matrix $A - \lambda I$ is called characteristic matrix of A.

$$A - \lambda I = \begin{bmatrix} a_{11} - \lambda & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} - \lambda & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{mn} - \lambda \end{bmatrix}$$

(ii) Characteristic Polynomial–

The determinent of $A - \lambda I$ is called characteristic Polynomial of A.

(iii) Characteristic Eqn–

$$|A - \lambda I| = 0$$

The roots of equation is called characteristic roots or eigenvalues of matrics A.

(iv) Eigenvectors–

If $\lambda = \lambda_1$, eigenvalues of non-$\mathcal{I}$ nxn matrix A. Then non-zero solution

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_n \end{bmatrix}$$

of equation $(A - \lambda I)X = 0$ is said to be eigenvector of matrix A, corresponding to eigenvalues $\lambda = \lambda_1$.

Jacobi Method–

Iteration method

⇓

Jacobi Iteration method

Problem;

$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$
$$x + y + 54z = 110$$

soln

$$|27| > |6| + |-1|$$
$$|15| > |6| + |2|$$
$$|54| > |1| + |1|$$

so we can use Jacobi method for given system of equation

$$x = (85 - 6y + z)\frac{1}{27}$$

$$y = (72 - 6x - 2z)\frac{1}{15}$$

$$z = (110 - x - y)\frac{1}{54}$$

$$x_0 = 0,\ y_0 = 0,\ z_0 = 0$$

First Iteration

$$x^{(1)} = \frac{85}{27} = 3.148$$

$$y^{(1)} = \frac{72}{15} = 4.8$$

$$z^{(1)} = \frac{110}{54} = 2.037$$

Second Iteration

$$x^{(2)} = \frac{1}{27}[85 - 6(4.8) + 2.037] = 2.157$$

$$y^{(2)} = \frac{1}{15}[72 - 6(3.148) - 2(2.037)] = 3.269$$

$$z^{(2)} = \frac{1}{54}[110 - 3.148 - 4.8] = 1.89$$

Third Iteration

$$x^{(3)} = \frac{1}{27}[85 - 6(3.269) + 1.89] = 2.492$$

$$y^{(3)} = \frac{1}{15}[72 - 6(2.157) - 2(1.89)] = 3.685$$

$$z^{(3)} = \frac{1}{54}[110 - 2.157 - 3.269] = 1.937$$

Fourth Iteration

$$x^{(4)} = \frac{1}{27}[85 - 6(3.685) + 1.937] = 2.401$$

$$y^{(4)} = \frac{1}{15}[72 - 6(2.492) - 2(1.937)] = 3.545$$

$$z^{(4)} = \frac{1}{54}[110 - 2.492 - 3.685] = 1.925$$

Fifth Iteration

$$x^{(5)} = 2.452$$

$$y^{(5)} = 3.853$$

$$z^{(5)} = 1.427$$

Sixth Iteration

$$x^{(6)} = 2.423$$

$$y^{(6)} = 3.57$$

$$z^{(6)} = 1.926$$

Seventh Iteration

$$x^{(7)} = 2.426$$

$$y^{(7)} = 3.574$$

$$z^{(7)} = 1.926$$

} Repeated Iteration

Sixth and Seventh Iteration give particularly same values therefore we can stop the Iteration.

Hence sol^n of eq^n

$x = 2.426$

$y = 3.574$

$z = 1.926$

Gaussian Elimination-

Solution of linear Algebric method-

(1) Direct method → Guass elimination method & Guass Jordan method

(2) Indirect method → Jacobi method & Guass sidel method → discussed earlier.

→ Guass Elimination method Problem:

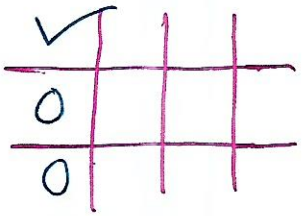
$$x - y + 2z = 3$$
$$x + 2y + 3z = 5$$
$$3x - 4y - 5z = 13$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 2 & 3 \\ 3 & 4 & 5 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 13 \end{pmatrix}$$

$$AX = B$$

$$A:B = \left[\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ \textcircled{1} & 2 & 3 & 5 \\ \textcircled{3} & 4 & 5 & 13 \end{array}\right]$$

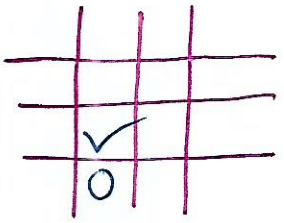
↓ Make zero



$R_2 \Rightarrow R_2 - R_1$

$R_3 \rightarrow R_3 - 3R_1$

$$(A:B) = \left[\begin{array}{ccc:c} 1 & -1 & 2 & 3 \\ 0 & 3 & 1 & 2 \\ 0 & -1 & -11 & -22 \end{array}\right]$$



$R_3 \longrightarrow 3R_3 + R_2$

$$(A:B) = \left[\begin{array}{ccc:c} 1 & -1 & 2 & 3 \\ 0 & 3 & 1 & 2 \\ 0 & 0 & -32 & -64 \end{array}\right]$$

$$x - y + 2z = 3$$
$$3y + z = 2$$
$$-32z = -64$$
$$z = \frac{-64}{-32}$$

$$\boxed{z = 2}$$

$$3y + 2 = 2$$
$$3y = 0$$
$$\boxed{y = 0}$$

$$x - 0 - 4 = 3$$
$$\boxed{x = 7}$$

Matrix Inversion-

→ Find A^{-1} of A

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 6 \end{bmatrix}$$

$$A^{-1} = \frac{adj\,A}{|A|} = \frac{1}{12-5}\begin{bmatrix} 6 & -1 \\ -5 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{7}\begin{bmatrix} 6 & -1 \\ -5 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$ — |A|

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$ — for Adj

$$A^{-1} = \frac{1}{|A|} \text{adj} A \Rightarrow$$

$$\text{adj}A = \begin{bmatrix} 2 & -1 & -1 \\ 0 & 3 & 0 \\ -1 & -1 & 2 \end{bmatrix}$$ — |A|

$|A| = 4 + 0 - 1$

$|A| = 3$

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 2 & -1 & -1 \\ 0 & 3 & 0 \\ -1 & -1 & 2 \end{bmatrix}$$

FLOWCHARTS AND ALGORITHM

Algorithm -

- Efficient procedures for solving large scale problems.
- Scalability.
- Classical data structures & Classical algorithms.
- Real Implimentations

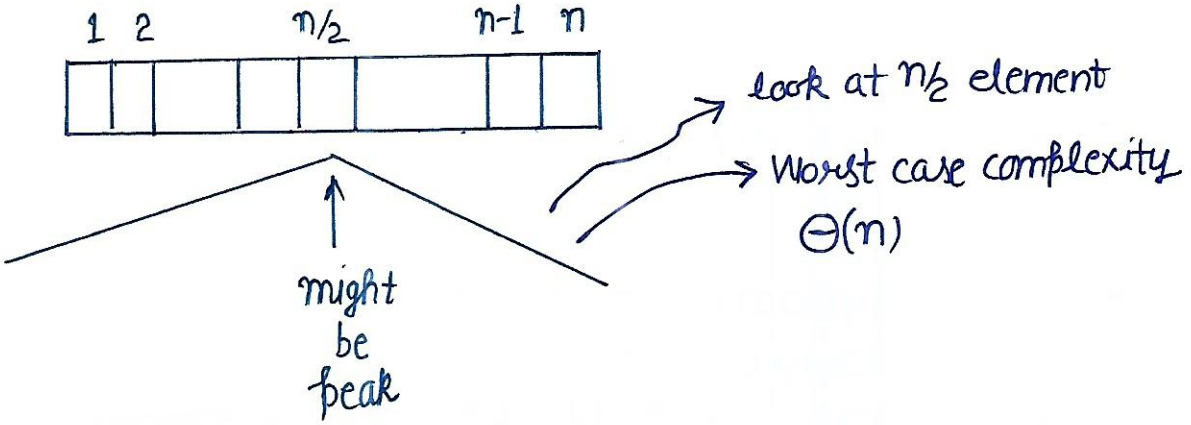
Algorithmic Thinking -

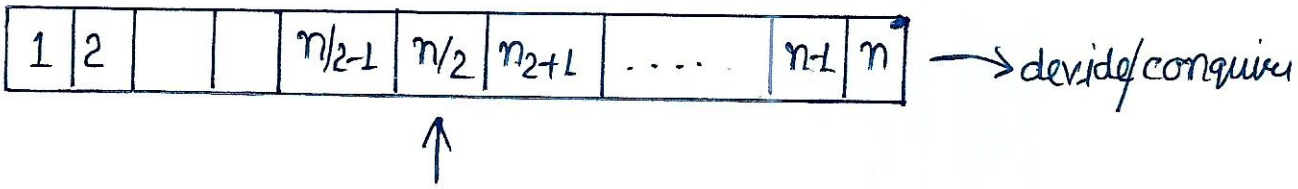
Peak Finding ⟶ 1D version

a	b	c	d	e	f	g	h	i
1	2	3	4	5	6	7	8	9

- a - i are numbers
- Position 2 is a peak If and only If
  $b \geq a$ and $b \geq c$
- Position 9 is a peak iff $i \geq h$

Problem : Find a peak if it exist -
start from left





→ If $a[n/2] < a[n/2-1]$ then only look at left half $1 \ldots n/2-1$ at look for a peak.

→ Else If $a[n/2] < a[n/2+1]$ then. :... $n/2+1 \ldots n$ for a peak.

→ Else $n/2$ position is a peak.

Definition of Algorithm-

Algorithm is a step by step procedure, which defines a set of instructions to be executed in a certain order to get a desired output. Algorithms are generally created independent of underlying language i.e. an algorithm can be implemented in more than one programming language.

Categories of Algorithms-

- Search
- Sort
- Insert
- Update
- Delete

These are only few algorithms.

Characteristic-

Not all the procedures can be called Algorithms. An algorithm should have the following characteristics.

- Unambiguous → must lead to only one meaning.
- Input → 0 or more well defined Inputs.
- Output → 1 or more well defined outputs and should match with desired output.

- Finitness → Must terminate after a finite numbers of steps.
- Feasibility → Shout be feasible with available resources.
- Independent → Should be Independent of any programming Language.

How to write Algorithms-

There is no well-defined standards for writing Algorithms. Rather, It is a problem and resource dependent. Algorithms are never written to support a particular programming Language.

As we know that all programming language share basic code construction like loops (do, for, while), flow control (If-else) etc. These common constructs can be used to write an algorithm.

We write algorithms in a step by step manner but it is not always a case. Algorithm writing is a process and is executed after the problem domain is well defined.

e.g-

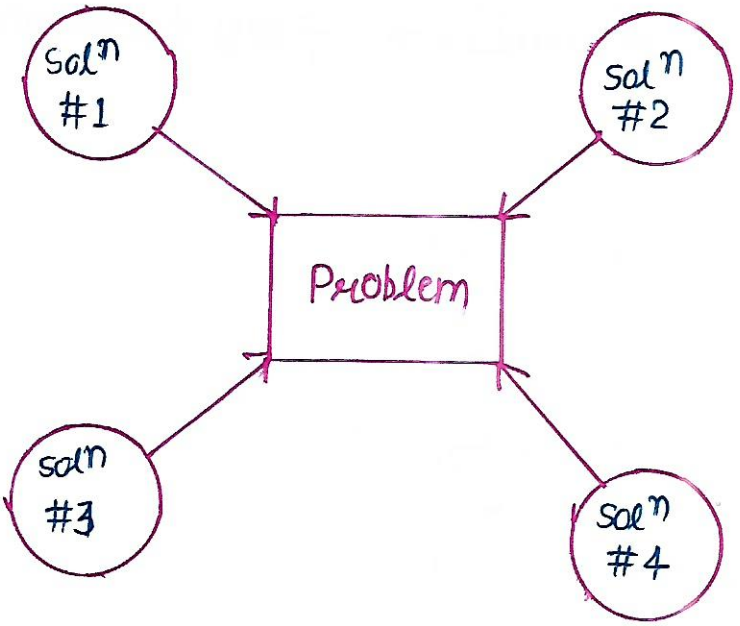
Problem- Design an algorithm to add two numbers and display the result-

1. START
2. declare three Integers.
3. define value of a & b.
4. add values of a & b.
5. Store output of step 4 to C.
6. Print C
7. STOP

Or

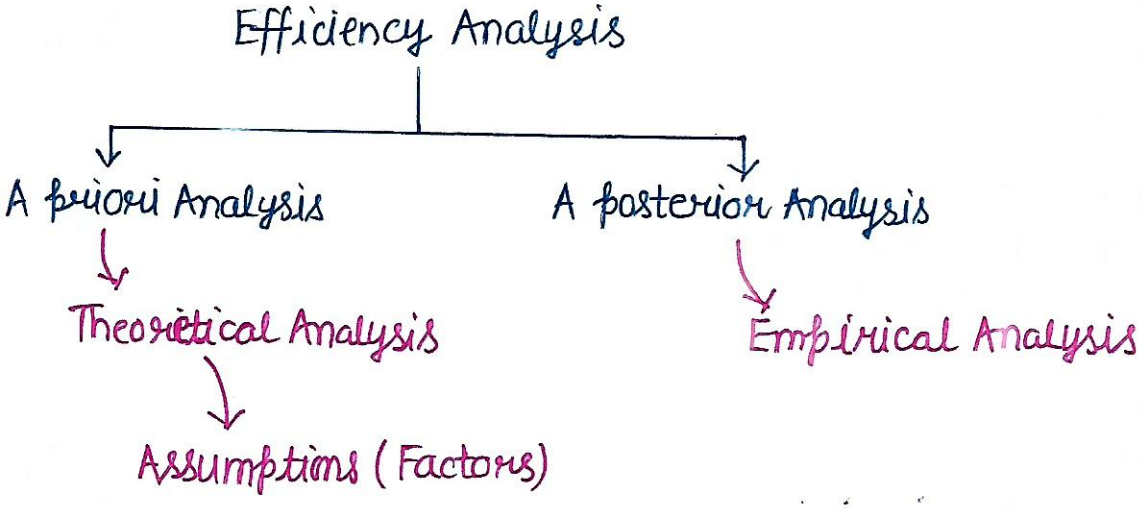
1. START ADD
2. get values of a&b
3. C ← a+b
4. display C
5. STOP

We design an algorithm to get a solution for a given problem. A problem can be solved in more than one ways.

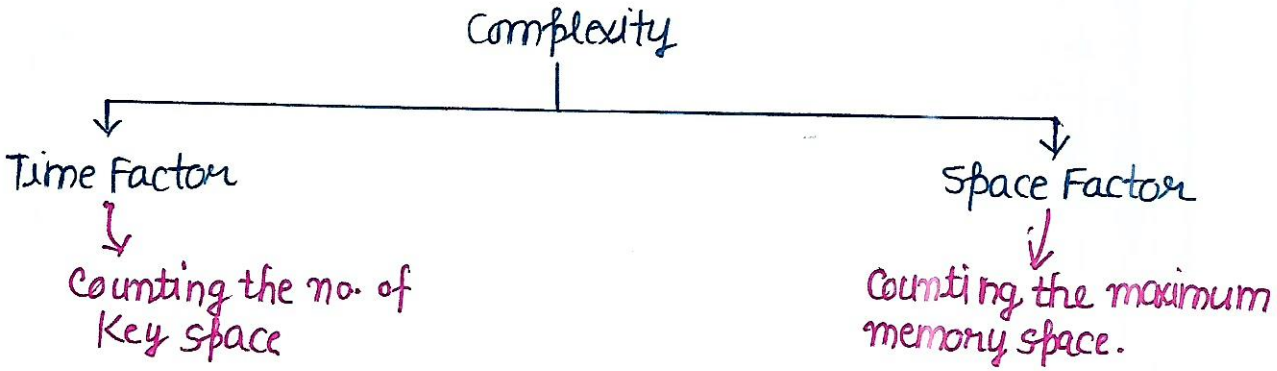


Hence many solution can be derived for a given problem.

Algorithm Analysis-



Algorithm Complexity-

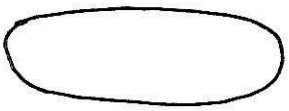


#Flowchart-

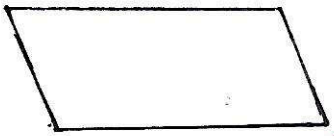
Flowchart is a graphical representation of Algorithms. Programmers often use it a program planning tool to solve a problem. It makes use of symbols which are connected among them to indicate the flow of Information and processing.

The process of drawing a flowchart for an algorithm is known as flowcharting.

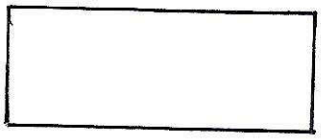
•Symbols-



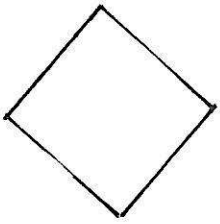
START
STOP
HALT
PAUSE



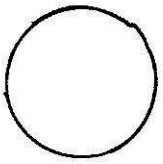
INPUT
OUTPUT



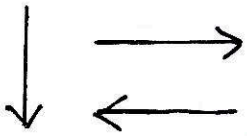
AIRTHMATIC INSTRUCTIONS
+
−
*
/



DECISION POINT
YES/NO
TRUE/FALSE
1/0
If/Else

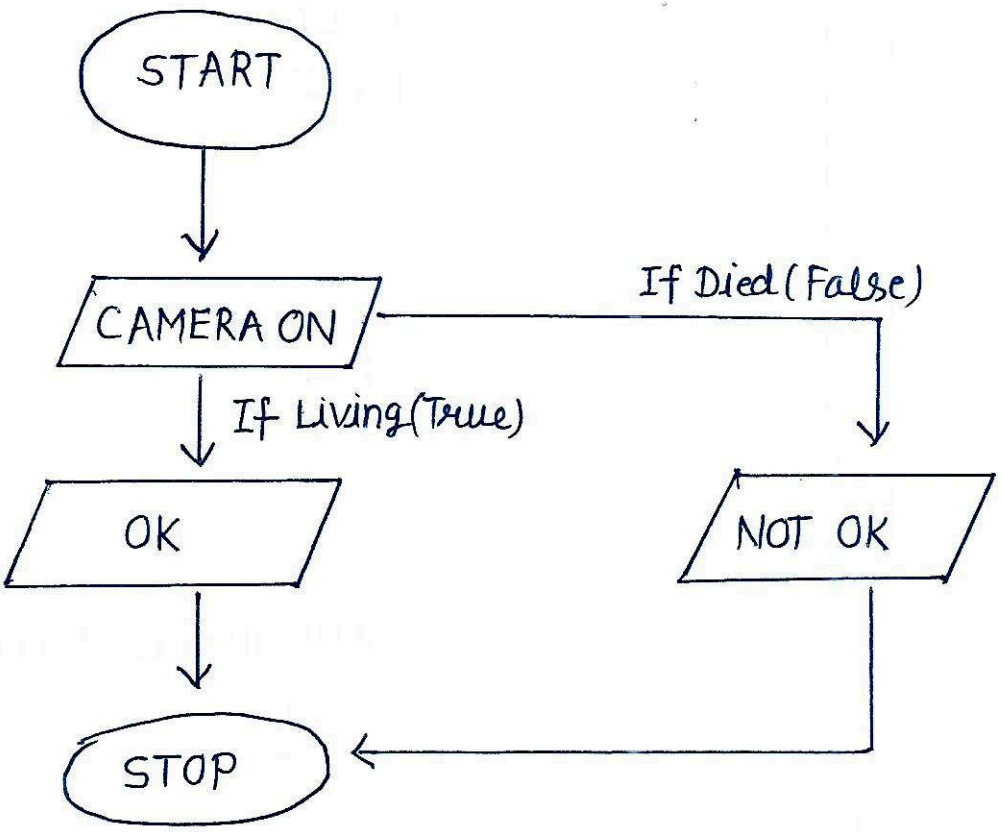


CONNECTOR FOR COMPLEX ALGORITHM



SEQUENCE OF EXECUSION

Example; SCHRODINGER CAT ANALYSIS



FORTRAN 90/95-

Fortran is one of the basic programming language available. The name fortran is the short of FORmula TRANslation, and this guide is based on Fortran 90, which is a version agreed in 1990. Fortran 95, a later standard was a minor revision of fortran 90. Fortran 2003, is in its final draft stage.

Fortran was developed for general scientific computing and is a very popular language for this purpose. In 1996 it was estimated that Fortran was employed for more than 90% of scientific computing. Fortran is not, however particularly suitable as a non-scientific general purpose language or for use in equipment control, commerce, text management or for use in (equipment) alternatives available.

Fortran 90 is available on the ITS Linux service, the UNIX service and on the networking PC service.

- Programming Basics-

A program is simply a set of Instructions that tell a computer to do a particular task. To create a program that work, work efficiently, you will need to do the following before you start writing the program.

→ Understand the Aim.
→ Make sure how to compute.

Adding two numbers-

- Uploaded on YouTube.

5 Logical Operations-

Assume variables

A ⟶ True

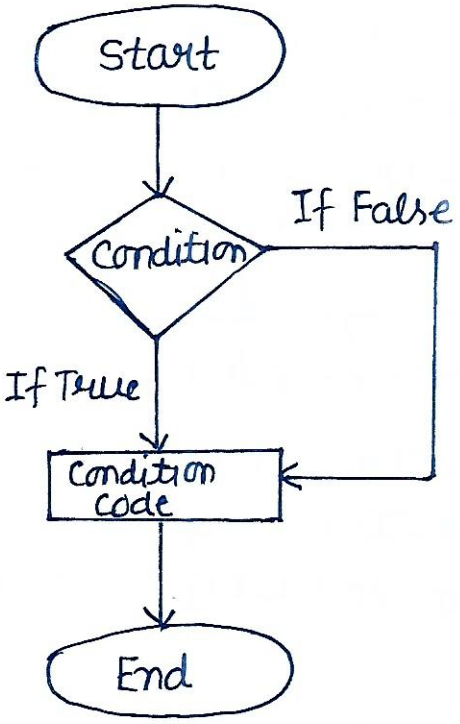
B ⟶ False

- Matrix Multiplication
- Sum of Values
- Roots of quadratic eqn
- Input output statement
- do while
- If, else
- Loop (definition)
- Use of Arrays (defn)

Operator	Description	Example
.and	• Called AND operator • If Both are non-zero then condition becomes true	A & B is False.
.or	• Called OR operator • If any one operator is non-zero, then condition becomes true	A or B is true.
.not	• Called NOT operator. • Use to reverse the operator • If operator is true then NOT operator is False.	A and B is true.
.eqv	• Called Logical equivalent. • Use to check equivalent of two logical values	A eqv. B is false.
.neqv	• Called Logical non-eqv. operator. • Called and used to check non-equvilant of two logical values.	A neqv. B is true

IF condition and Flowchart-

Decision making structure require that the programmer specify one or more conditions to be evaluated or tested by the program, along with a statement or statements to be executed, if the condition is determined to be true, and Optionally, other statements executed if the condition is determined to be false.



- Other→ If
  If Else
  Nested If
  Select
  Nested Select

If statement consists of a logical expression following by one or more statements and terminated by an end If statement.

- Syntax-

```
If (logical expression) then
   Statement
End If
```

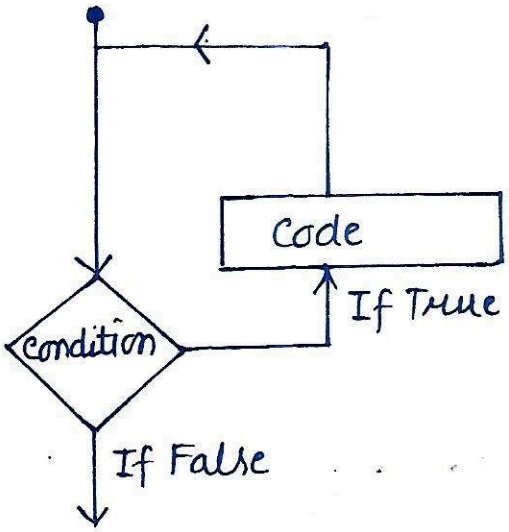
However you can give a name to the if block, then the syntax of the named If statement would be

```
[Name:] If (logical statement) then
        ! Various statements
        ....
        End If [Name]
```

#Loop- There may be a situation, when you need to execute a block of code several numbers of time. If general statements are executed sequentially. The First statement in a function is executed first, then by the second, and so on.

Programming Languages provides various control structures that allow for more complicated execution paths.

A loop statement allows us to execute a statement or group of statements multiple time and following in the general form of a loop statement in most of the programming Language.



Type-

- do
- do while
- nested

DO -

The do loop construct enables a statement, or a series of statements, to be carried out iteratively, while a given condition is true.

```
do var = start, stop [, stop]
    ! Statement(s)
    ...
end do
```

SUM OF 1 to 10 using do-Loop-

```
integer i, n, sum
sum=0
do 10 i=1,n
      sum= sum+i
    write(*,*) 'i=', i
    write(*,*) 'sum=', sum
continue
```

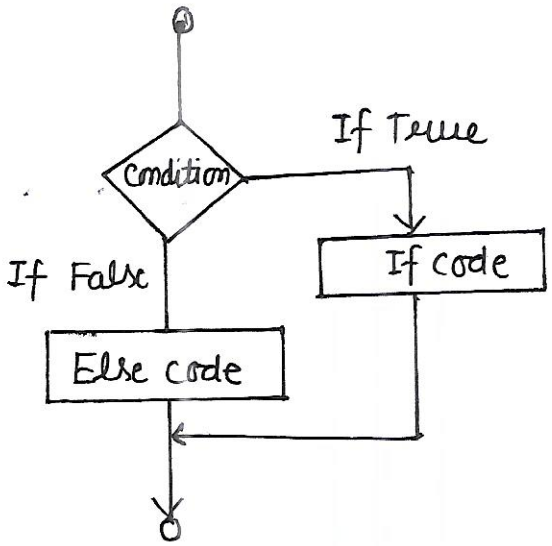
} Add values 1 to 10, 1 to n, 1 to 100 . . . .

If else -

An If..then statement can be followed by an optional else statement, which executes when the logical expression is False.

- Syntax -

```
If (logical expression) then
   statement(s)
else
   other statement(s)
end If
```



#Advantage and Limitation of Flowcharting-

- Advantages-

(1) Visual clarity
(2) Instant communication
(3) Effective coordinates
(4) Efficiency Increase
(5) Effective Analysis
(6) Problem solving
(7) Proper Documentation

- Limitations-

Flowcharts have many limitations.

(1) Complex logics
(2) Alteration and modifications.
(3) Reproduction.
(4) The essential of what is done can easily be lost in the technical details of how it is done.
(5) Factors that affected the sequence are not included.

#C Language-

- Codez club
- Tutorialspoint

C Language is a general purpose, procedul, imperative computer programming language developed in 1972 by Dennis M. Ritchie at the Bell Telephone Labratory, to develop the UNIX OS. C is most widely used computer Language. It keeps flactuating at number one scale of popularity along with JAVA programming Language, which is equally popular and most widely used among modern software programmers.

Why ? because

- Easy to learn.
- Structured Language.
- Efficient program.
- Low level activity. → It is a programming language that provides little or no abstraction from a computer instruction set architecture.
- Compiled on a variety of computer platform.

- **Facts-**
  - C was invented to write Unix OS.
  - C is a successor of B language which was introduced in 1970s.
  - The Language was formalized in 1988 by ANSI.
  - The UNIX OS was totally written in C
  - It is most popular language.
  - Today's most popular Linux OS and RDBMS MYSQL have been written in C.

- **Terminal-**

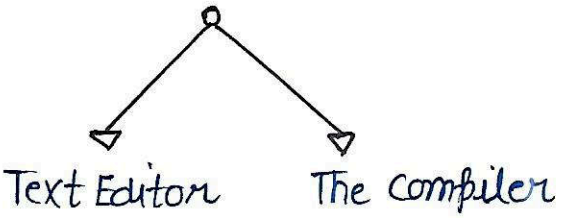
```
Cd Desktop/
touch File.c
gcc File.c -o Out*
./Out*
```

- **Applications-**

C is initially used for system development works. Some examples are used in C are-

(1) OS
(2) Language compilers
(3) Assemblers
(4) Text Editors
(5) Point Spoolers
(6) Network drivers
(7) Modern programs.
(8) Databases
(9) Language Interpreters
(10) Utilities

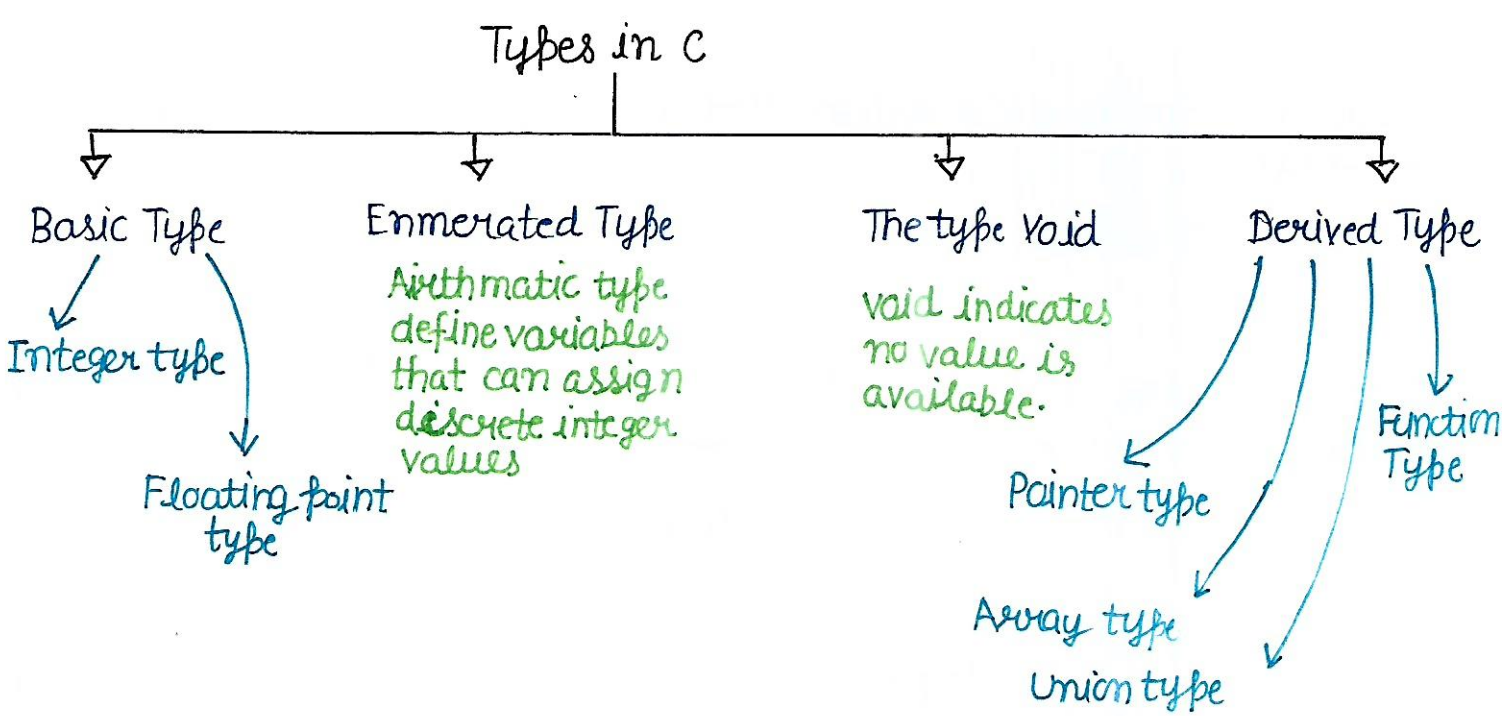
- **Pre-requirements-**



- **Basic Program-**

A C program basically consists of following parts.

→ Preprocessor commands → #include <Stdio.h>
→ Functions → int main ( )
→ Variables
→ Statement and Expression → Maths
→ Comments → /* ___ */



- Integer Type-

  # Char
  # Unsigned Char
  # Signed Char
  # int
  # unsigned int
  # short
  # Unsigned short
  # long
  # Unsigned long

- Floating Type-

  # Float
  # Double
  # Long double

- Variables Type- Just a given name to storage Area.

  # char
  # int
  # float
  # double
  # void

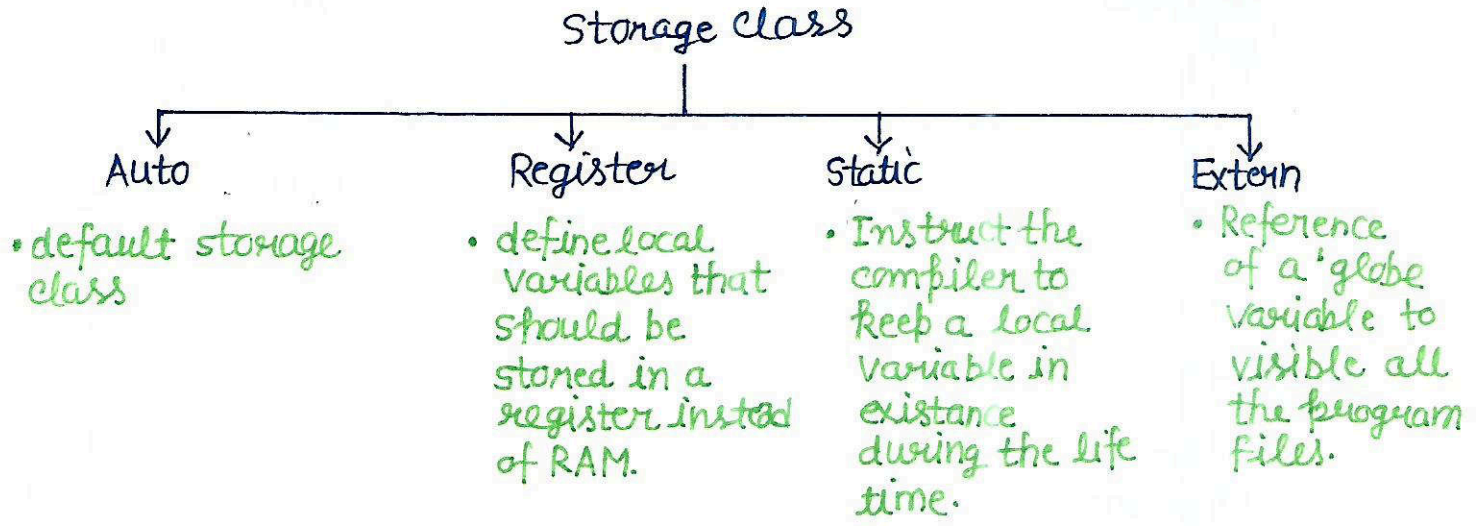
#Constants-

Constants refers to a fix values that program not alter during its execution.

define

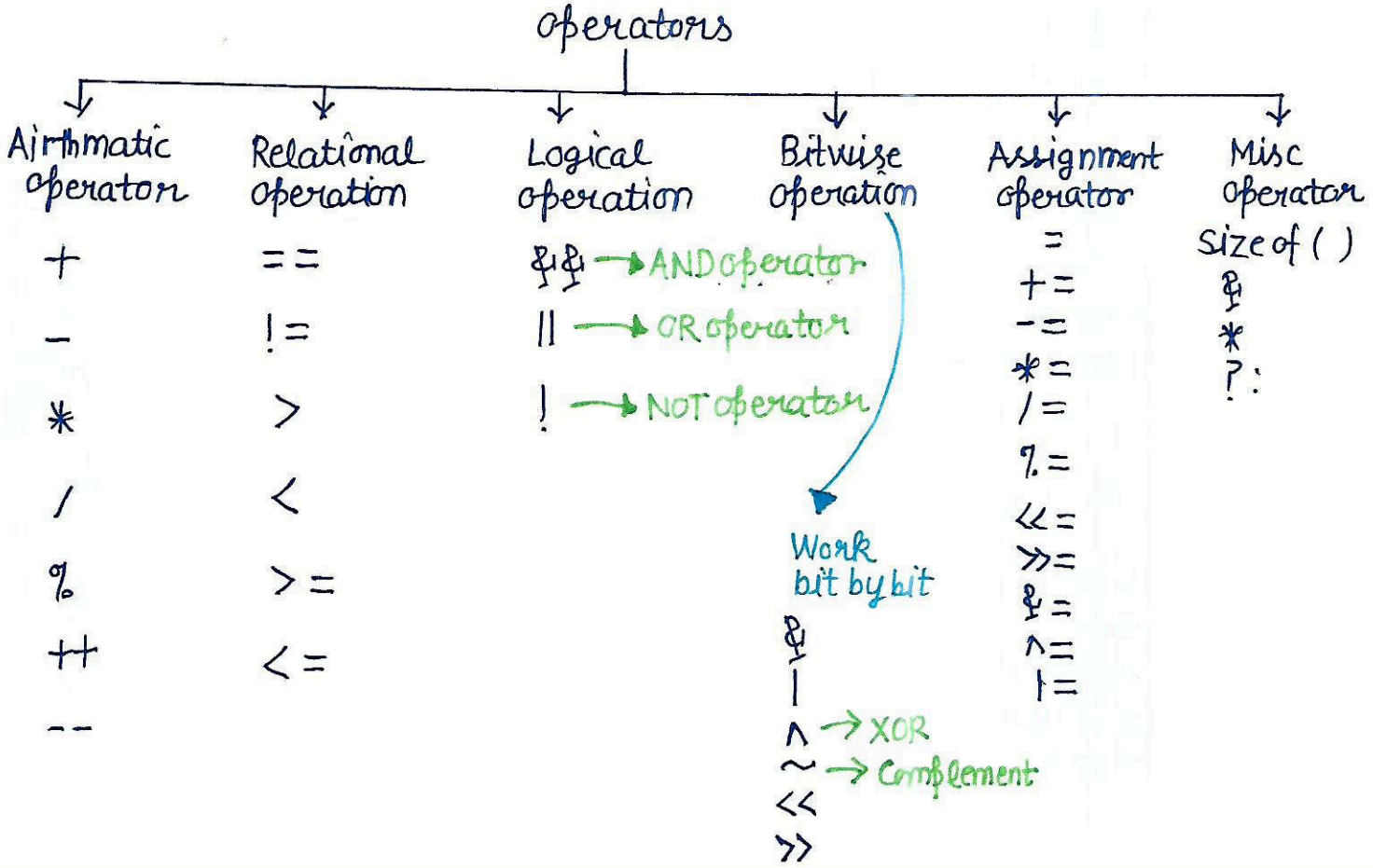
#Storage Class-

A storage class defines the scope (visiblity) and life time of variables and/or functions within a C program.



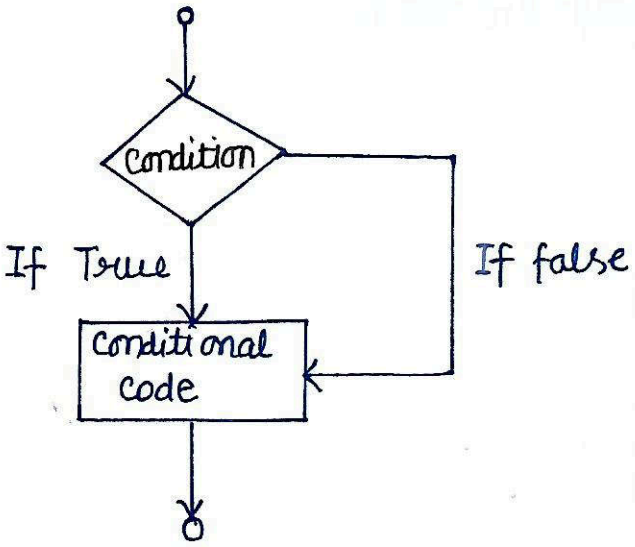
#operators-

An operator is a symbol that tells a compiler to perform specific math or logical Function.



#Decision Making-

- Specifies one or more conditions to evaluated or tested
- Statement or statements to be executed.



- C Language assumes non-zero values as true.
- Assume Zero as False.

→ **If-** If statement consist of a Boolean expression followed by one or more statement.

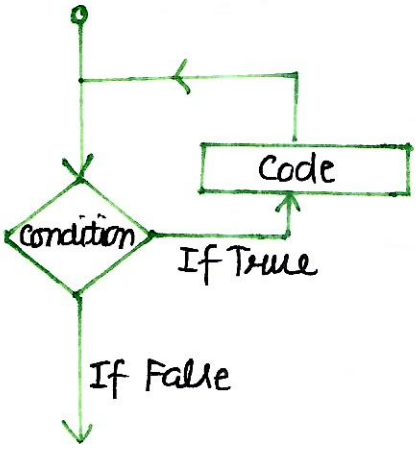
```
If (boolean expression)
{
 /* Statement will execut If the boolean expression is true. */
}
```

→ **If Else-**

```
If (boolean expression)
   {
     /* Statement is true*/
   }
else
   {
     /* False*/
   }
```

Loop -

- When a block code needs to executed several times.
- A loop statement allows us to execute a statement or group of statement multiple times.



Types -

- While → Repeatedly execute a target statement.

```
while (condition)
{ statement(s);
}
```

- For → Repetition control structure that allows us to effectively write a loop that needs to execute a specific numbers of time
- Do while → A do while loop is similar to a while loop, except the fact that it is gaurenteed to execute at least one time.
  - Used in scenario where your loop should execute at least one time.

```
do
{
statement(s);
}
while (Condition);
```

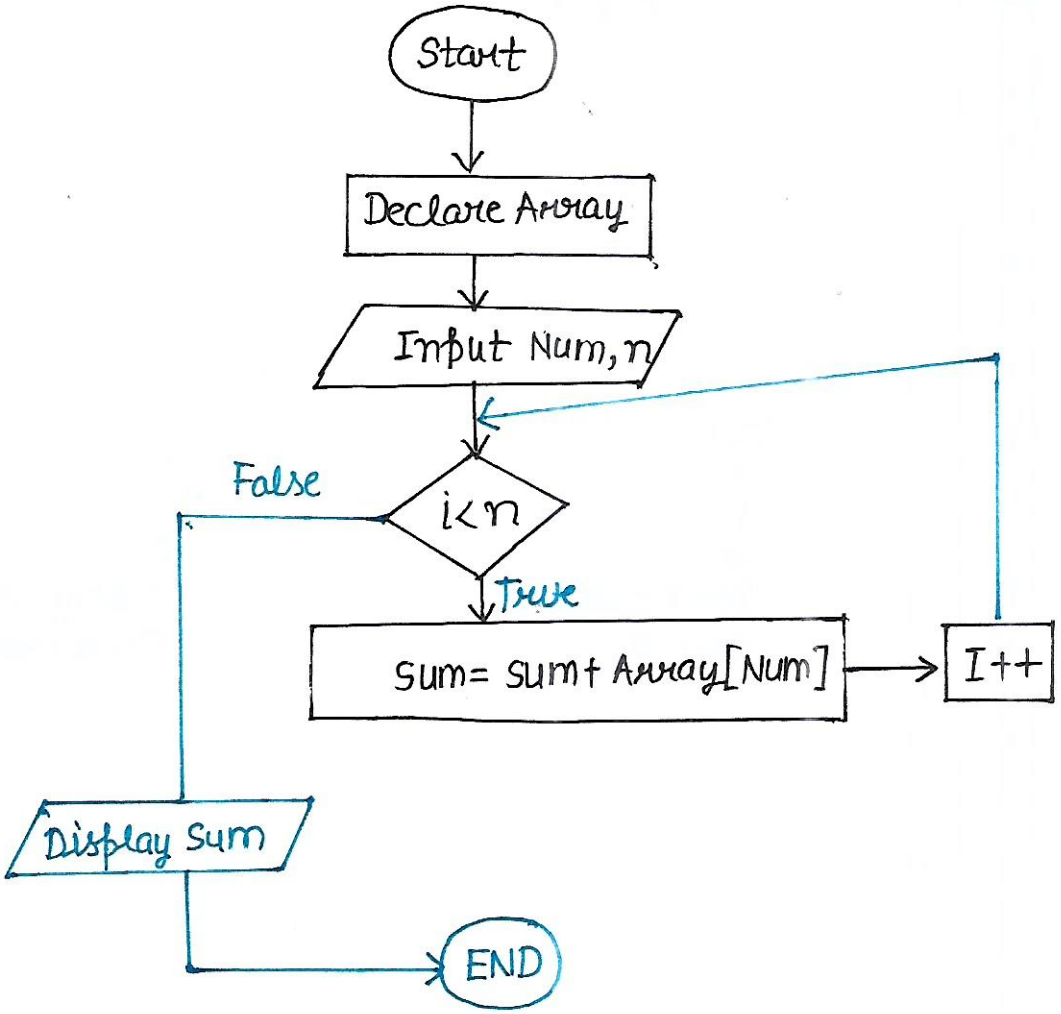
Loop Control -

- Break statement
- Continue statement
- Goto statement

• Array-

Array is kind of data structure that can store a fixed-size sequential collection of elements of same type. An array is used to store a collection of data, but is often more useful to think of an array as a collection of variables of same type.

- An array is used to store a collection of data.



• Recursion-

Recursion is the process of repeating items in a self similar way. In programming language, if a program allows you to call a function inside the same function, then it is called a recursion of a function.

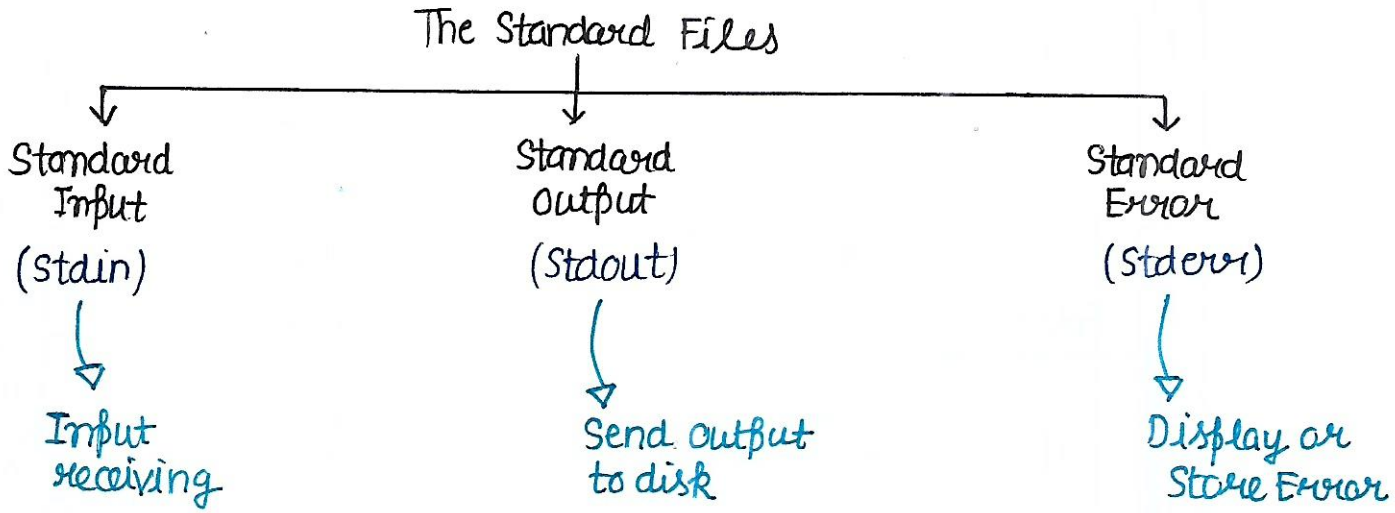
```
Void recursion( )
{ recursion( ); /* Function calls itself */
}
int main( )
{
 recursion( );
 }
```

The C programming language supports recursion, i.e. a function to call itself. But while using recursion, programmers need to be careful to define an exit condition from the function, otherwise it will go into an infinite loop.

Advantages-

→ Reduce unnecessary calling of function.
→ Extreme useful when applying the same solution.

- I/O Statement in C-



1. getchar()
2. Putchar()
3. gets()
4. puts()
5. getch()
6. getche()

1. Printf
2. Scanf

ELECTRONICS-B

UNIT- 1

Modulation, Demodulation, Transmission & Receiver

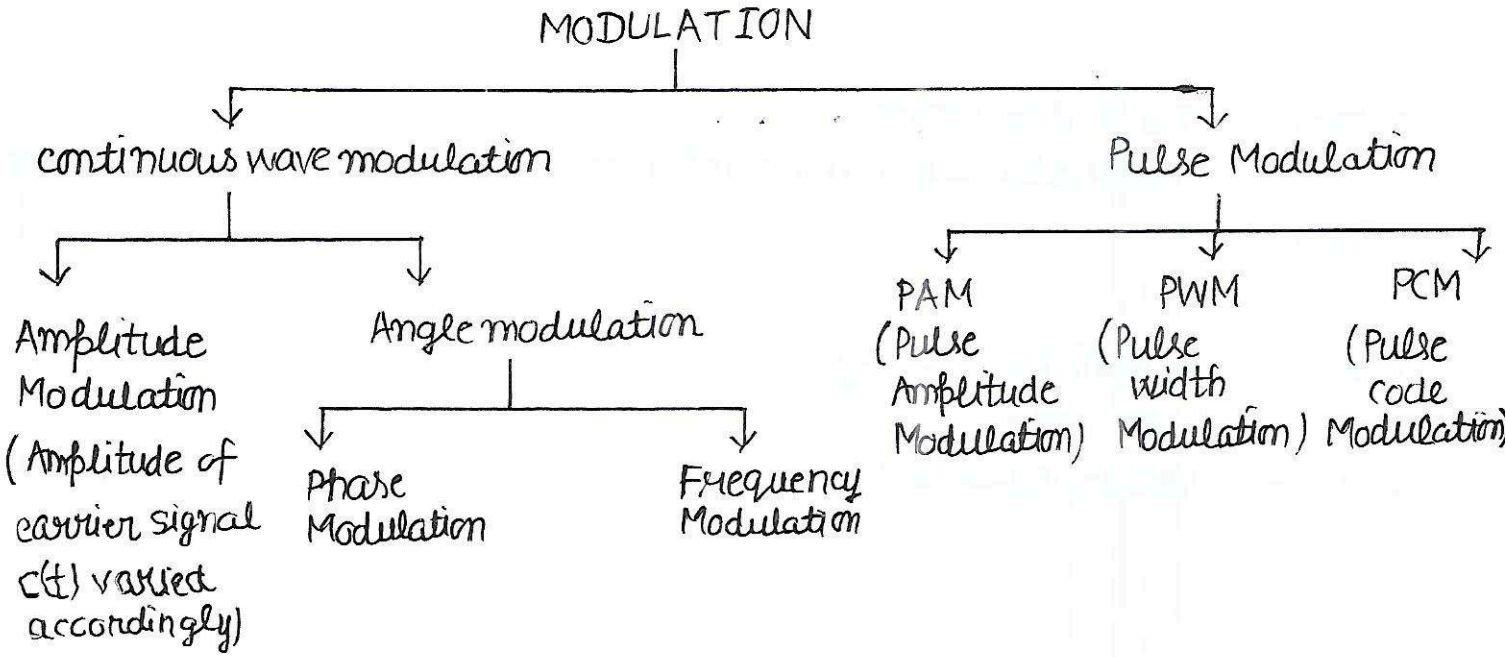
Need of Modulation-

- To transmit vaice signal a large size antenna is required as antenna length is proportional to half of wavelength. The size of Antenna will be more the distance between transmitter and receiver.
- Again when more than one transmitter is involved, all station will overlap in one frequency band.
- For those above reasons, we choose a carrier, which is a high frequency radio wave, can travel long distance without attenuation and as the frequency is high, smaller antenna is required.
- Selecting different carrier frequency for different transmitting stations can eliminate overlapping of frequency band.

Modulation-

- Modulation is a technique for impressing information on a radio frequency carrier wave by varying one or more characteristics of the wave in accordance with the intelligence signal. There are various forms of modulation.
- The signal which caracteristics are being varied is known as carrier signal having frequency f_c.
- The condition of modulation

$$f_c > f_m$$



• Amplitude Modulation –

AM is a modulation technique where the amplitude of a high frequency sine wave is varied in direct propotion to the modulated signal $m(t)$. The modulated signal contains the inteneded message or information.

The high frequency sinosoidal waveform is modulated by combining it with the message signal using a multiplier or mixer.

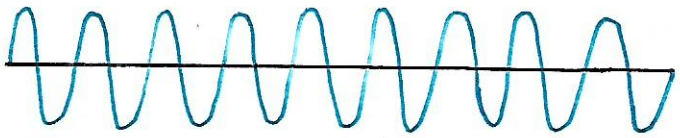
$$c(t) = A\cos\omega_c t$$

Let Phase $\rightarrow 0$

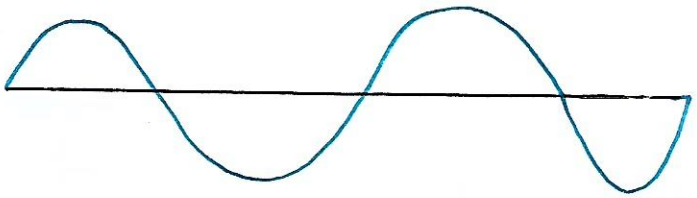
$$A \propto x(t)$$

$$s(t) = x(t)\cos\omega_c t + A\cos\omega_c t$$

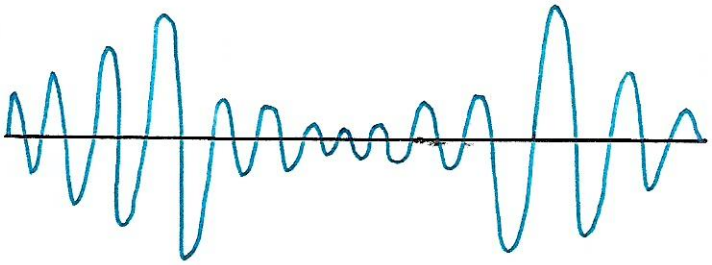
$$s(t) = \underbrace{[x(t) + A]}_{\varepsilon(t)}\cos\omega_c t$$



Carrier signal



Modulated Sine wave signal



Amplitude modulated signal

Advantages –

- Very simple to implement.
- Can be demodulated using a circuit consisting of very few components.
- Inexpensive

Disadvantages –

- Not efficient with power usage.
- High level of noise.
- Not efficient in bandwidth.

- **Frequency Modulation-**

The FM signal $s(t)$ defined by

$$s(t) = A\cos\left[2\pi f_c t + 2\pi K_f \int_0^t m(\tau)d\tau\right]$$

Is a nonlinear function of the modulated signal $m(t)$, which makes frequency modulation a non linear modulation process.

Consequently, the spectrum of a FM signal is not related in a simple manner to that of the modulated signal - Its analysis is much more difficult than that of an AM signal.

Spectral Analysis of an FM signal
- A narrow FM signal.
- A wideband FM signal.

consider $m(t) = A_m \cos(2\pi f_m)$

Then

$$f_i(t) = f_c + K_f m(t)$$

$$f_i(t) = f_c + K_f A_m \cos(2\pi f_m)t$$

Instantaneous frequency of resulting FM signal

$$\boxed{f_i(t) = f_c + \Delta f \cos 2\pi f_m t}$$

$\Delta f = K_f A_m$ is called frequency deviation, representing the maximum departure of the instantaneous frequency of a FM signal from the carrier frequency f_c.

Angle $\theta_i(t)$ of the FM signal

$$\theta_i(t) = 2\pi f_c t + 2\pi K_f \int_0^1 m(\tau)d\tau$$

$$\theta_i(t) = 2\pi f_c t + 2\pi K_f A_m \frac{\sin \omega_m t}{\omega_m}$$

$$\theta_i(t) = 2\pi f_c t + 2\pi \frac{K_f A_m}{f_m} \sin(f_m t)\, 2\pi$$

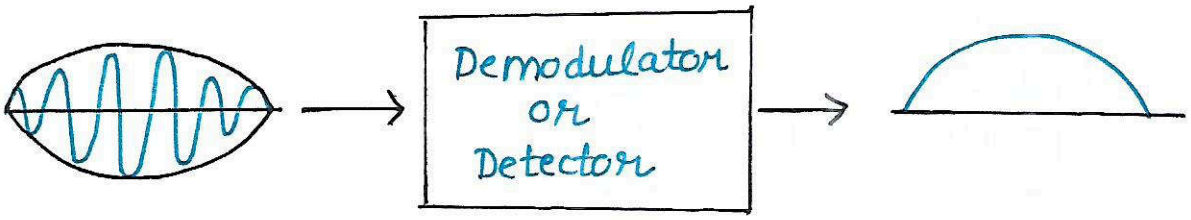
$$\theta_i(t) = 2\pi f_c t + \frac{\Delta f}{fm} \sin(2\pi f_m t)$$

The ratio of the frequency deviation Δf to the modulation freq. fm is commonly called the modulated Index of the FM signal.

$$\beta = \frac{\Delta f}{fm}$$

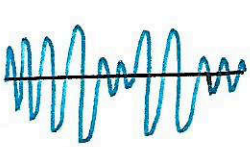
$$\theta_i(t) = 2\pi f_c t + \beta \sin(2\pi f_m t)$$

- Demodulation –
  - The recovery of the message signal from the modulated signal is called demodulation or detection.
  - Demodulation occurs at the receiver.

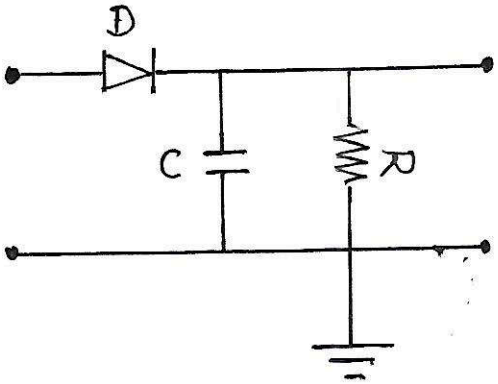


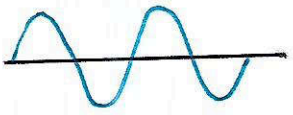
- The Envalope diode Detector –

There are various ways to measure or detect the amplitude of a waveform. Here we will consider one of the simplest, used by most portable radios, the Envalope detector.



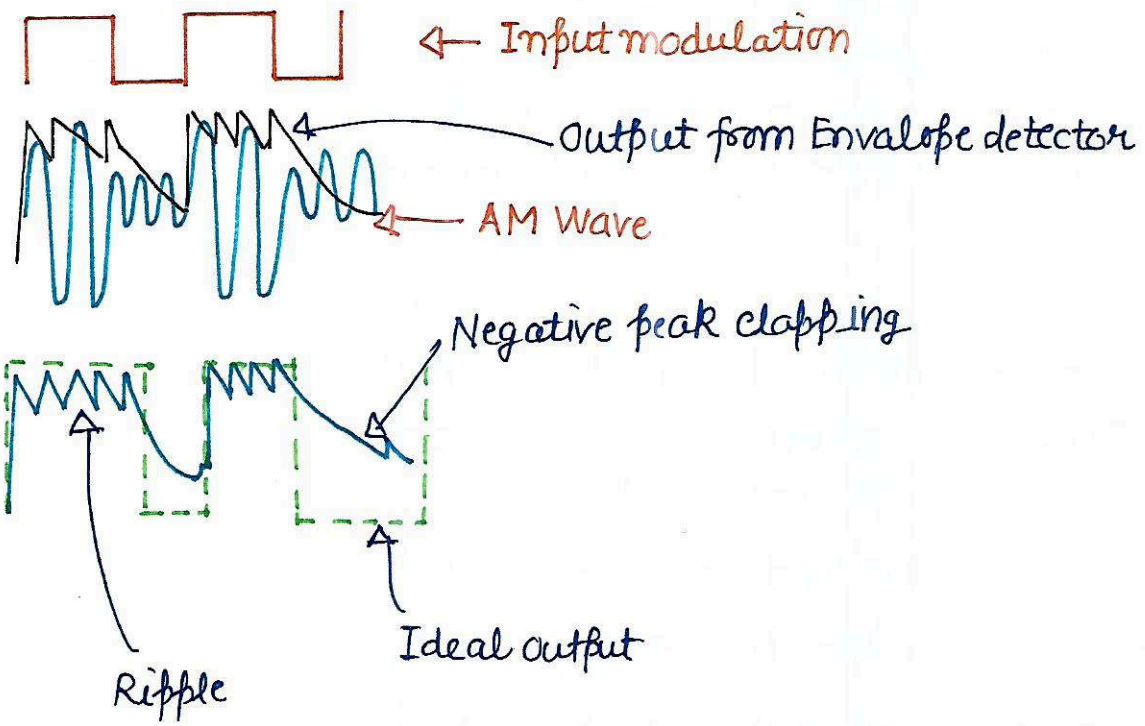
Input $s(t)$





Output $m(t)$

This is essentially just a halfwave rectifier which charges a capacitor to a voltage $\approx$ To the peak voltage of the incoming AM waveform, $s\{t\}$.



$$T = \frac{1}{f_c}$$

$$\tau = RC$$

$$V'_{Peak} = V_{peak} e^{-T/\tau}$$

$$T \ll\ll \tau$$

$$\Delta V \approx \frac{V_{peak} T}{\tau} = \frac{V_{peak}}{f_c \tau}$$

$$V_{droop} = V_{peak} e^{-t/\tau}$$

$$\tau \ll\ll t_m$$

$$t_m = \frac{1}{f_m}$$ ← highest modulated frequency

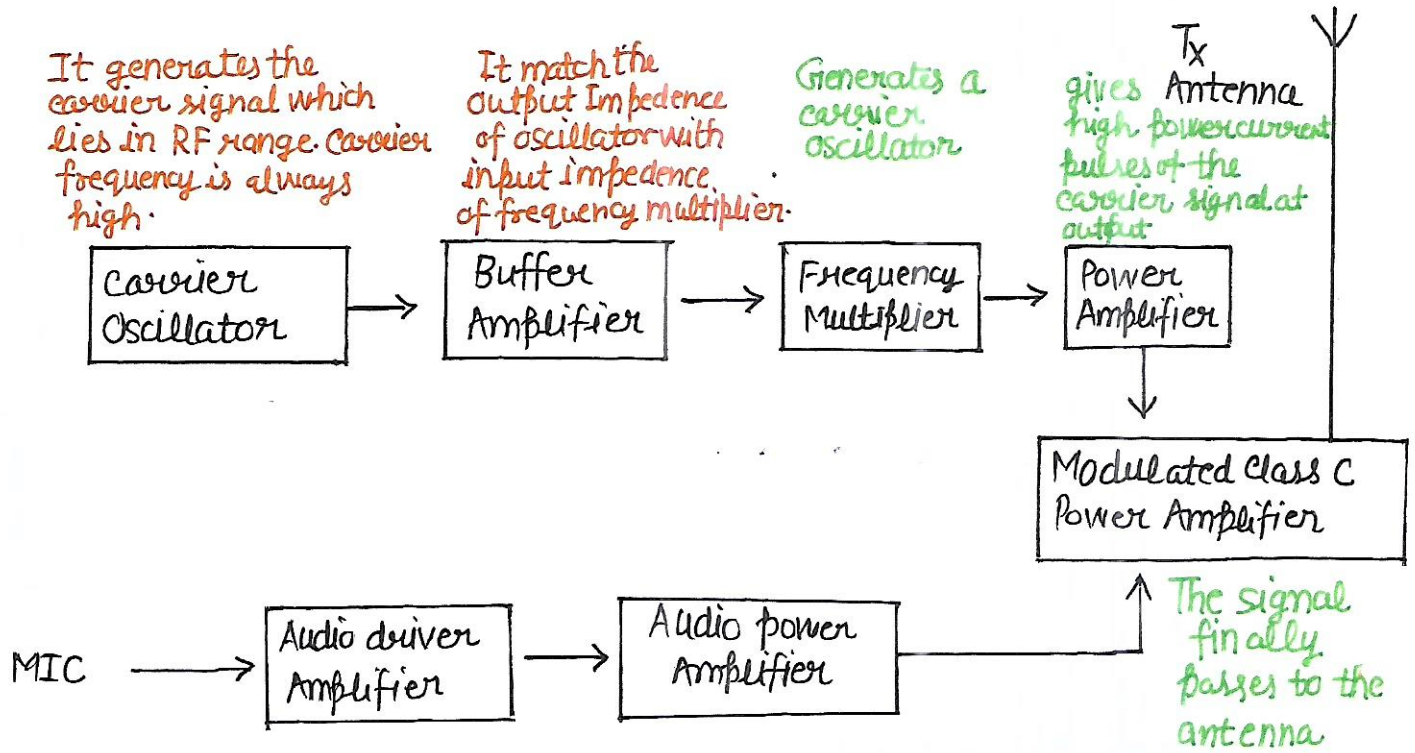
$$1/f_m \gg \tau \gg 1/f_c$$ ← In practical

- **Ratio Detector–**

- The ratio detector is a type of detector circuit, commonly used in radio detetor or receiver for demodulating the FM signal.
- The ratio detector is a varient of foster-Seeley discriminator but one diode conducts in an opposite direction, and using a tertiary winding in the proceding transformer.

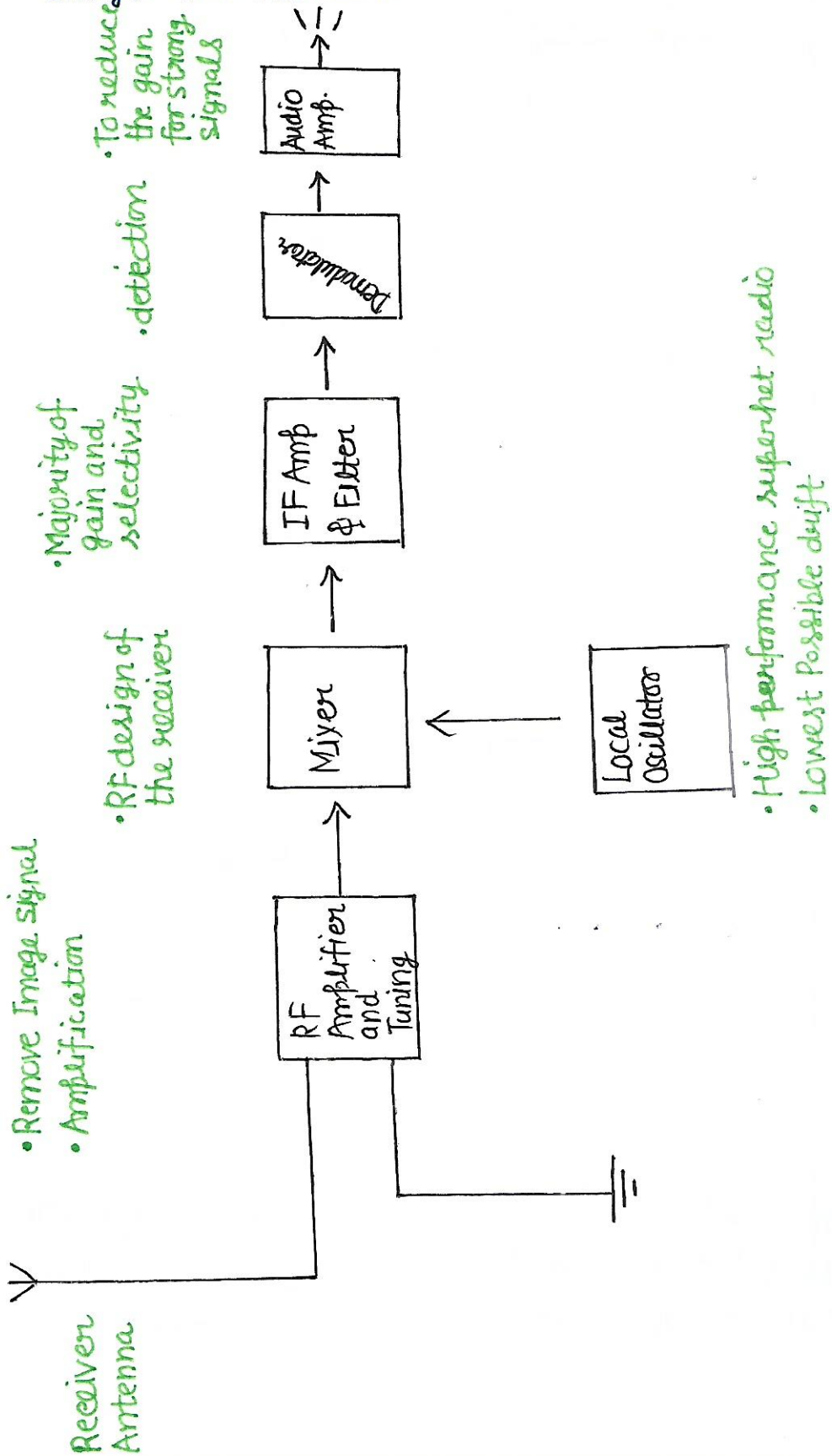
#AM Transmitter–

- Transmitter that transmit AM signals are known as AM transmitters
- These transmitters are used in medium wave and short wave frequency bands for AM broadcast.
- Medium Wave band frequencies 550 KHz to 1650 KHz
- Short wave band frequencies 3 MHz to 30 MHz.
- The two types of AM transmitters that are used based on their transmitting powers are
  - Low Level
  - High Level



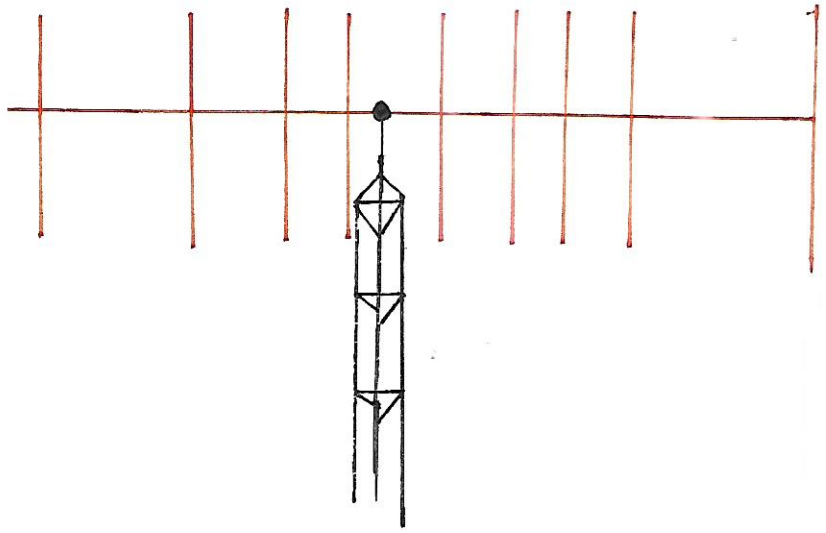
Superheterodyne Receiver-

- The superhet radio receiver is used in many forms of radio broadcast reception, two way radio communication and the like.
- It is useful to have an understanding of the different signal blocks, their function and the overall signal flow, not only for the RF circuit design but also for the operational point of view. It is possible to get the best performance by understanding its internal RF design and Function.

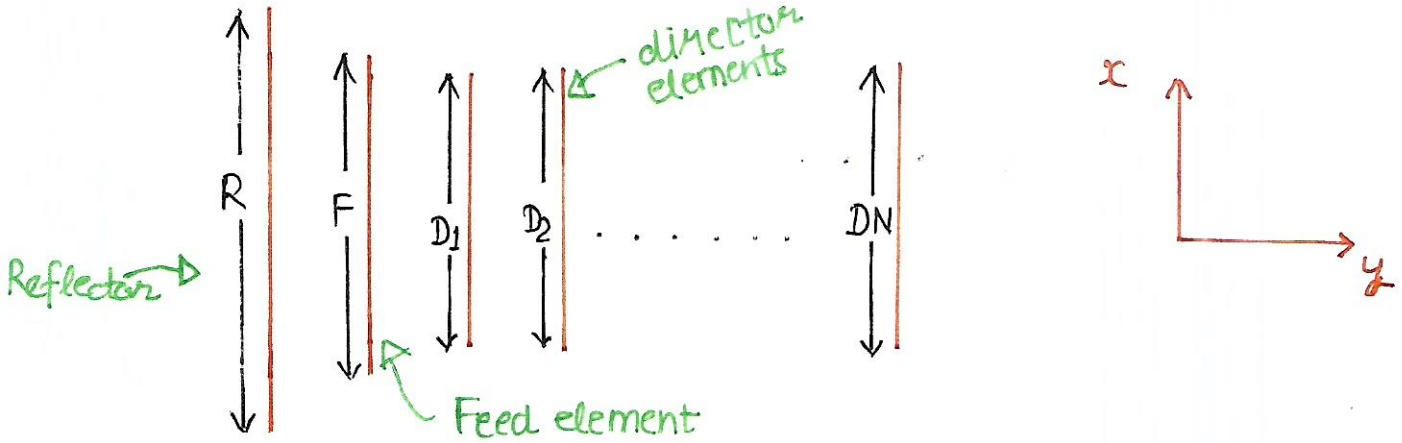


#Yagi Antenna-

Yagi Antenna is one of the most brilliant antenna design. It is simple to construct and has a high gain, typically greater than 10dB. The Yagi Antennas typically operates in the HF to UHF bands (about 3MHz to 3GHz), although their bandwidth is typically small, on the order of few percent of the center frequency. You are very familiar with this antenna, as they sit on the top of roof everywhere.



The basic geometry of a Yagi-Uda Antenna is shown below.



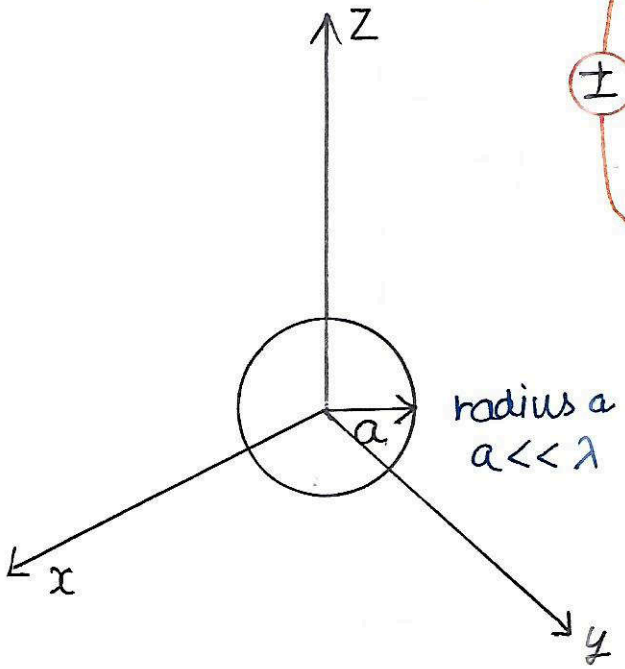
The Yagi Antenna consists of a single feed or driven element, typically a dipole or a folded dipole antenna. This is the only member of the above structure that is actully excited (a source of V is applied).

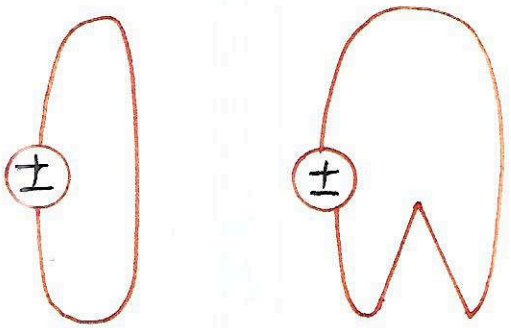
The rest of the element are parasitic - they reflect or help to transmit the energy in a particular direction.

- Invented in Japan.
- In 1926.
- By Shintaro Uda

Loop Antenna -

The Loop Antcna is a small closed loop.





→ Loop lies in x-y plane.

These antennas have low radiation resistance and high reactance, so their impedence is difficult to match to a transmitter. As a result, these antennas are most often used as receiver antennas, where impedence mismatch loss can be tolerated.

Since the loop is electrically small, the current within the loop can be approximated as being constant along the loop, so that $I = I_0$

Field

$$E_\phi = \frac{\eta k^2 a^2 I_0 e^{-jkr} \sin\theta}{4r}$$

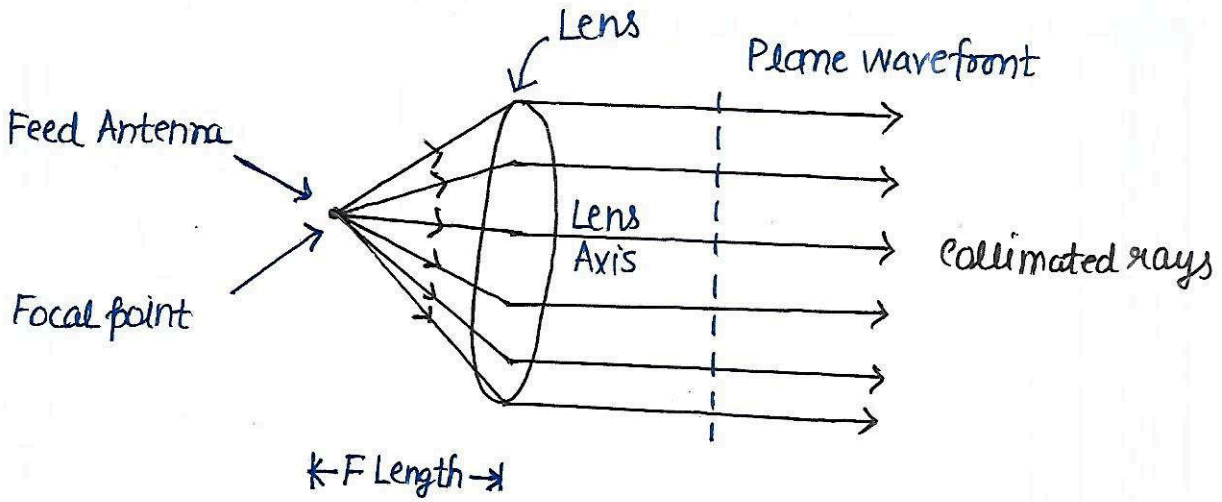
$$H_\theta = \frac{-E_\phi}{\eta}$$

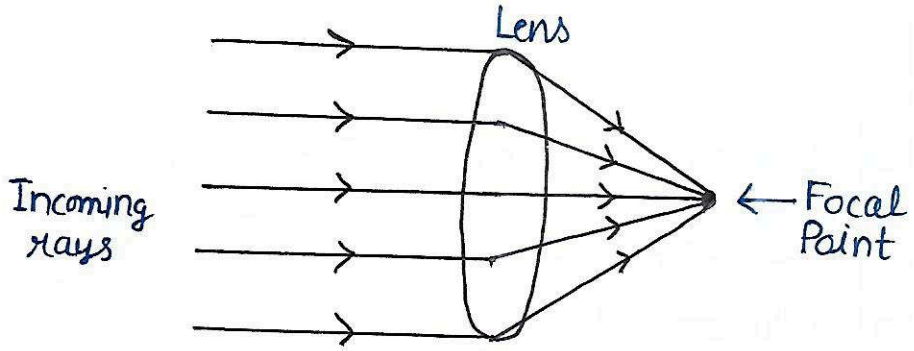
Radiation resistance

$$R_r = \left(\frac{177NS}{\lambda^2}\right)^2$$

Lens Antenna-

- Lens Antenna is an antenna which consists of an Electromagnetic lens with a feed.
- Lens Antenna is a 3D EM device whose refractive index is different from unity. It acts as a glass lens in optics.
- A lens antenna in association with a primary feed antenna in transmitting mode is shown in figure below.



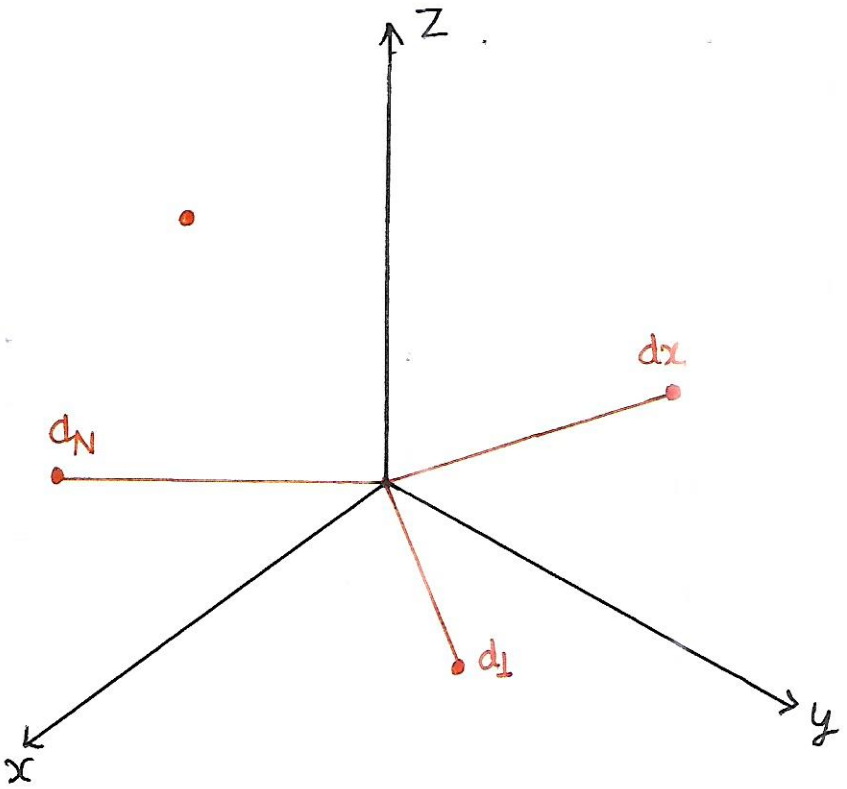


Lens Antenna

- Dielectric Lens (Delay Lens)
  - Outgoing EM waves or rays are collimated and are retarded or delayed by a lens material or media.
    - Metallic
    - Non-Metallic
- Metal Plane Lens
  - Outgoing wavefront is speeded up by the lens material.

Antenna Array-

- An Antenna array is a set of N spatially separated anteenas. The no. of antennas in an array can be as small as 2, or as large as several thousand. In general, the performance of an array increases with the no. of antennas in the array.
- The drawbacks of course is increased cost, size and complexity.
- Location $d_n = [x_n, y_n, z_n]$
- The position of the elements in the phase array are illustrated in the following figure.



Standing wave ratio and reflection cofficient-

For a radio (tx and Rx) to deliver power to an antenna, the impedence of radio and transmission line must be well matched to the antenna's impedence. The parameter VSWR is a measure that numerically describe how well the impedence matched to the radio or transmission line it is connected to.

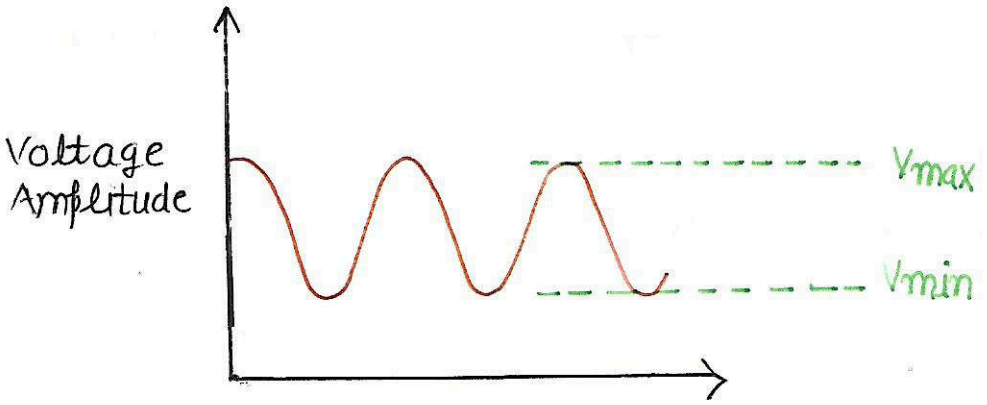
VSWR stands for Voltage standing wave ratio, and is also referred to as standing wave ratio(SWR). VSWR is a function of the reflection cofficient is given by Γ, then the VSWR

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|}$$

- Bandwidth Requirement -

100 - 200 MHz VSWR < 3

The VSWR is less than 3.0 over the specified frequency range; This VSWR specifications frequency also implies that the Reflection cofficient is less than 0.5 (i.e. $\Gamma < 0.5$) over the quoted frequency range.



VSWR	Γ	Reflected Power(%)	R.P(dB)
1	0	0	$-\infty$
5	0.6	44.0	-3.52
8	0.7	60.5	-2.18
9	0.800	64.0	-1.94
10	0.818	66.9	-1.74
15	0.875	76.6	-1.16
20	0.905	81.9	-0.87
50	0.961	92.3	-0.37

UNIT - 3

PROPAGATION OF RADIO WAVE

Radio wave -

- Radio wave are a type of EM Radiation with wavelengths in Electromagnetic spectrum longer than infrared light.
- 30 Hz to 300 GHz
- Radio waves are generated artificially by Tx and Rx by using antennas.
- First predicted by Maxwell in 1867.
- Marconi developed first practical radio wave around 1894-1895.
- Radio waves in a vaccum travel at the speed of light (c).

• Propagation characteristics - Radio waves are more widely used for communication than other Electromagnetic waves mainly because of their desirable propagation property, stemming from their large wavelength.

* Line of sight refers to radio waves that travel in a straight line.
* Radio waves can reach points beyond the line of sight by diffraction and reflection. It is called Indirect propagation.
* Ground waves; at lower frequencies below 2 MHz in the medium waves and longwave bands, due to diffraction vertically polarized radio wave can bend over hills and mountains, and propagate beyond the horizon, travelling as surface waves which follow the contour of the Earth.

• Biological and Environment Effect -

- Non Ionization Radiation
- RW does not have enough energy to separate e⁻ from atoms which causing chemical reactions or DNA damage.
- The main effect of absorption of radio waves by material is to heat them.
- Looking into a source of radio wave can pentrate the eye and heat the lens enough to cause cataracts
  ↑ Clouding of Lens.

Eccles and Larmor Theory –

Let the ionization medium contains N_e electrons for cubic centimeter, each of charge e and mass m.
When the EM wave travel through such a medium, the motion of the e^- are influenced by $\vec{E}$ force of the passing wave.

Influened – $\vec{E}$ field

Fluctuating Intensity $E = E_0 \sin pt$

(p ↑ Angular frequency)

Electrons will viberate. Eq^n of motion of an e^- as :

$$m\left[\frac{d\xi}{dt}\right] = e E_0 \sin pt$$

($\frac{d\xi}{dt}$ ↑ Instantaneous velocity of the e^- in the direction of $\vec{E}$ Field.)

This motion is integrated

$$\xi = \left(\frac{e}{mp}\right) E_0 \cos pt$$

Multiply by $N_e \cdot e$ in both sides

$$N_e \cdot e \cdot \xi = \left(\frac{E_0 N_e e^2}{mp}\right) \cos pt$$

(N_e ↑ N_e is no. of e^- per cm^3.)

dielectric constant → $K\left(\frac{\delta E}{4\pi}\right) dt = K\left(\frac{E_0}{4\pi}\right) p \cos pt$

$$\left[K - \left(\frac{4\pi N_e e^2}{mp^2}\right)\left(E_0 \frac{p}{4\pi}\right) \cos pt \right]$$

The presence of the electrons has thus the effect of reducing the dielectric constant of the medium from K to $\left(\frac{K - 4\pi N_e e^2}{mp^2}\right)$. In empty space, $K=1$.

$$\mu = c/\sqrt{1-(4\pi N_e e^2/m\omega^2)}$$

$$\mu = \frac{c}{\mu}$$

$$\boxed{\mu = \sqrt{1-(4\pi N_e e^2/m\omega^2)}}$$

Appleton-Hartee Formula-

Appleton first applied the theory in 1925, and in 1927 he generalized Lorentz's treatment and derived formula for dispersion.
According to magneto-ionic theory, the complex reflection index M and the state of polarization R of the wave given by

$$M^2 = [\mu - (jck/\omega)]^2$$

$$= 1 + 2/[2(\alpha+j\beta) - \gamma T^2/(1+\alpha+j\beta) \pm \frac{\sqrt{\gamma T^4 + 4\nu L^2}}{(1+\alpha+\beta j)^2}$$

$$\text{And } R = \frac{hz}{hy} = \frac{-j}{\gamma L[1/(M^2-1) - (\alpha+j\beta)]}$$

μ = Refractive Index
K = Index of attenation
C = Velocity of EM wave in free space
N_e = Electron density
H = Magnetic Field in Guass
ω = Angular frequency
γ = Collision frequency

$$\alpha = -\frac{\omega^2}{\omega_0^2}$$

$$\beta = \frac{\omega\gamma}{\omega_0^2}$$

$$\nu = \omega\omega_H \cdot \omega_0^2$$

$$\nu_{L,T} = \beta\beta_L \cdot T/\beta_o^2 \cdot \beta_o^2 = 4\pi N_e \cdot e^2/m$$

Then

$$\mu^2 = 1 + \frac{1}{[\alpha - \gamma_T^2/2(1+\alpha)]} \pm \sqrt{\frac{\gamma_T 4}{4(1+\alpha)^2 + \gamma_L}}$$

In case of no external $\vec{B}$ Field

$$\gamma_T = \gamma_L = 0$$

$$\mu^2 = 1 + \frac{1}{\alpha} + 0 \approx 1 = \frac{\beta_o^2}{\beta^2}$$

i.e.

$$\frac{\beta^2}{\beta_o^2} = 1 = \frac{\beta_o^2}{\beta^2}$$

$$4\pi N_e e^2/m\beta^2 = 1$$

$$4\pi N_e e^2/m(2\pi f_c)^2 = 1$$

or $$\boxed{N_e = N_{max} = \pi m f_c^2/e^2 = 1.24 \times 10^{10} f_c^2 e/m^3}$$

Chapmann's Theory of Layer Formation-

A theoretical distribution of ionization as a function of height produced solely by the absorption of solar radiation by a single atmospheric constituents.

Sydney chapmann presented in 1931 a simple mathematical model for the formation of ionization layers which are based on the fact that energatic photons from the sun split air molecules into e- and positive ions.

Energy absorbed along the radiation path.

$$\frac{1}{\sec(\chi)} \frac{dI}{dz} = I_\infty \sigma \eta e^{-I\sec(\chi)}$$

If η e⁻-ion pairs are produced per unit of energy absorbed, the ion production rate

$$q(\chi, z) = I_\infty \sigma \eta n(z) e^{-I\sec(\chi)} \quad \ldots\ldots (1)$$

$$\frac{dq}{dz} = 0$$

$$\tau = \frac{1}{\sec(\chi)} = \cos(\chi)$$

$$q_m(\chi, z_m) = \frac{I_\infty \eta}{eH \sec(\chi)}$$

$$\boxed{q_m(0, z_m) = \frac{I_\infty \eta}{eH}}$$

Chapman variation $z' = \frac{(z - z_{m_0})}{H}$

$$\eta = \eta_0 e^{-\frac{z}{H}}$$

$$\eta = \eta_0 e^{\left(\frac{-z_{m_0}}{H}\right)} \cdot e^{\frac{(z - z_{m_0})}{H}}$$

$$\eta = \eta_0 e^{\frac{z_{m_0}}{H}} e^{-z'}$$

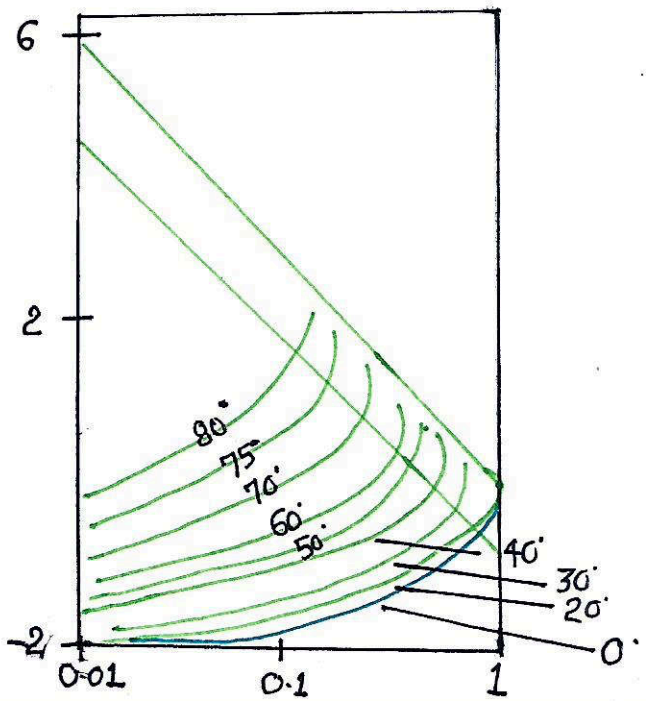
Put in eq^n (1)

$$q(\chi, z) = \frac{I_0 \eta}{H} \sigma \eta H e^{-\sigma \eta H \sec(\chi)}$$

$$\sigma \eta_0 H e^{\left(\frac{-z_{m_0}}{H}\right)} = 1$$

$$q(\chi, z') = \frac{I_0 \eta}{eH} e^{(1 - z' - \sec(\chi) e^{-z'})}$$

$$\boxed{q(\chi, z') = q_{m_0} e^{(1 - z' - \sec(\chi) \cdot e^{-z'})}}$$



• Pulse method for measuring the height of ionosphere region-

-The original idea hypothesized quite naturally that the Doppler width of the returned spectrum would be of order

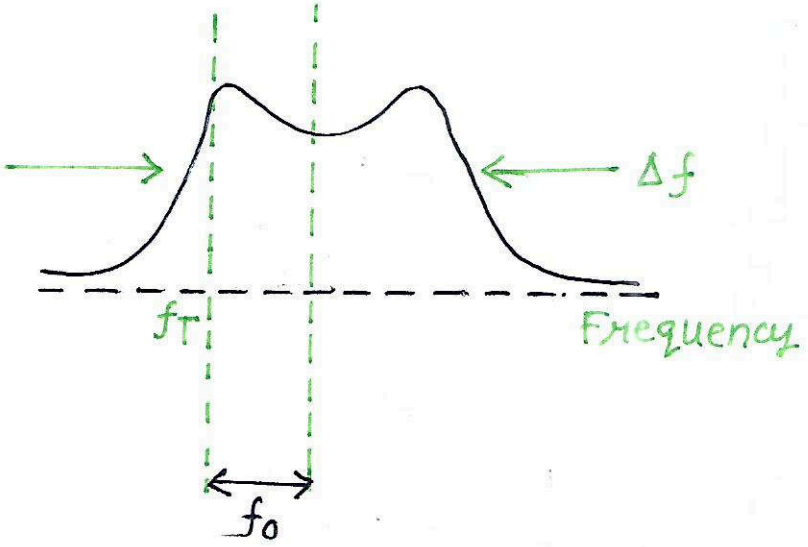
$$\delta f / f \cong \langle V \rangle_e / C$$

↳ Mean thermal electron velocity.

-The First experiment by Bowles showed the Doppler width to be of order $\langle V_i \rangle / C$ $\langle V_i \rangle$ is the ion thermal speed.

- This is explained by the fact that, when the probing wavelength is greater than the debye length, the e^- are linked to the ions through the debye cloud around them, and the part of the scattered signal is characterised by properties

$$E_\perp = B \times V_i$$



- The radar scatters from the irregularities where

$$K_T = K_S + K_m$$

K_T → Transmitted Wave

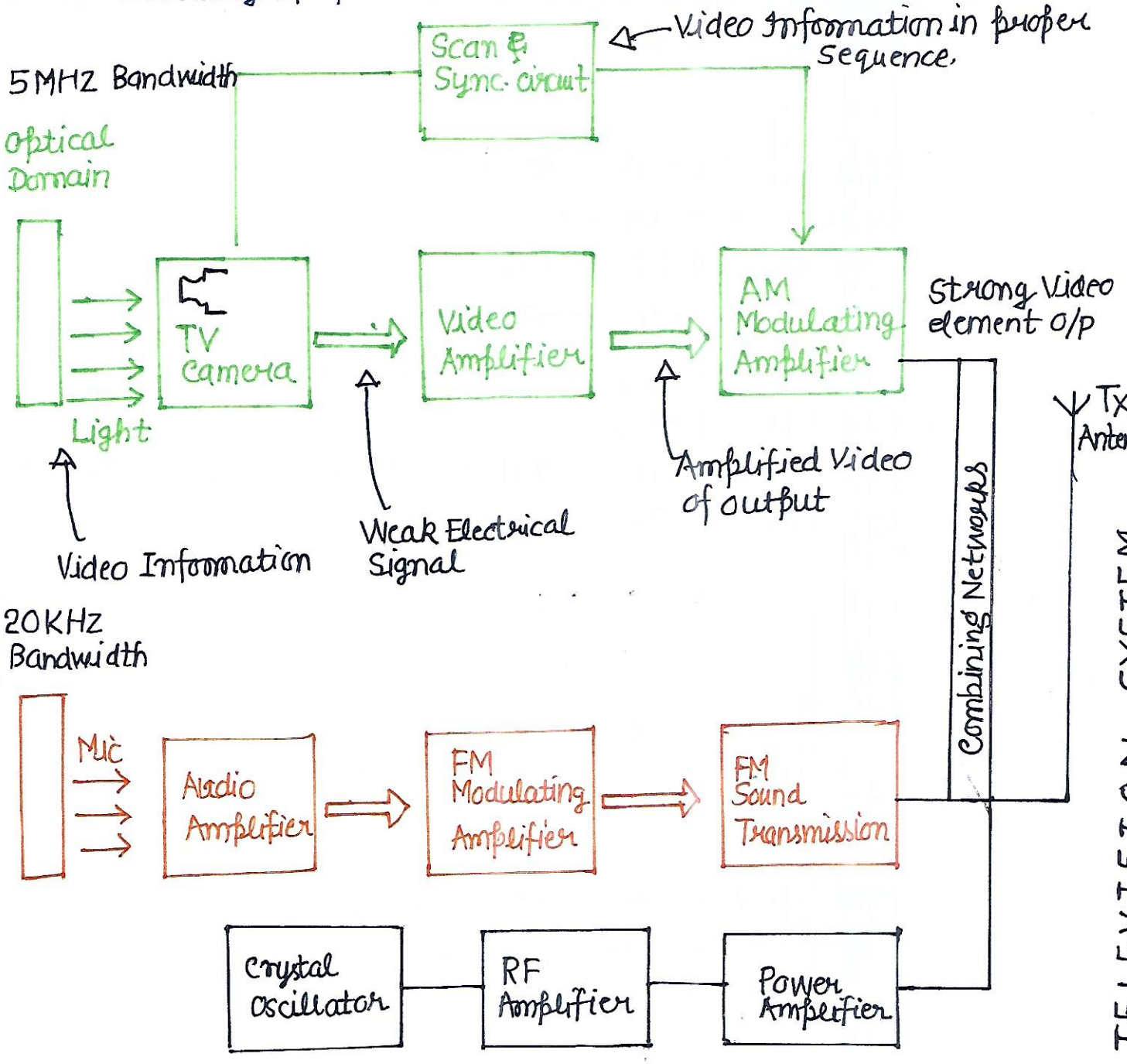
K_S → Scattered Wave

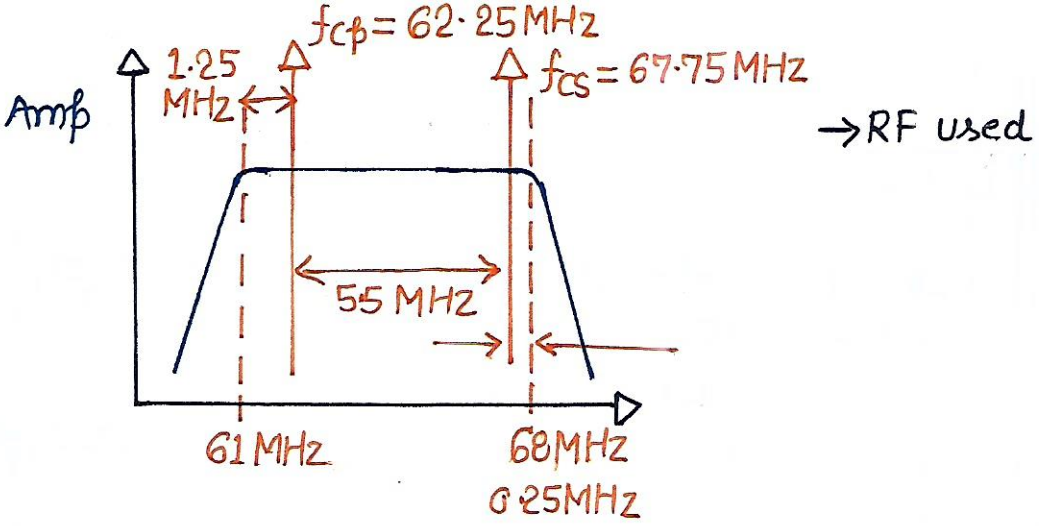
$$K_m = 2K_T$$

$$\lambda_m = \frac{4\pi}{\lambda}$$

#General Principle of Image transmission-

- There are almost infinite elements/Pixels in any picture, so information of picture is very complex.
- Each pixels has different level of brightness.
- Information is given as a function of two variables which are space and time.
- Ideally, there are infinite pixels of information in optical domain.
- practically, conversion of optical element into electrical form is done, and its transmission is carried out element by element.
- Scanning of pixels is done very fastly.





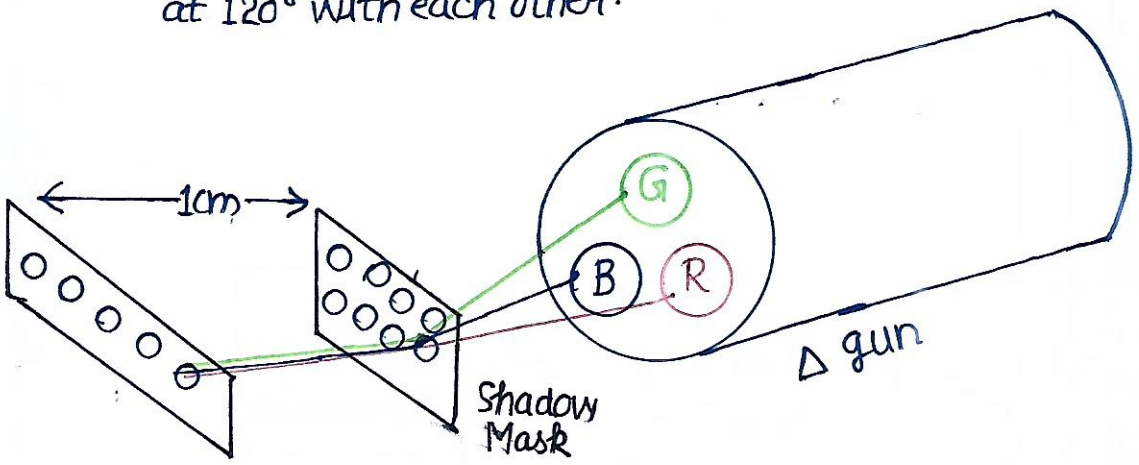
- **Basic parameters–**
  - It has a video information, in between black and white with shades of gray.
  - It transmits channels in VHF and UHF.
  - Picture and sound signals are modulated on RF carrier to reduce antenna size.

#Colour Television Picture Tube–

- Receiver of colour TV receives Y, B-Y, R-Y video signals.
- Y is luminance signal.

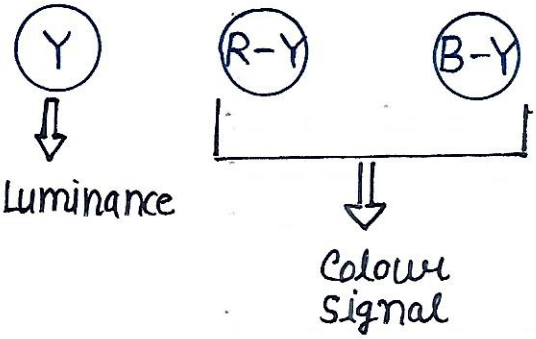
$$Y = 0.3R + 0.59\alpha + 0.11\beta$$

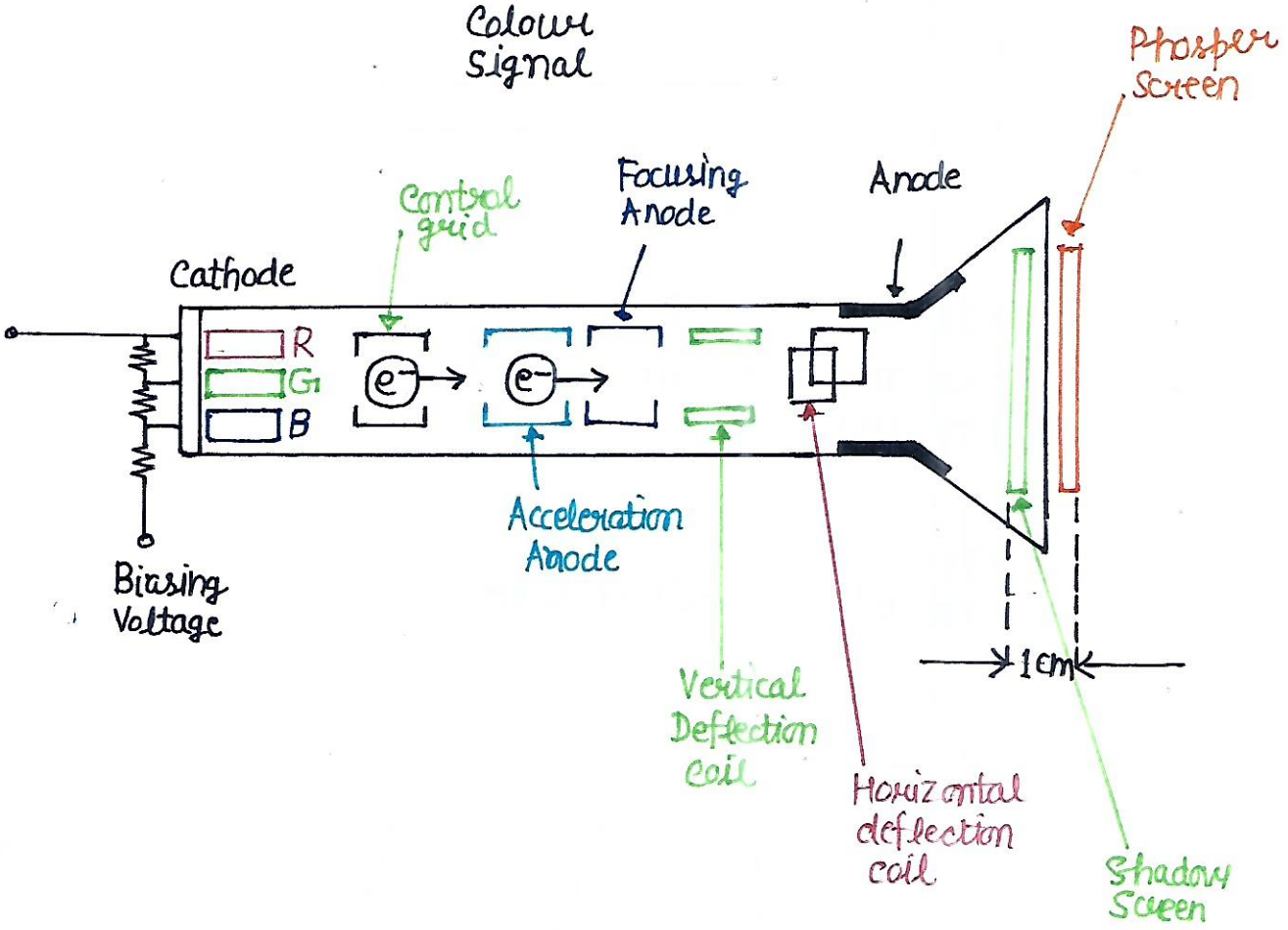
- B-Y, R-Y is having information regarding colour.
- Monochrome TV only sends information of Luminance by sending Y signal only.
- At Receiver of colour TV, in delta gun, three guns are placed at 120° with each other.



- The tube employs a screen where three colour phosphor dots are arranged in groups known as triads.

- The triads are repeated and depending on the size of the picture tube, approximately 1 million such dots forms 3.33 lac triads, deposited in glass tube.
- Ratio of electron passing through the shadow mask is 20%. Other 80% electrons are dissipated as heat.
- Signals

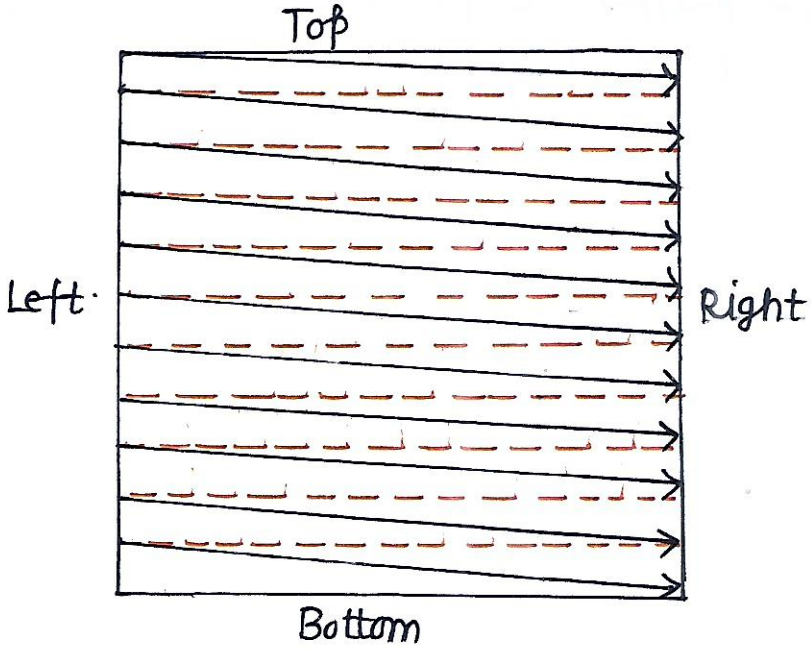




- Sawtooth waves are given to the coils.
- With vertical deflection, frequency will be 50 Hz and 15625 Hz for horizontal deflection.

Image Scanning Sequence-

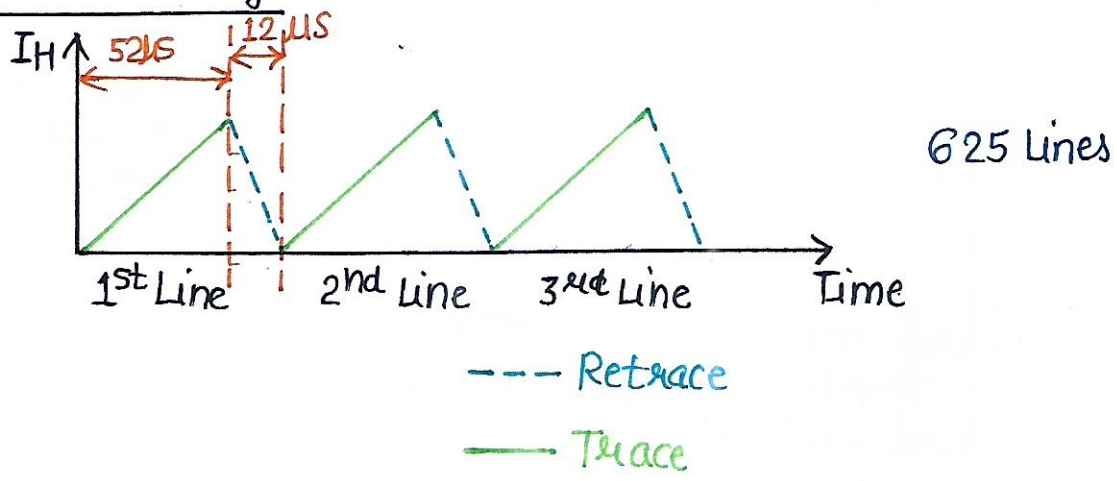
- TV picture element of the frame produced by a scan process so that illusion of continuity is created.



- 16 fps scanning is required.
- Human eyes can't separate two frames if scanning with 1/16th second per frame.

- Parameters-
  - 25 fps
  - Horizontal and Vertical scanning

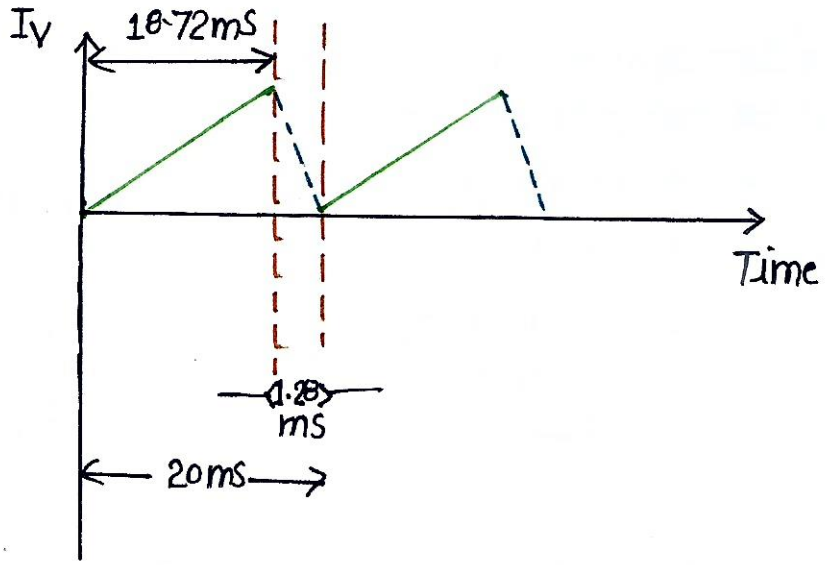
- Horizontal Scanning-



$$f_H = \frac{1}{T_H}$$

$$f_H = \frac{1}{64} = 15625\ Hz$$

• Vertical Scanning -



$$f_V = \frac{1}{T_V}$$

$$f_V = \frac{1}{20ms}$$

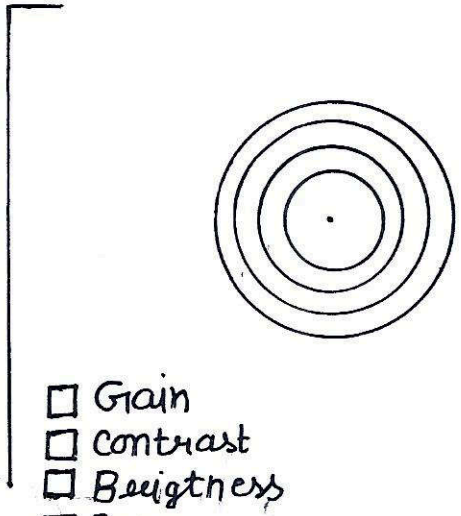
$$\boxed{f_V = 50Hz}$$

#RADAR-

- Radar is a detection system that uses radio waves to determine the range, angle or velocity of objects.
- It can be used to detect the aircraft, ships, spaceship, guided missiles, motor vehicles, weather formations and terrain(TRF)
- A radar system consists of a transmitter producing Electromagnetic waves in the radio or microwave domain, a transmitting antenna, a receiver antenna and a processor to determine properties of object(s).
- Radio waves (pulsed or continuous) from the transmitter reflect off the object and return to the receiver, giving information about the object's location and speed.
- It was secretly developed by many countries during the second world war.
- The term RADAR was given by US Navy in 1940.
- Radio Detection and Ranging.

• Working-

Radar sends an electromagnetic pulse, transmitted through the atmosphere. Object which are called targets in the path of pulse will scatter most of the energy but some will be reflected back to the RADAR. Reflected radiation will go to the receiver. If there are more targets than scattered signal combine to form a stronger signal. The value of range, position, direction and altitude is measured by the scattered signals.

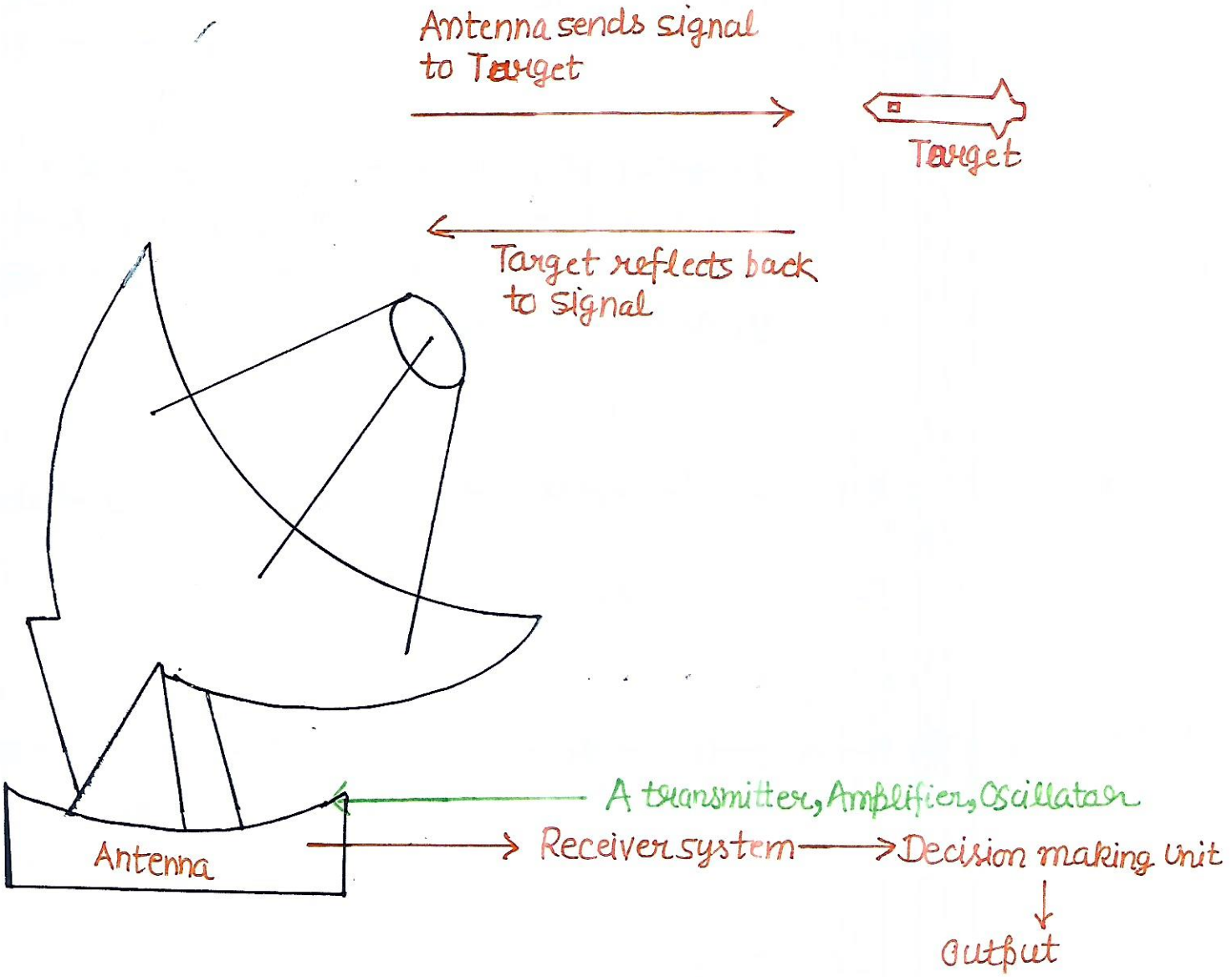


RADAR SYSTEMS

- **Use or Applications :**

(1) It determine the target position in military.
(2) It warns aircraft about obstacles in their path, in aviation.
(3) It warns to prevent the collisions of space(craft) in marine and aviation.
(4) Meteralogists and in weather forecast, it determine the weather possibilities.
(5) It is used by cops to determine the vehicle speed.
(6) Geologists to determine the composition of earth's crust.

- **BASIC ARRANGMENT OF RADAR SYSTEM :**



→ Transmitter – It generate a signal using a waveform generator and then amplified it using power amplifier.

→ Waveguide – The waveguide are transmission lines for transmission of the Radar signals.

→ Antenna – Anteena can be a parabolic reflector, planer array, or electronically steered phase array.

→ Duplexer – A duplexer allows the antenna to be used as a transmitter or a receiver. It can be gaseous device that would produce a short circuit at the input to the receiver when transmitter is working.

→ Receiver – It can be a superhetrodyne receiver or any other which consists of a processor to process the signal and detect it.

→ Threshold Decision – The output of the receiver is compared with a threshold to detect the presence of any object or target. If the output is below to threshold, the presence of noise is assumed.

Range measurement of Radar –

Range is the distance from the radar sight to the target measured along the line of sight.

$$v = \frac{S}{t} \quad \ldots\ldots (1)$$

$$C_0 = \frac{2R}{t} \quad \ldots\ldots (2)$$

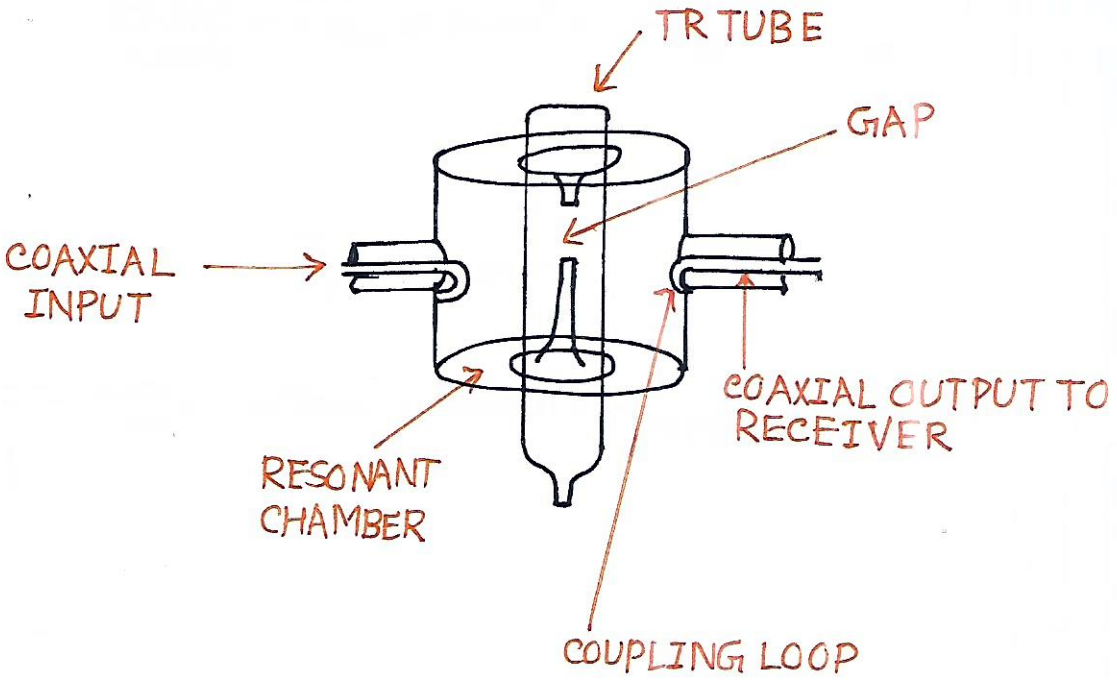
The factor of the two eqn comes from the observation that the radar pulse must travel to the target and back before detection or twice the range.

$$R = \frac{C_0 t}{2}$$

$$C_0 = 3 \times 10^8 \text{ m/s}$$

#DUPLEXER-

- The duplexer permits the same antenna to be used for both transmission and receiver.



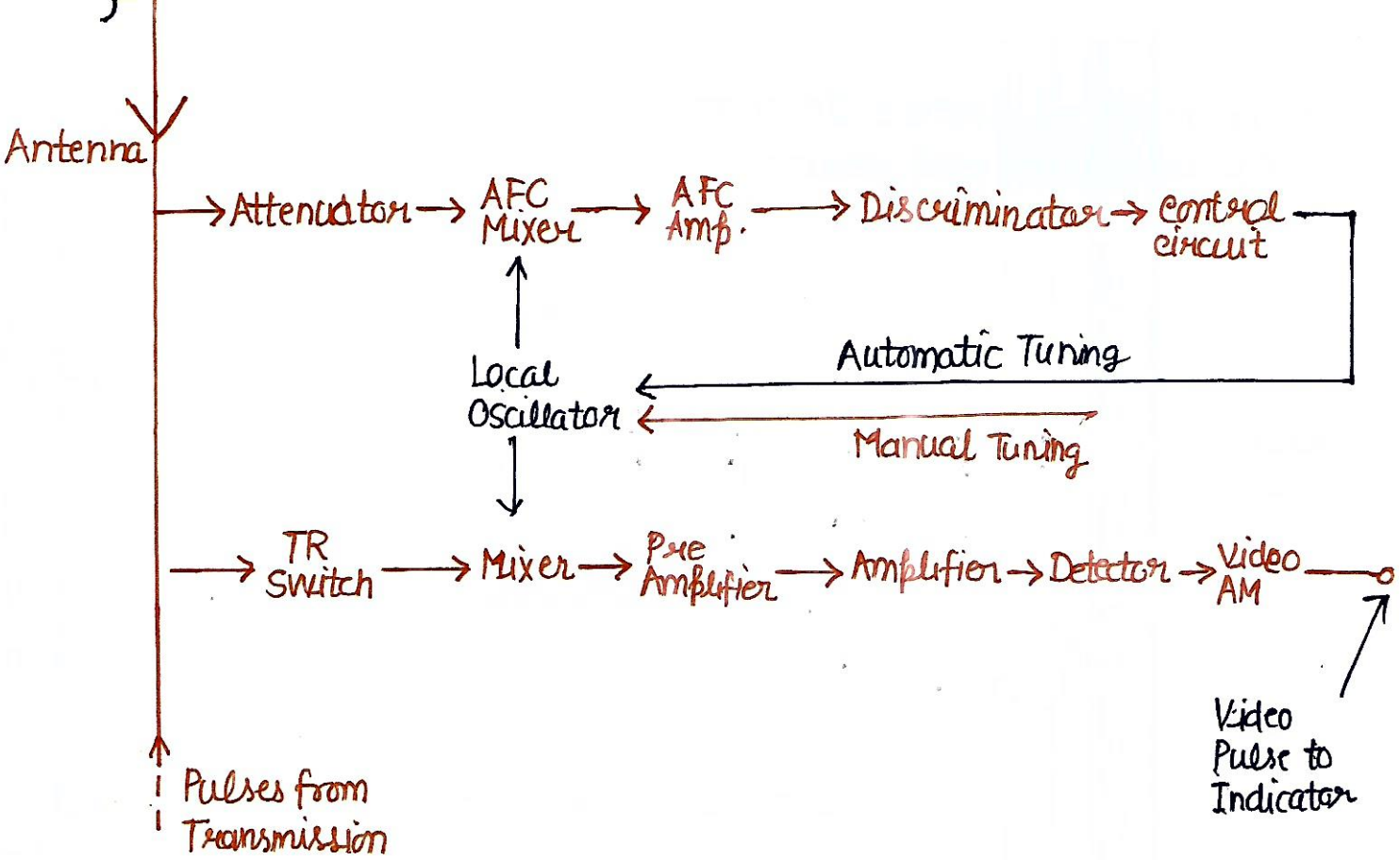
- Figure shows a typical TR switch for use in coaxial circuits. It consists of a resonant cavity associated with a special cold cathode tube. The tube itself consists inert gas at low pressure.
- The switch acts almost as a short circuit and permits little transmission of energy from input to output line.

#RADAR Receiver-

- Radar receivers are basically conventionally UHF or microwave super-phetrodyene receiver having low noise figure and have appropriate high intermediate frequency and video bandwidth necessary to reproduce the radar pulse. Typical values of intermediate frequency are 30 MHz to 60 MHz.

- It is desirable to use an AFC system which system automatically control the frequency of the local osillator.




Antenna
Attenuator
AFC Mixer
AFC Amp.
Discriminator
control circuit
Automatic Tuning
Local Oscillator
Manual Tuning
TR Switch
Mixer
Pre Amplifier
Amplifier
Detector
Video AM
Video Pulse to Indicator
Pulses from Transmission

LASER Physics – B

- UNIT 1
  MODULALION OF LIGHT
- UNIT 2
  OPIICAL SOURCE AND DETECTORS
- UNIT 3
  FIBRE OPTICS
- UNIT 4
  HOLOGRAPHY

The Electo-optic Effect

#Electro-optic Effect-

Light propagation through Isotropic and Anisotropic Medium. When light propagate through Anisotropic media, we found that, in general, the state of polarization of the light beam may change as it propogates through this medium. If light propagate through crystals in the presence of External electric Field, it alter the refrective index of the crystal and thus byfringence induces. This Effect is known as Electro-optic Effect.

When an $\vec{E}$ is applied across an optical medium, the distribution of e- within it is distorted so that the polirizability and hence the refrective index of the medium changes anisotropically.

The change in refrective index as a function of the applied Field can be obtained from an equation of form

$$\Delta\left(1/n^2\right) = \underbrace{r E + P E^2}_{\text{Pocket Effect}}$$

r → Linear cofficient

P → Kerr Effect (↳ quardratic cofficient)

Pocket effect, the precise effect of the applied $\vec{E}$ depends on the crystal structure and symmetry of the material under consideration.

Thus

$$\Delta\left(\frac{1}{n^2}\right) = -\frac{2\Delta n}{n^3} = r_{63} E_z \qquad \left(\because \begin{array}{l} n_x = n_o + \frac{n_o^3}{2} r_{63} E_z \\ n_y = n_o - \frac{n_o^3}{2} r_{63} E_z \end{array}\right) \quad \ldots\ldots(1)$$

Kerr Constant P was very small where r_{63} is appriciate electro-optic Effect cofficient in KDP

Let us consider a beam of plane polarized light propagating in z direction through a crystal such as KDP with a plane of polarization at 45° to the induced axes x', y' as.

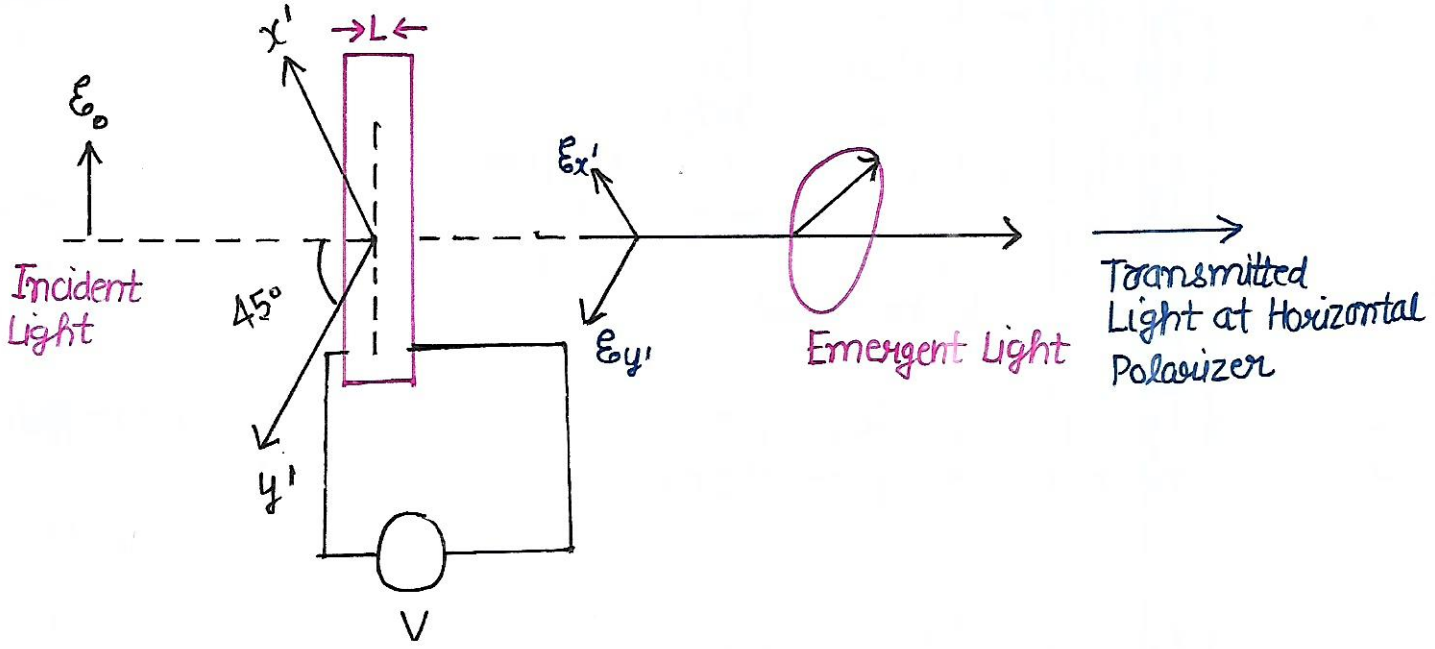
If the incident beam

$$\mathcal{E} = \mathcal{E}_0 \cos(\omega t - Kz)$$

$$\mathcal{E}_x = \frac{\mathcal{E}_0}{\sqrt{2}} \cos(\omega t - Kz)$$

$$\mathcal{E}_y = \frac{\mathcal{E}_0}{\sqrt{2}} \cos(\omega t - Kz) \qquad \ldots\ldots\ldots\ldots (1)a$$

Thus If a crystal of L thickness, the phase change of the two components will be



The induced birefringence occurs in a plane normal to the applied $\vec{E}$.

$$\Phi_{x'} = \frac{2\pi}{\lambda_0} n_{x'} L$$

$$\Phi_{y'} = \frac{2\pi}{\lambda_0} n_{y'} L$$

use eq^n(1)

$$\phi_{x'} = \frac{2\pi}{\lambda_0} L n_0 \left(1 + \frac{1}{2} r n_0^2 \mathcal{E}_z\right)$$

$$\phi_{y'} = \frac{2\pi}{\lambda_0} L n_0 \left(1 - \frac{1}{2} r n_0^2 \mathcal{E}_z\right) \qquad \ldots\ldots\ldots (1)b$$

$$\phi_{x'} = \phi_0 + \Delta\phi_0$$

$$\phi_{y'} = \phi_0 - \Delta\phi_0$$

Where

$$\Delta\phi_0 = \frac{\pi}{\lambda_0} L r n_0^3 \varepsilon_z = \frac{\pi}{\lambda_0} r n_0^3 V \quad \ldots\ldots\ldots (2)$$

In eq^n(2), we have taken ε_z to equal V/L where V is applied voltage.

Net retardation

$$\Phi = \phi_{x'} - \phi_{y'} = 2\Delta\phi_0 = \frac{2\pi}{\lambda_0} r n_0^3 V$$

and the emergent light will be in general elliptically polarized.

From eq^n's 1(a) & 1(b)

$$\varepsilon_{x'} = \frac{\varepsilon_0}{\sqrt{2}} \cos(\omega t + \Delta\phi)$$

$$\varepsilon_{y'} = \frac{\varepsilon_0}{\sqrt{2}} \cos(\omega t - \Delta\phi)$$

$\Delta\phi$, Phase shift depends directly on the applied voltage V.

Transmitted Electric Field

$$\varepsilon = -\frac{\varepsilon_0}{2}[\cos(\omega t + \Delta\phi) - \cos(\omega t - \Delta\phi)]$$

$$\varepsilon = -\varepsilon_0 \sin\Delta\phi \sin\omega t$$

And

$$I = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} \varepsilon^2 dt \quad \left[\because T = \frac{2\pi}{\omega}\right]$$

$$I = I_0 \sin^2\Delta\phi = I_0 \sin^2\left(\frac{\Phi}{2}\right)$$

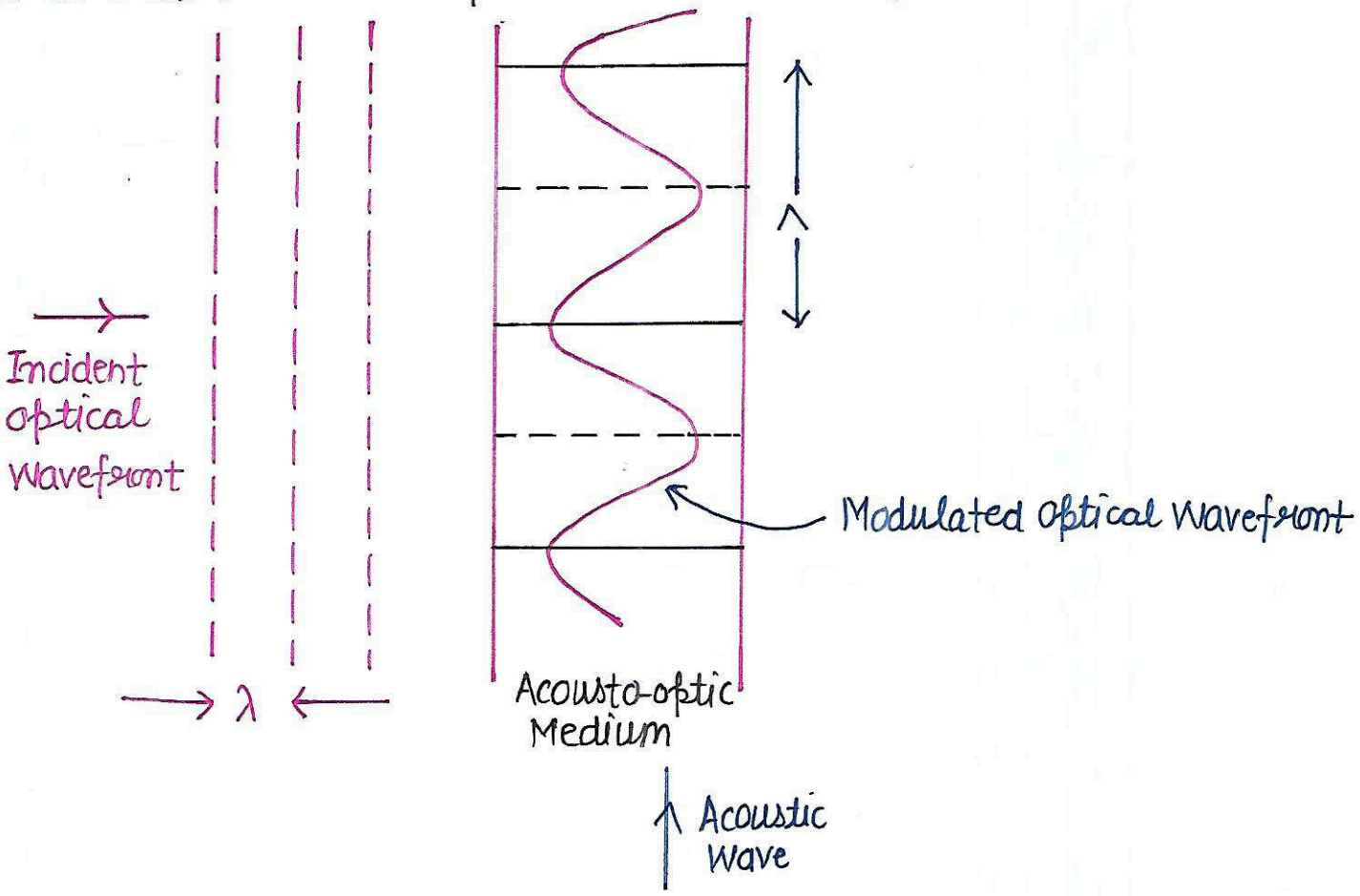
↳ Incident on Electrooptic Crystal

$$\boxed{\frac{I}{I_0} = \sin^2\left(\frac{\pi}{\lambda_0} r n_0^3 V\right)} \Rightarrow \text{Such system is Electrooptic.}$$

#Acousto Optic Effect -

The Acousto optic effect is the change in the refractive Index of a medium caused by the mechanical strains accompanying the passage of a surface acoustic (strain) wave along the medium. The strain and hence the refractive index varies periodically with a wavelength Λ equal to that of a acoustic wave. The refractive index changes are caused by the photo-elastic Effect, which occurs in all materials.

Let us consider the case of a monochromatic Light wave.



The solid horizontal lines represent Acoustic wave peaks and dashed horizontal lines represent acoustic wave troughs. The portions of the wavefront near to a pressure peak will encounter a high refractive index and therefore advance with a lower velocity than those portion of the wavefront which encounters pressure minima. The acoustic wave velocity is much less than the light wave velocity.

As light wave propagates in the direction normal to the local wavefront, almost all the wave element will suffer a change in the direction leading to a redistribution of the light flux, which concentrate near regions of

compression. In effect, the acoustic wave setup a diffraction pattern grating within the medium so that optical energy is diffracted out of the incident beam into the various orders.

There are two main cases of interest.

(A) Raman Nath Regime

(B) Bragg Regime.

(A)

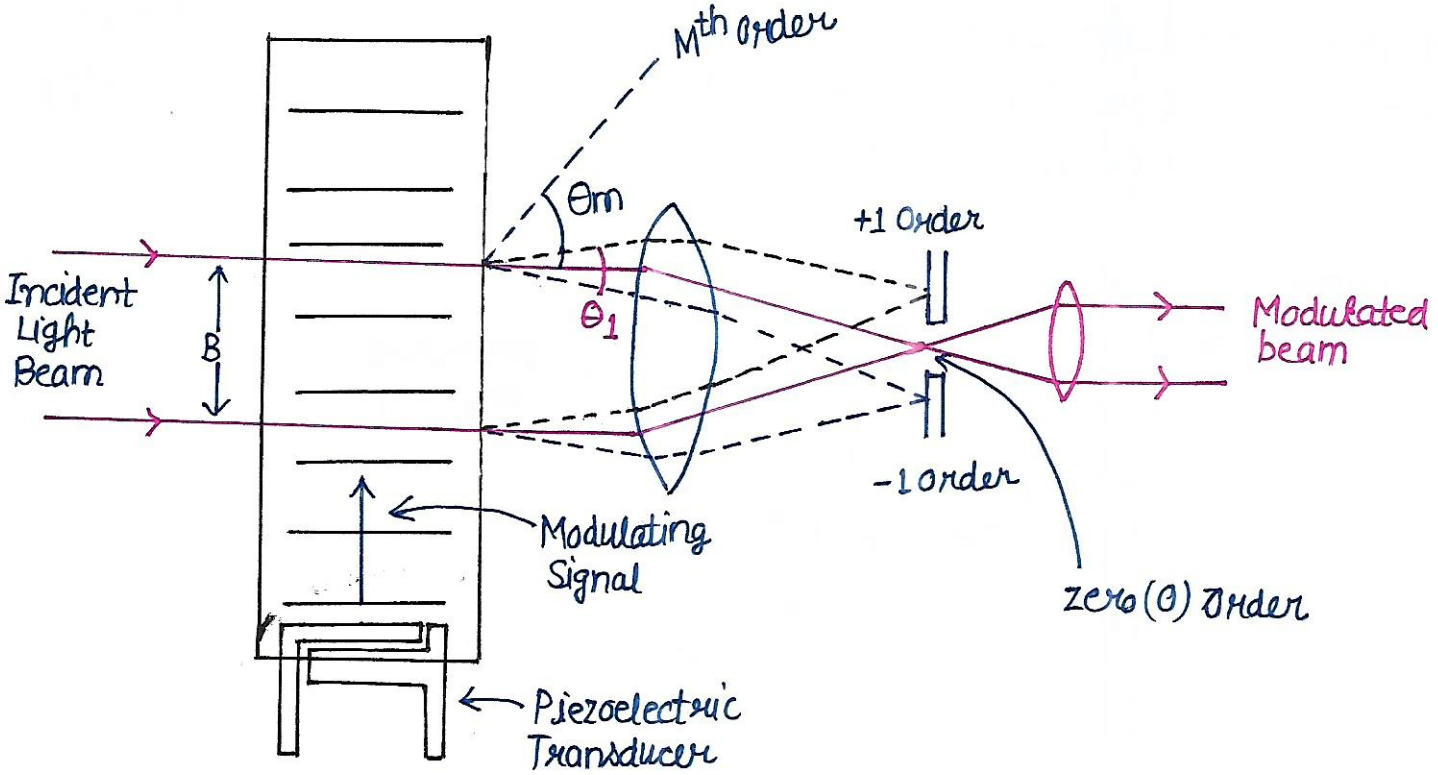


Figure (Acousto Optic Modulator)

In the raman nath regime, the width of the acoustic beam is so small that is diffracted light suffers no further redistribution before leaving the modulator. The light is then diffracted as though from a simple plane grating such that

$$m\lambda = \Lambda \sin\theta_m$$

where $m = 0, \pm 1, \pm 2$

θ_m = Angle of diffraction

The irradiance I of the light in these orders depends on the ruling depth of a acoustic grating which is related to the amplitude of the

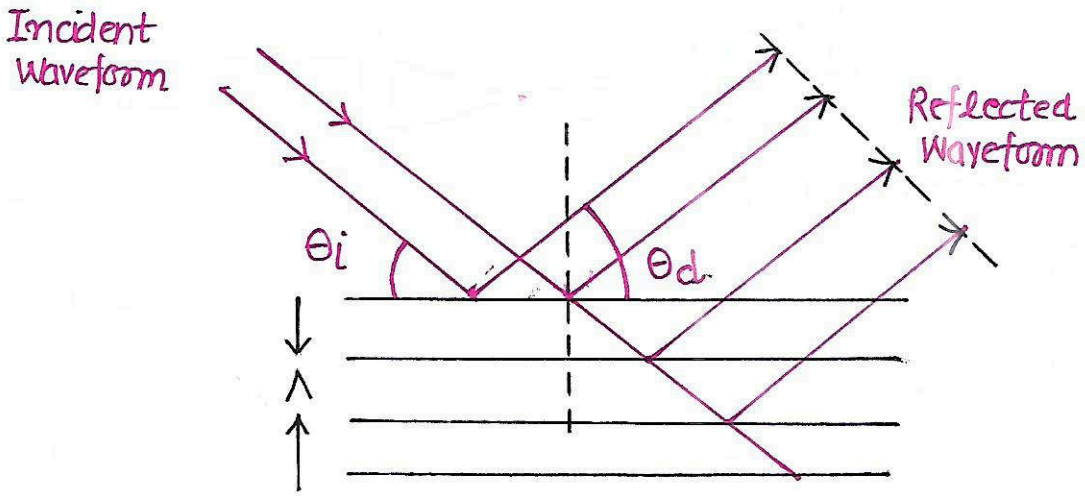
acoustic grating. The fraction of light removed from the zero order beam.

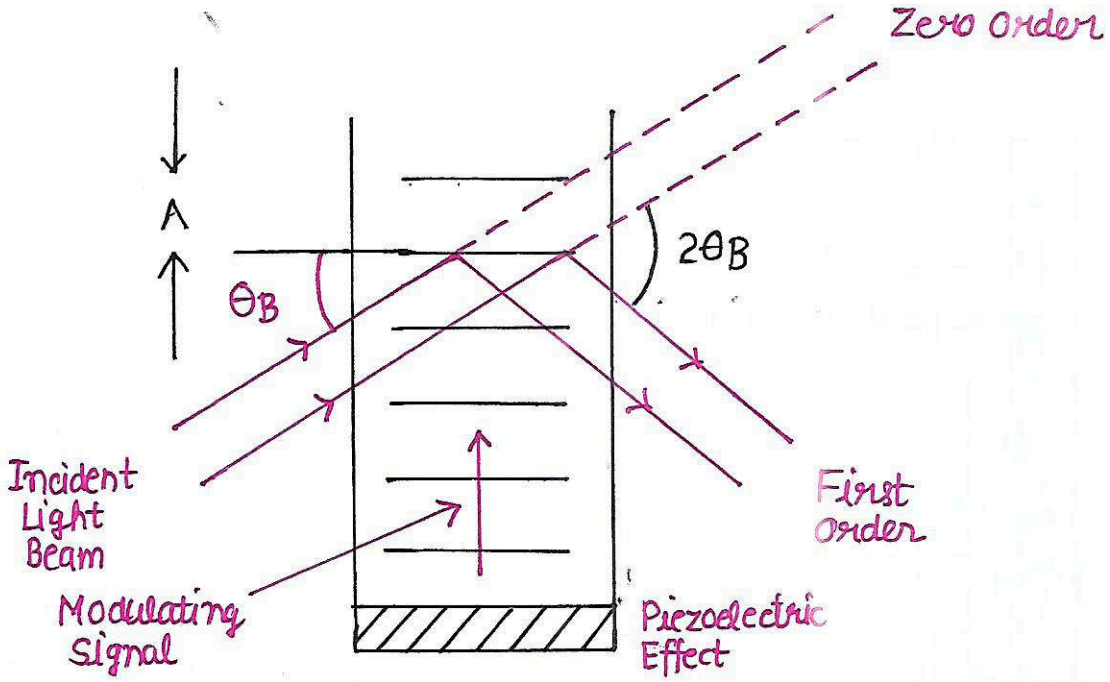
← Transmitted Irradiance.

$$\eta = \frac{I_o - I}{I}$$

(B)

The Physical basis of the Bragg regime is that light diffracted from the incident beam is extensively rediffracted before leaving the acoustic field. Under these conditions, the acoustic field acts very much like a thick diffraction grating, that is a grating made up of planes rather than lines. The situation is then very similar to that of Bragg diffraction of X-rays from planes of atoms in a crystal.





Conditions ;

(i) $\theta_i = \theta_d$

(ii) $\sin\theta_i + \sin\theta_d = \frac{m\lambda}{\Lambda}$

$$m = 0, 1, 2, \ldots\ldots$$

$$\sin\theta_i = \sin\theta_d = \frac{m\lambda}{2\Lambda}$$

The diffraction is similar to that obtained with a plane grating, but only for special angles of incident.

Although in the simplified theory given above strong scattering can take place when m is equal to any positive Integer, a more rigorous treatment.

The modulation depth $(I - I_0)/I_0$ or diffraction efficiency η, in this case can theoritically equal 100% in contract to about 34% for the Raman Nath case.

$$\eta = \sin^2\left(\frac{\theta}{2}\right)$$

$$\eta = \frac{\sin^2\phi}{2}$$

where $\phi = \frac{2\pi}{\lambda} \cdot \frac{\Delta n L}{\cos\theta_B}$ (Δn: Amplitude of Refractive Index fluctuation; L: Length of modulator)

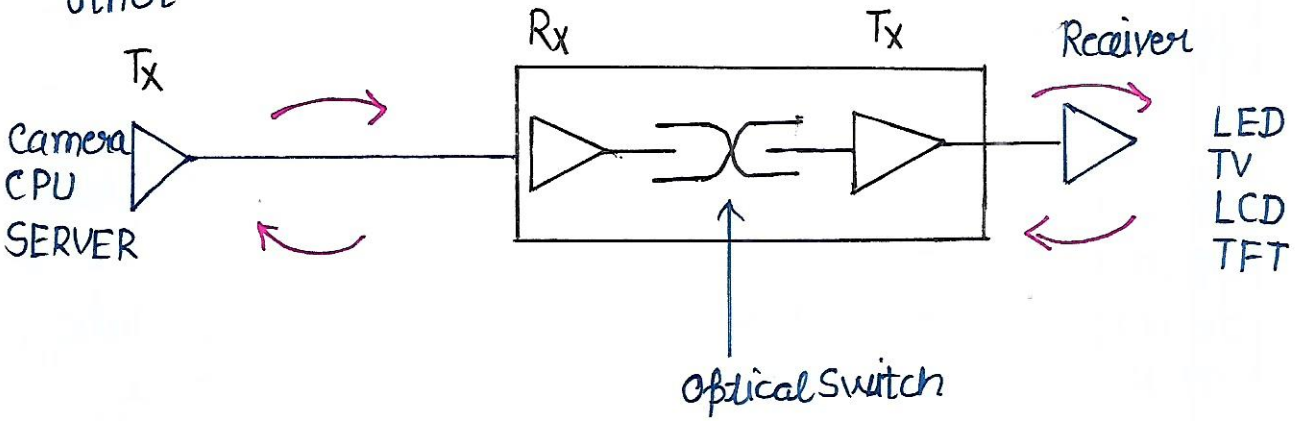
The acoustic waves, which create the diffraction grating, are of course moving through the medium, and, as consequence, the diffracted wave behaves as if it had been reflected from a mirror moving with the same velocity as the grating, and therefore appears to originate from a (mirror) source moving as twice the mirror velocity. Thus the frequency is of reflected beam is changed by the Doppler Effect, is given by

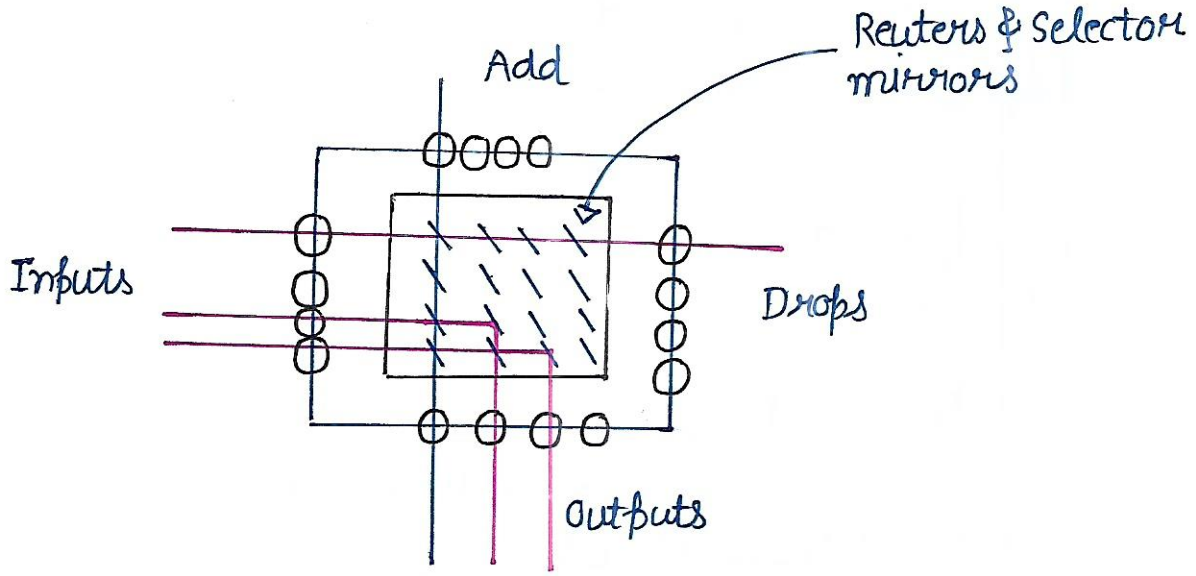
$$\nu' = \nu_0\left[1 \pm 2v_x\left(\frac{c}{n}\right)\right]$$

Frequency shift $\Delta\nu = \nu' - \nu_0 = \pm 2\nu_0 v_x \frac{n}{c}$

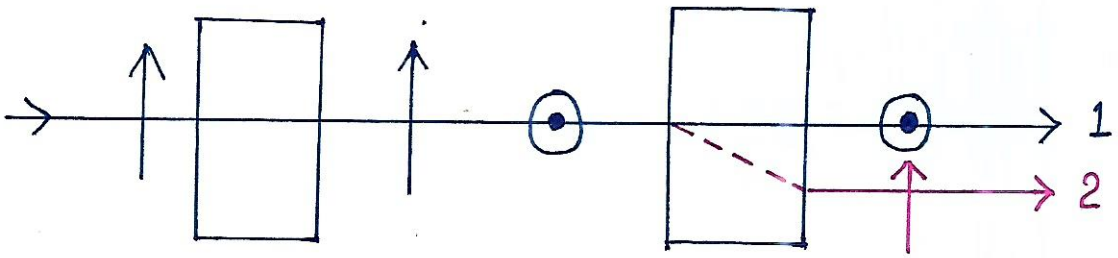
Optical Switches-

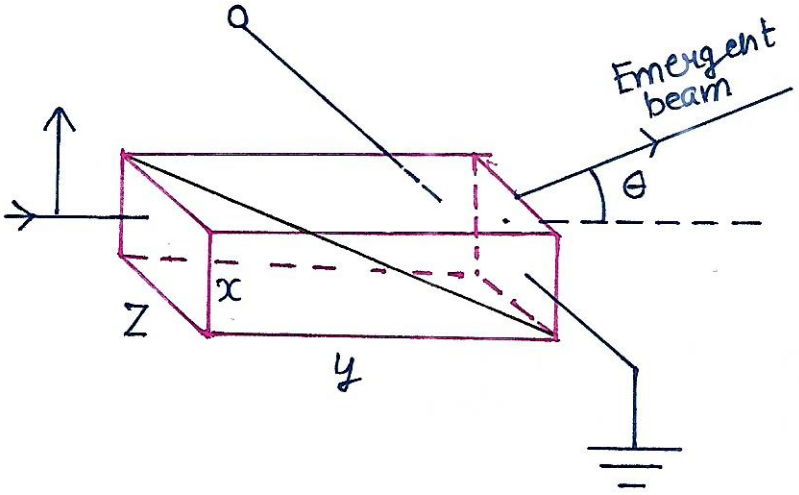
An Optical Switch is a device that selectively switches optical signal from one channel to another. The switching can be temporal or spatial. The former is known as optical switch or optical modulator, (which) while the latter is called an optical space switch or an optical router. In general, optical modulators and routers can be made for each other.





An Optical Switch may operate by mechanical means, such as physically shifting an optical fiber to drive one or more alternative fibers or by Electro-optic effect, Magneto-optic Effect or other methods. Slow optical switches, such as those using for alternate routing of an optical switch transmission path such as routing around a fault. EOE, MOE used for logic operations.





The arrangement shows that it can be used for beam switching. Here we have two similar prism of KDP but with opposite orientations. A ray entirely in the upper prism travels in a medium of refractive index.

$$n_2 = n_o - \frac{n_o^3}{2} r \varepsilon_z$$

While a ray entirely in the lower prism travels in a medium where the efforts of the applied field is reversed so that

$$n_1 = n_o - \frac{n_o^3}{2} r \varepsilon_z$$

The arrangement leads to a deflection of the beam. The rays 1 and 2 will transverse the crystal in times t_1 and t_2.

$$t_1 = \frac{Ln}{C}$$

$$t_2 = \frac{L(n+\Delta n)}{C}$$

Using snell's Law, the angle of beam diffection θ measured outside the crystel is given by

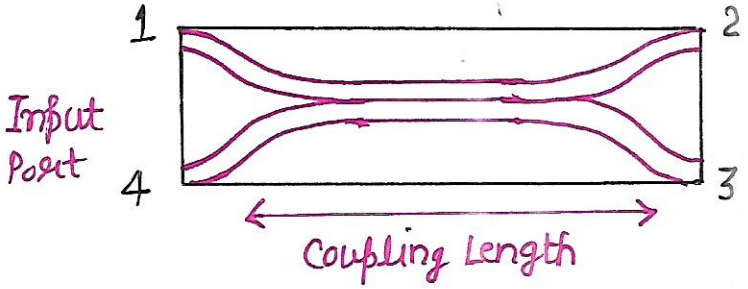
$$\theta = \theta' \times n = \frac{n \Delta y}{W}$$

$$\theta = \frac{L \Delta n}{W}$$

$$\boxed{\theta = \left(\frac{L}{W}\right) n_0^3 r \varepsilon_z}$$

Optical Directional Coupler-

Couplers are devices used to interconnect adjacent waveguides to combine or devide light with very low loss. A directional coupler allows the light to pass through one direction only, In a 1×2 directional coupler,



It devides the input power applied to the port (1) into two outputs at port (2) and (3). In ideal condition no light reaches at port (4). For efficient coupling, it requires that the propagation vectors are the same for each guide. The light coupling can be varied from 0 to 100 percent by suitable choosing the coupling length.

Various coupling Functions :

1. Combiner- Combine a number of Input into one or more outputs.
2. Splitter- Devide input into two or more outputs.
3. Polarizer Splitter- Input devide into two polarized light.
4. Monitor- Couple some portion of light to the monitor port.

Magneto-Optic Effect :

The presence of Magnetic Field may also affect the optical properties of some substances thereby giving rise to a number of useful devices. In general, however, as electric field are easier to generate than magnetic field, electro-optic devices are usually preferred to magneto-optic devices.

- **Faraday Effect –**

This is simplest magneto-optic effect and the only one of the real intrest for optical modulators. It concerns the changes in refractive index of a material subjected to a steady magnetic field $\vec{B}$. Faraday found that when a beam of plane polarized light passes through a substance subjected to a magnetic field, its plane of polarization is observed to rotate by an amount proportional to the $\vec{B}$ component parallel to the direction of propagation. This is very similar to optical activity. In the Faraday effect the sense of rotation of the plane of polarization is independent of the rotation of propagation. Thus the rotation can be doubled by reflecting the light back through the faraday effect device.

The rotation of the plane of polarization is given by

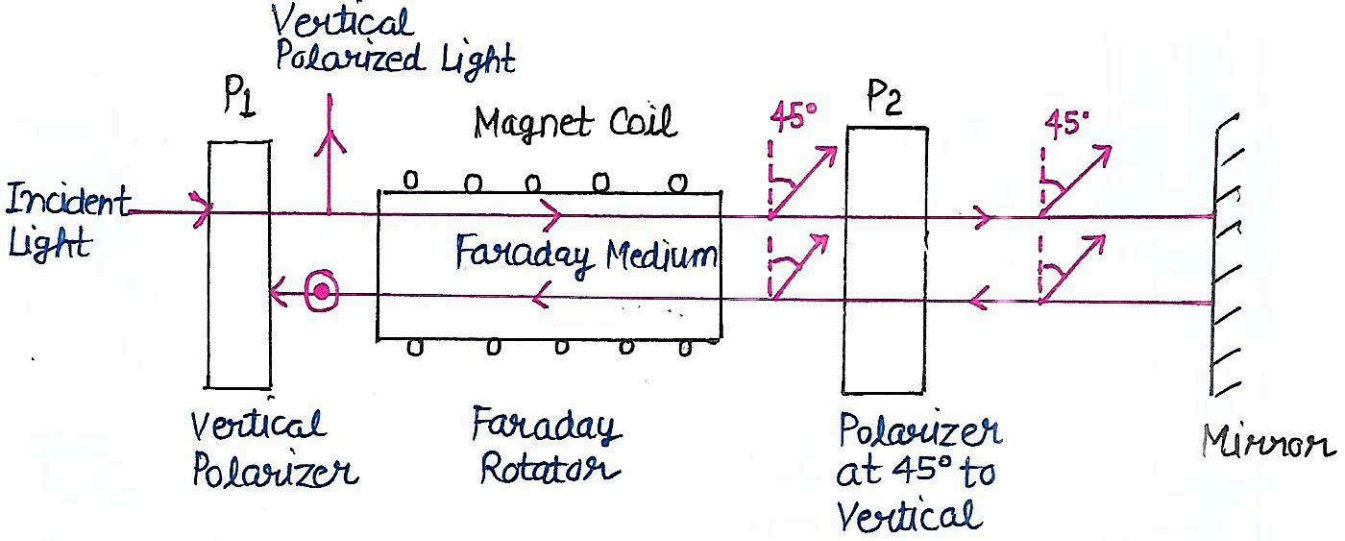
$\theta = VBL$
- L → Path length in the material
- B → Magnetic Flux density ∥ direction of propagation
- V → Verdet Constant

The Faraday Effect is small and wavelength dependant.

$$\theta = \frac{\pi}{\lambda_0}(n_T - n_I)L$$

A faraday rotator used in conjunction with a pair of polarizers acts as optical isolator which allows a light beam to travel through it in one direction but not in opposite one.

The construction of a typical isolator is shown below.



Light passing from Left to right is polarized in the verticle plane by polarizer P_1. The faraday rotator is adjusted to produce a rotation of 45° in a clockwise sense. The second polarizer P_2 is set at 45° to P_1 so that it will transmit light emerging from the rotator. However, a beam entering from the right will be plane polarized at 45° to the verticle by P_2 and then have its plane rotation by 45° in the clockwise direction by the rotator. It will therefore be incident on P_1 with its plane of polarization at right angles to a plane of transm. and be eliminated.

One potential application of magneto-optics currently receiving attention is large capacity computer memory. Such memories must be capable of storing very large amount of information in a relatively small 's area. Writing may be achieved by heating the memory elements on the storage medium to a temperature above the curie point using laser beam. To read the information, laser beam is direction to the memory elements.

Systems, incorporating, have enabled information to be stored, read and erased at rate >1 Mbps.

OPTICAL SOURCES AND DETECTORS

Optical Source :

- Emission of Light can take place in the form of Photon.
- Energy must be supplied to boost the e- from low energy state to a higher energy state.
- The energy can come from many sources : Heat (Incandesent Light), Electrical discharge (D_2, Hg, Na, Lamps), Electrical current (LED, LD), Biolumisence (Fire Fly- Luciferase Engyme).

Optical source requirement for performance-

- Physical dimensions
- Narrow Radiation Pattern
- Linearity
- Ability to be directly modulated by current
- Fast response time
- Adequate output power into fiber
- Line Width
- Efficiency
- Stability
- Driving circuit Issue
- Reliability and Cost.

Optical Sources of Light-

→ Tungustan

→ Deuterium

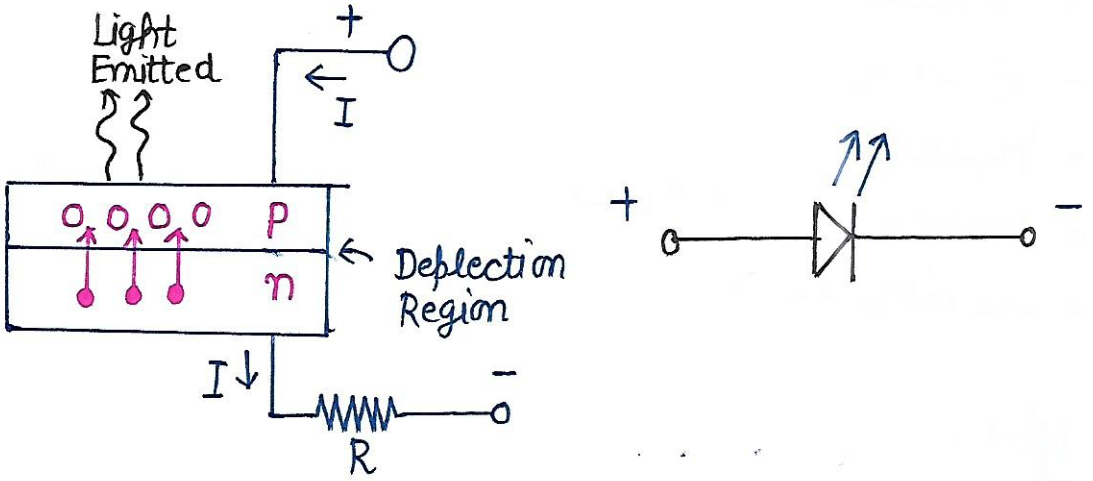
→ Mercury

→ Hollow cathode Lamp

#LED Structure & Materials -

A pn junction acts as the active or recombination region. When a pn junction is forward biased, e^- and holes recombine either radiatevely (emitted Photon) or non-radiatievely (emitted heat). This is simple LED operation. Emitted wavelength depends on bandgap energy.

Material	Wavelength Range (um)	Bandgap Energy (eV)
AlGaInP	0.61 - 0.68	1.82 - 1.94
GaAs	0.9	1.4
AlGaAs	08 - 0.9	1.4 - 1.55
InGaAs	1.0 - 1.3	0.95 - 1.24
InGaAsP	0.9 - 1.7	0.73 - 1.35

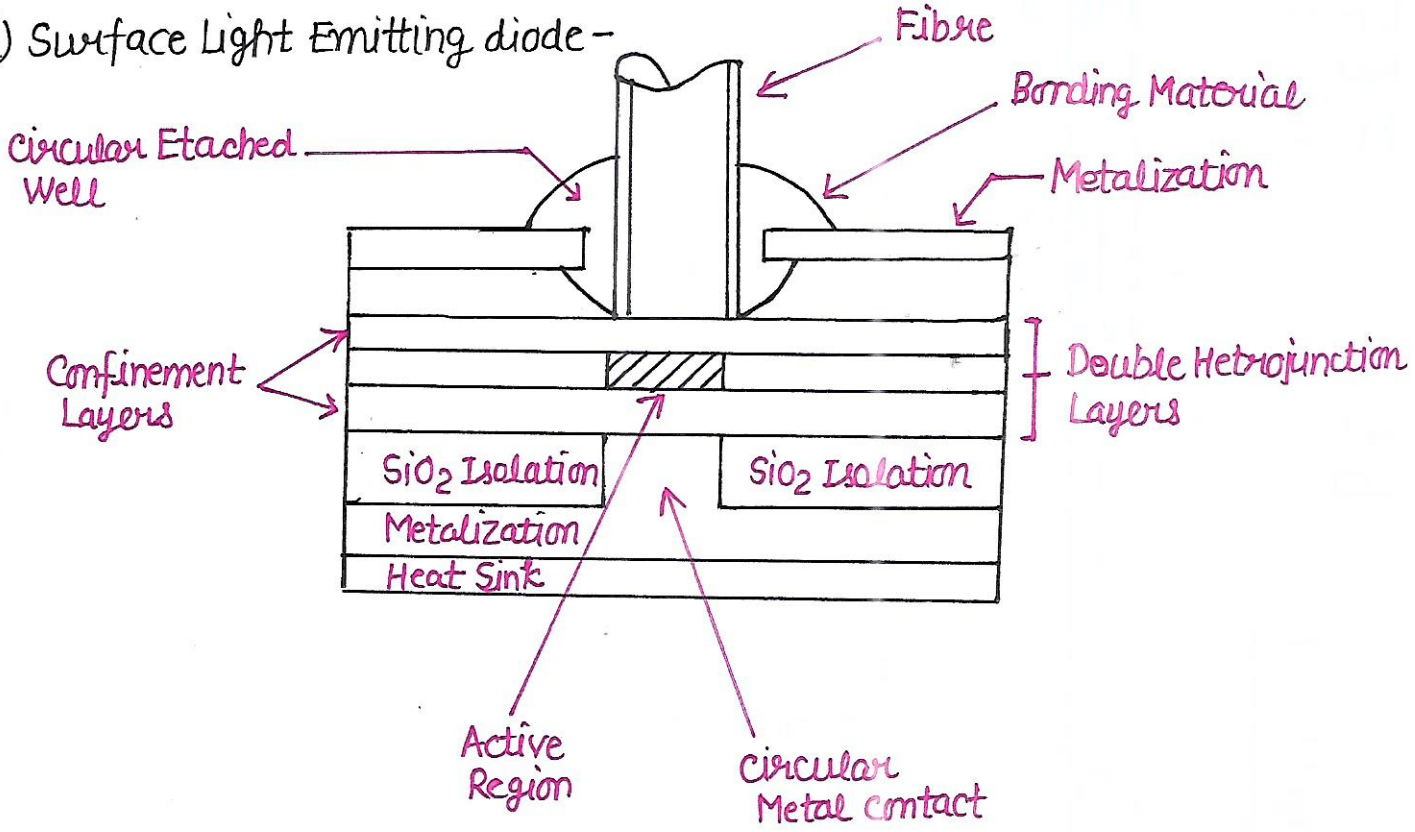


Light is emitted at site of carrir recombination which is primarily close to the junction.
However the amount of radiative recombination and the emission area within the structure is dependant upon the semiconductor material used and the febrication of the devices.
It emits coherent light through spontaneous emission.

Types of the LED structures

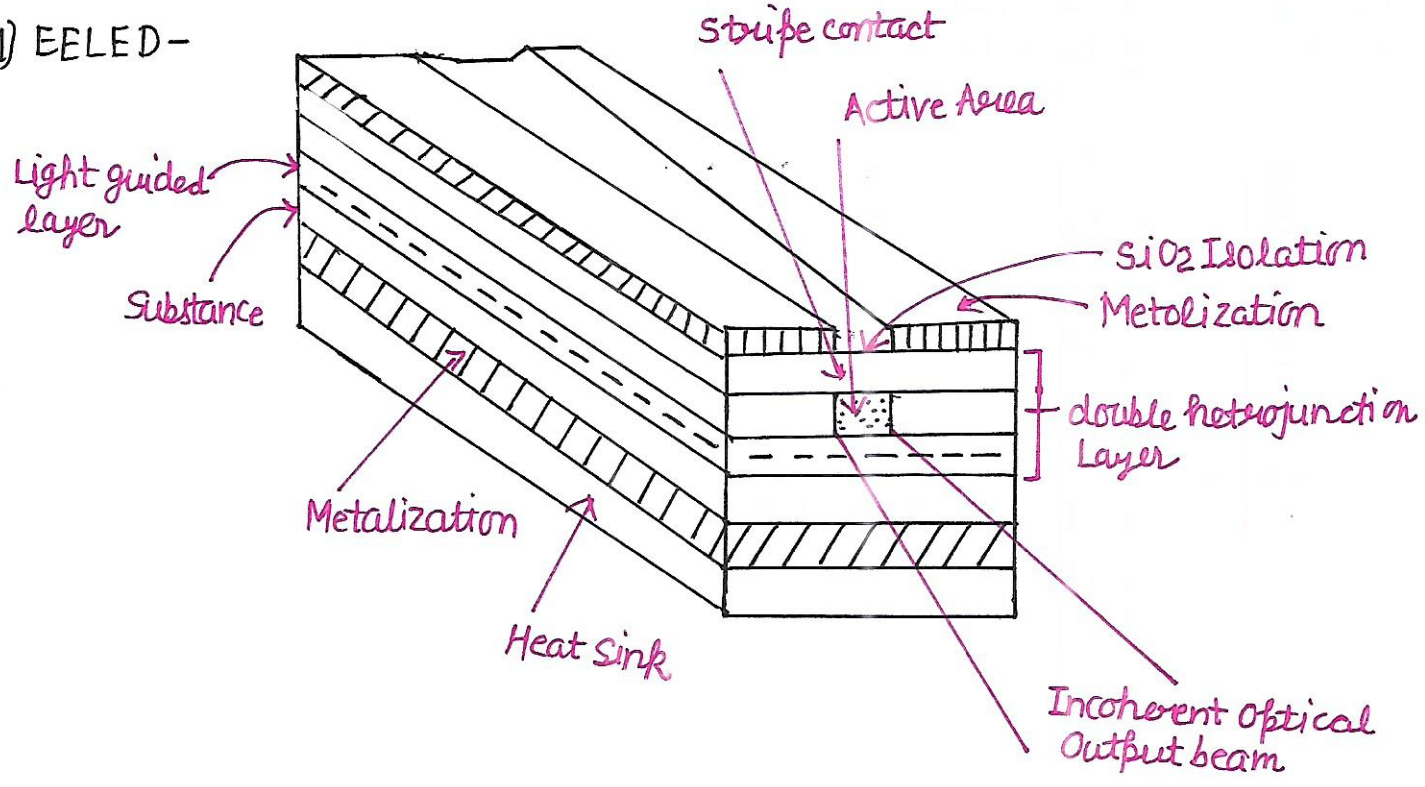
- SLED — Surface Light Emitting Diode
- EELED — Edge Emitting LED
- SLD — Subluminescent LED

(1) Surface Light Emitting diode –



The active region is limited to a circular section that has an area compatible with a fiber core and face.

(11) EELED –



Edge Emitted double hetrojunction LED's, the output beam is lambertian in the plane of the pn junction (θ_{11}) and highly directional perpendicular to the pn junction. It has advantage of transparent guiding layers with a very thin active layer (50 to 100 μm). This reduces self absorption and narrow the beam divergence. Light is emitted at one end face only.

(iii) Superluminescent LED-

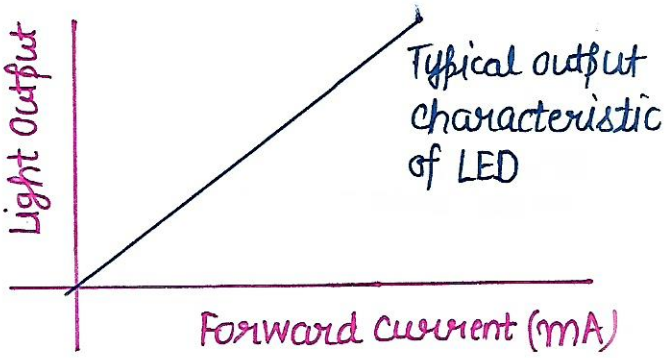
Diagram is quite similar to EELED. But It has significant benefits over both SLED and ELED for OFS.

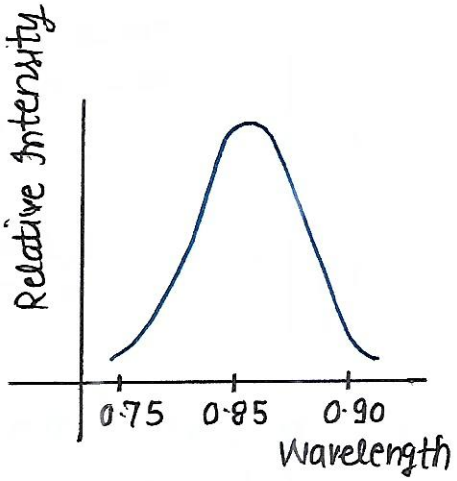
It provides;

* High output power
* Directional output beam
* Narrow spectral linewidth

One end is made optically lossy to prevent reflections and thus suppress lasing.

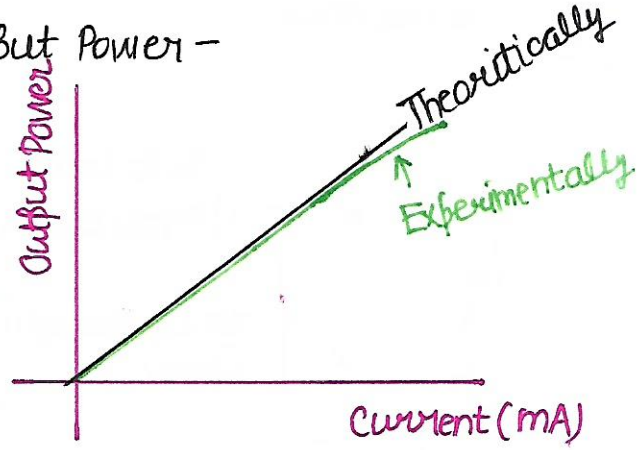
- Characteristics-



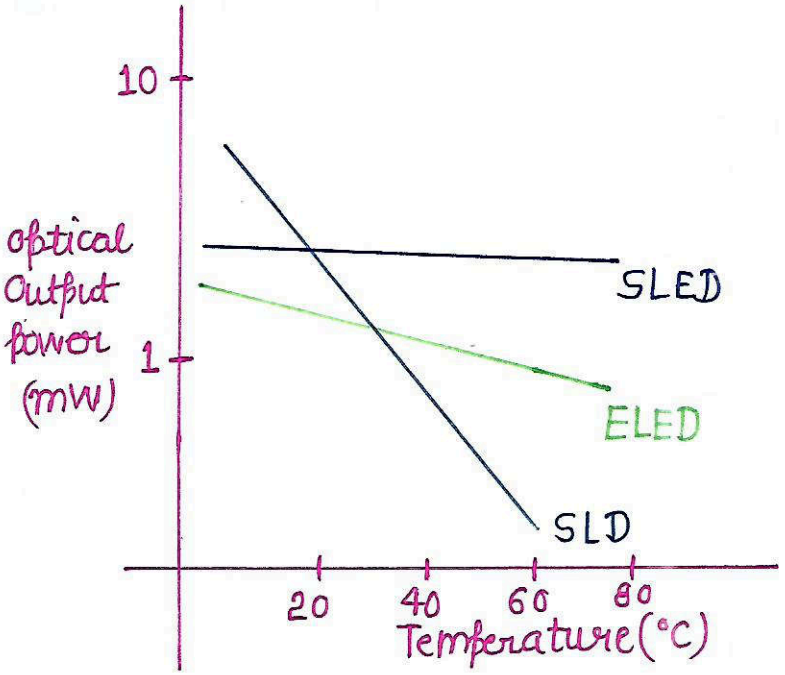


The light output is quite linearly proportional to the current, so it can be preciously modulated.

- Output Power -



• Temperature Dependance -



This characteristic show the variation in output at a specific drive current over the temperature range for all the devices. To utilize the high power potential of such devices at elevated temperature, the use of thermo-electric cooler may be necessary.

Liquid Crystal diode -

It is a passive type of display. LCD is commonly known as liquid crystal display. Here one of the prime requisites is for low power consumption, particularly so for the digital watch because of the necessarity low capacity of the power source.

There two types of basic LCD available. These are -

(a) Reflective - Require front Illumination.

(b) Transmissive - Requires rear Illumination.

The Liquid crystal state is a phase of matter which is exhibited by a large number of organic materials overs a restricted temp. range.

At the end of the temperature range, the material becomes a crystalline solid, whilst at the upper end of the temperature range, it changes into a clear liquid. Within this range it has a milky yellowish appearance and combines some of the optical properties of solids with the fluidity of liquids. A major characteristic of the all liquid crystals commponds are the rod-like shape of their molecules.

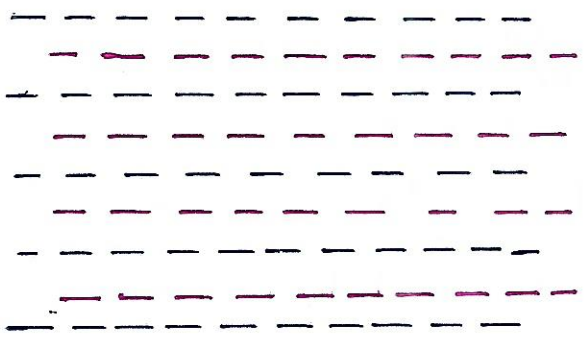
When they are in crystal liquid phase, these molecules can take up certain orientations relative both to each other and to the liquid crystal surface.

Ordering of Liquid crystals

(a) Nematic (b) Cholesteric (c) Smectic

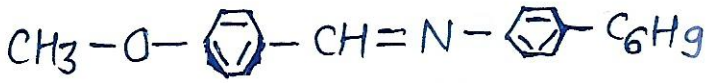
(a)

Nematic and Cholesteric are more important in display devices at present and Illustrated.



In Nematic Ordering (above figure), the molecules are aligned parallel to each other, but apart from remaining properties. A nematic liquid crystal molcule usually consists of two benzene reings linked with a central group.

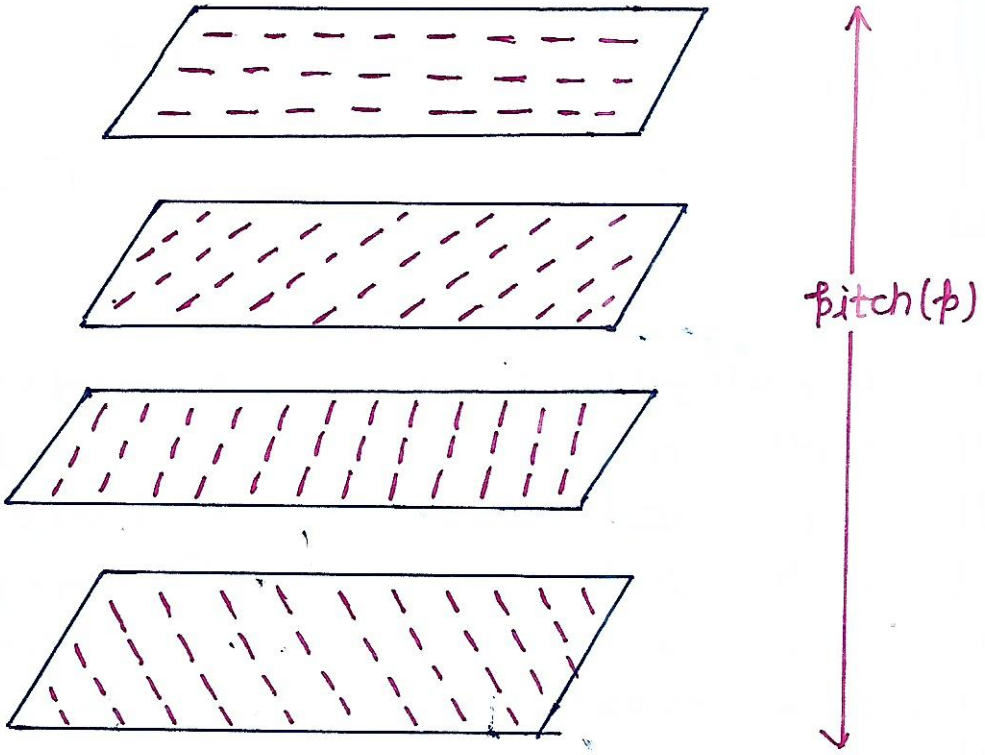
4-methoxybenzylidene-4-butylanaline (MBBA)



It shows liquid crystal behaviour over temperature range 20°C to 47°C.

(b)

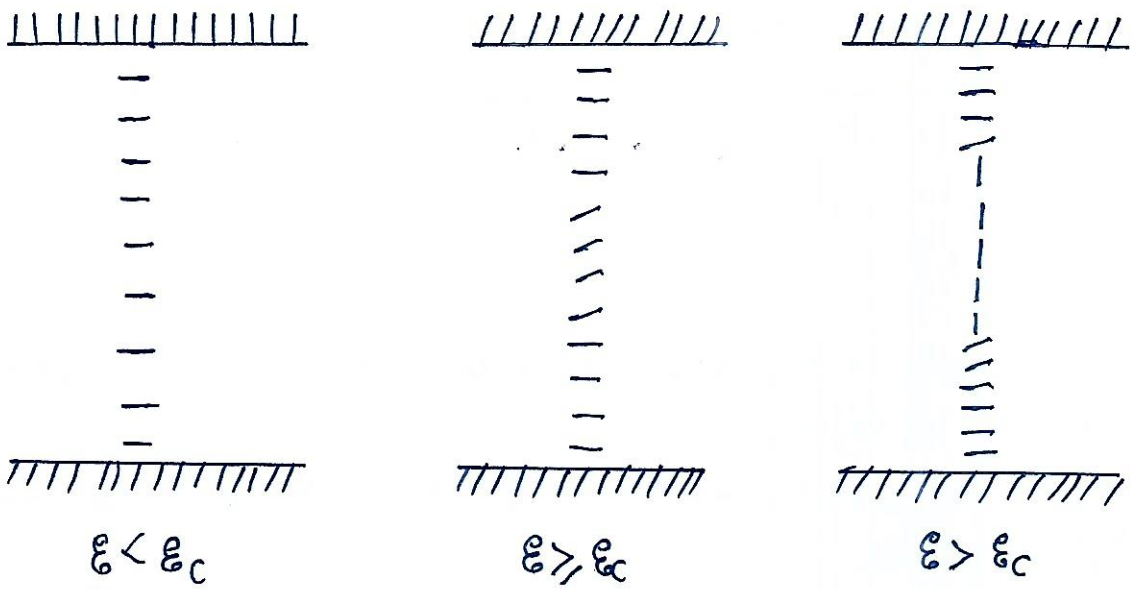
In the Chlesteric phase, we may regard the material as being made up from a large number of planes each having a nematic like structure but each plane showing a progressive change in the director direction from the one below. The distance between planes having the same director direction is called pitch p. This type of displays exhibit some interesting colour effects.

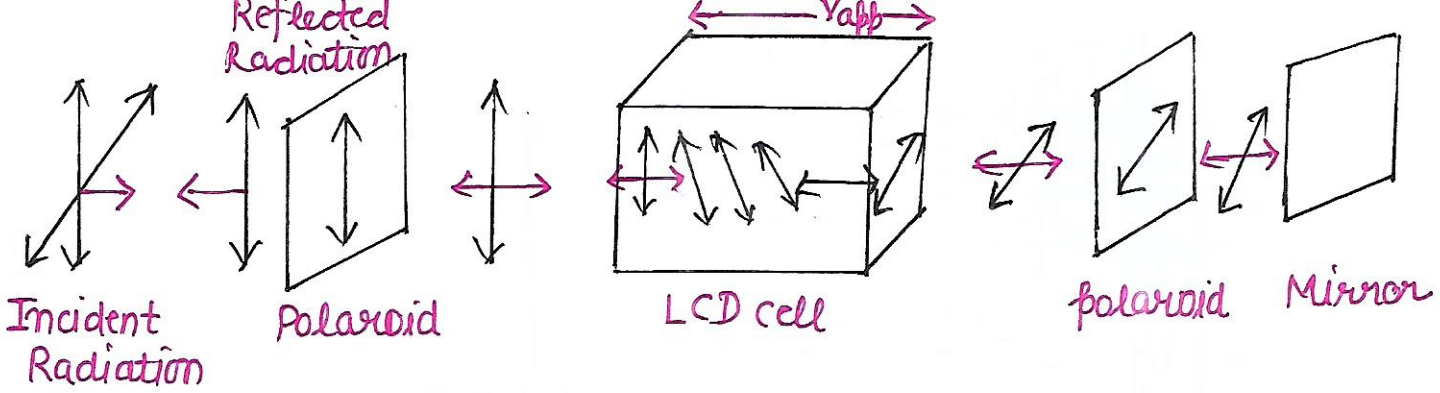


One of the most important electrical characteristic of liquid crystal display material is that they show different dielectric constants $\varepsilon_{\parallel}$ and $\varepsilon_{\perp}$.

If $\varepsilon_{\parallel} > \varepsilon_{\perp}$ we refer it as positive material, which tends to make the molecules lie along the $\vec{E}$, since they will tend to minimize their energy.

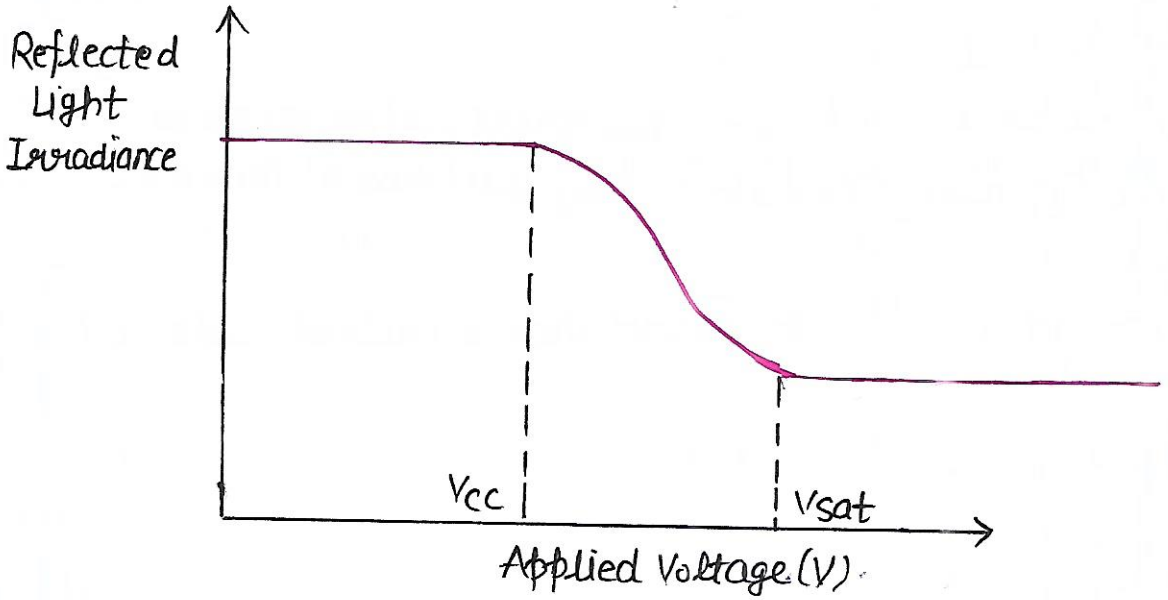
This transition is found to take place above a critical field (ε_c)





In operation, the cell is sandwitched b/w two pieces of polaroid whose polarizing directions corresponds to the director ordering direction of the particular cell surface they are next to. In the reflected mode, a reflector is placed behind the back sheet of polarizod. The arrange-ment has been shown and traces the behaviour of a polarized beam as it transverse the system with no voltage applied. Further process is as shown in figure.

The amount of light reflected foom an LCD as a function of applied voltage is shown.



PIN Photodiode-

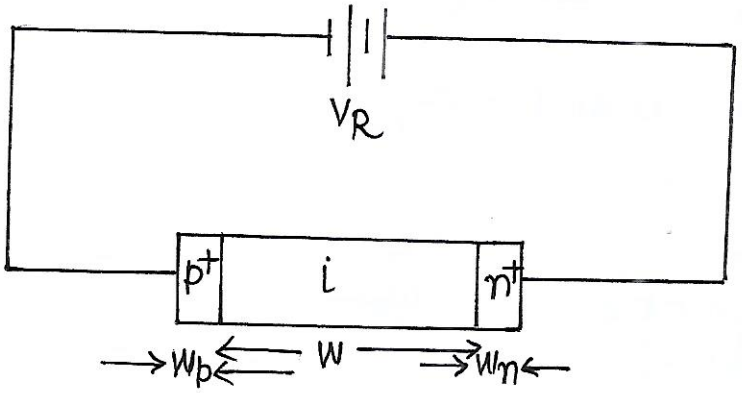
PIN photodiode is a kind of photo detector. It converts optical signal into electrical signals.

This technology was invented in 1950's. There are three regions in this type of diode.

Regions :

1. p-region
2. Intrinstic region
3. n-region

The p-region and n-region are comparatively heavily doped than than the usual p-n diode's p and n-regions. The width of the intrinstic region should be larger than the space width of a normal p-n junction



The p-n photodiode operates with an applied voltage (reverse bias) and when the reverse bias is applied, the space charge region must cover the intrinstic region. Electron hole pairs are generated in space charge region by photo absorption. The switching speed of frequency response of photodiode is inversly proportional to the life time.

The switching speed can be enhanced by a small minority carrier life time. For the photo detector application where the speed of response is important, the depletion region width should be made large as possible as a result the switch speed also increase

Applications :

(1) RF and microwave switches.
(2) RF and microwave variable attenuators
(3) photo detector
(4) photovoltaic cell.

Avalanch Photodiode-

An Avalanch photodiode (APD) is a highly sensitive semiconductor photodiode that exploits the photoelectric effect to convert light into electricity.

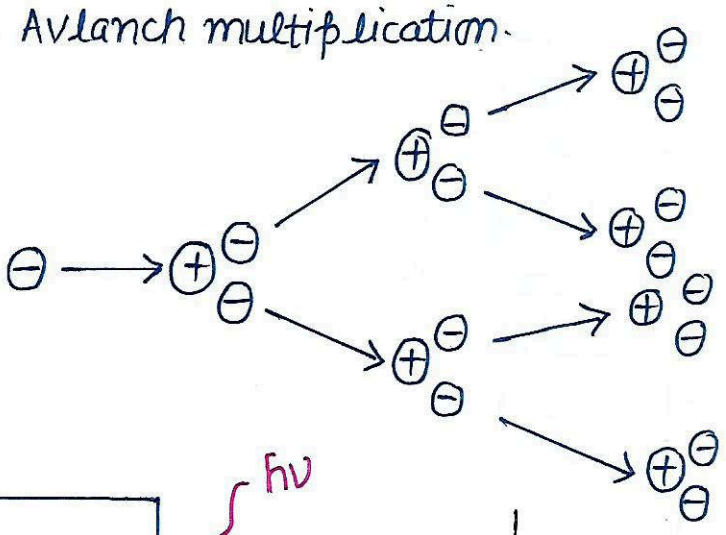
One of the main working parts of any optical receiver is the photodetector. The device that transforms optical signal into electrical signal.

$T_X \longrightarrow R_X$

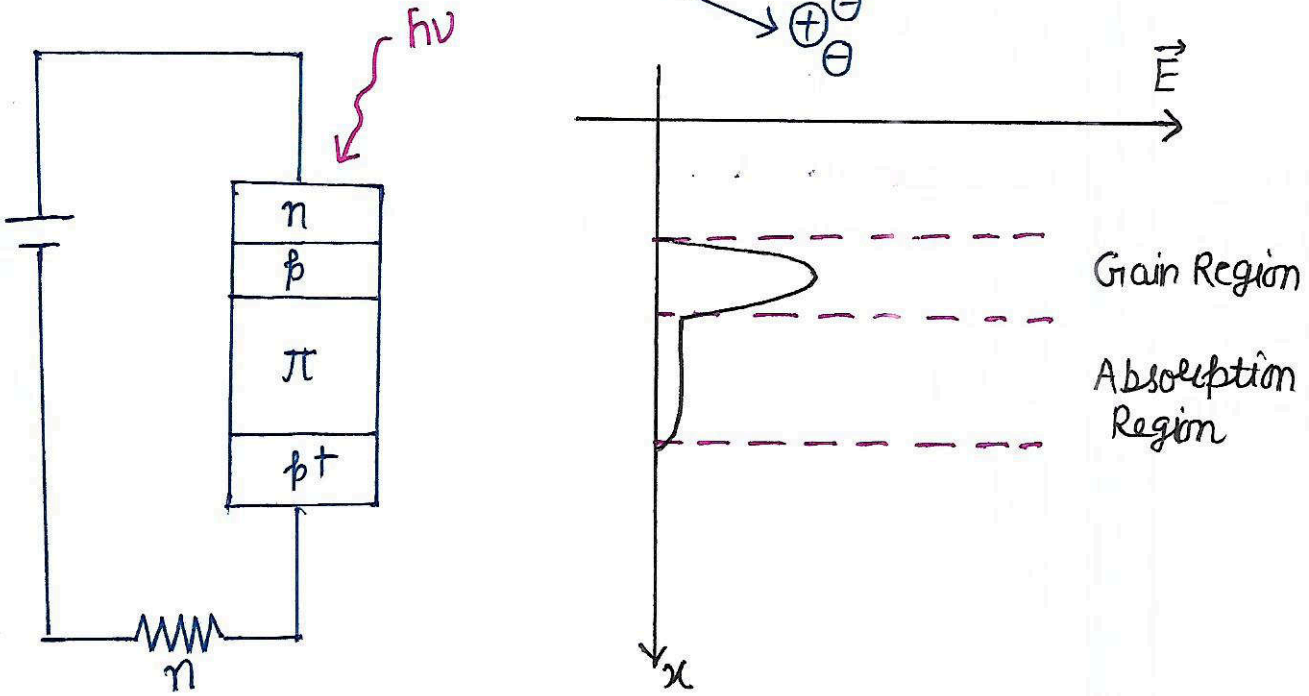
The photodiode can be operated in one of the two modes;

- Photoconductivity (Reverse-bias)
- Photovoltaic (Zero-bias)

They are the photodetectors that provides built in first stage gain through Avlanch multiplication.



→ Generation of free carriers that are created by absorbed photons.



(1) In APD, an extremely high electric field is created around $3 \times 10^5 V cm^{-1}$

(2) The generation of high field region causes holes ⊕ & electrons ⊖ to aquire sufficient energy to excite new electron-hole pairs. This cause avalance breakdown in ordinary reverse bias condition. This process is known Impact Ionization.

→ APD are used in optical receiver before the signal is given to the receiver circuitry, the photon is multiplied with photo current and this increase the sensitivity of the receiver.

→ It requires high operating voltage and produces high noise. The O/p is not linear.

→ It is used as counterpart of PIN diode that is often used.

#Photomultipliers-

In the photomultipliers, the photoelectrons are accelerated towards a series of electrodes (dynodes) which are maintained at successively higher potential with respect to cathode. On striking a dynode surface, each e^- causes the emission of several other electrons which in turn are accelerated towards the next dynode and continue the multiplication process. Thus if on avarage δ secondary e^- are emitted at each dynode surface for each incident e^- and there are N dynodes overall, then the total current amplification factor between the cathode and Anode is given by

$$G = \delta^N \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

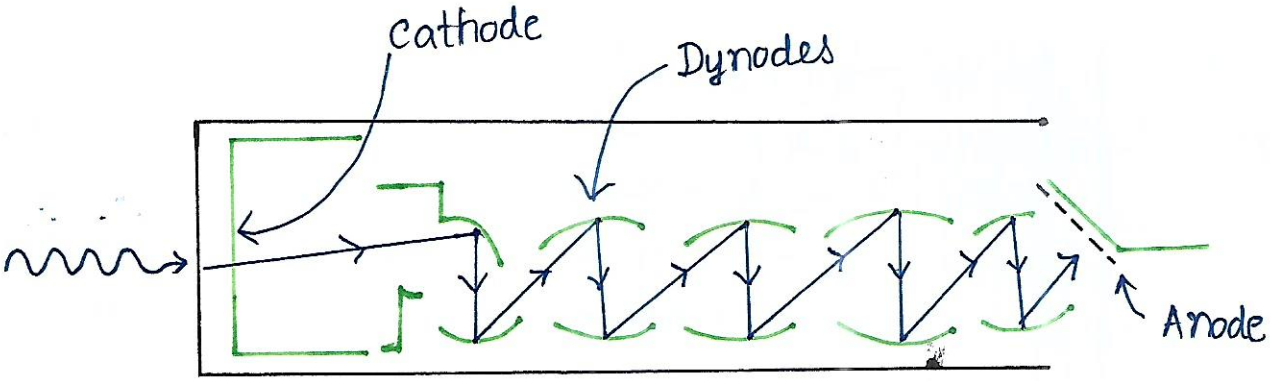
e.g.

$\delta = 5$

$N = 9$

$G = 5^9 = 1953125 = 1.9 \times 10^6$

$Gain \simeq 2 \times 10^6$

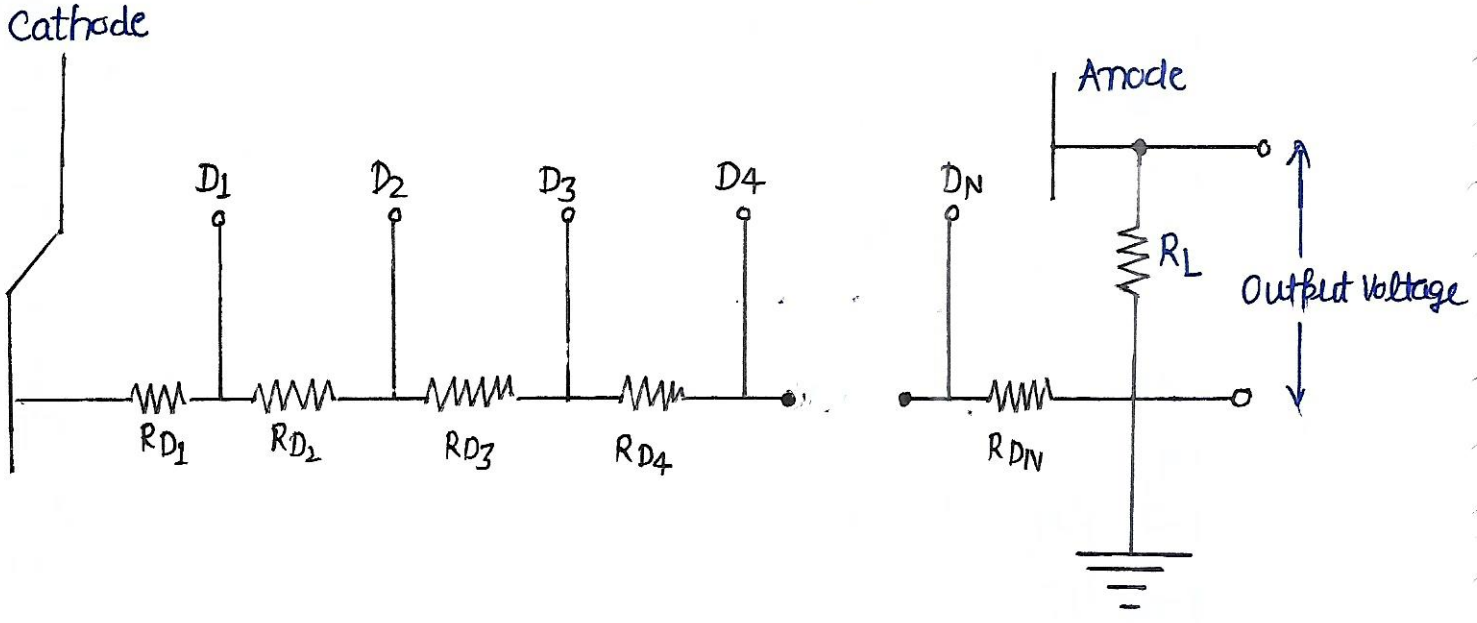


The Photoelectrons are emitted from the opposite side of the cathode layer to that of the incident radiation. Obviously, in this arrangement the thickness of the photocathode is very critical. If It is too thick, few photons will penetrate to the e⁻ emitting side, whilst if it is too thin few photons will be absorbed.

Types of dynode made PMT

- Venetial blind Type — Inexpensive, Very suitable
- Box and guid type — Similar to VBT
- Linear Focused
- Circular cage focused

In Linear Focused and circular cage, some degree of e⁻ focusing is obtained by careful shaping and positioning of the dynodes.



The dynode potential are usually provided by means of this circuit. Care must be taken to ensure that the voltage b/w the cathode and the first dynode is large enough to maintain proportionality b/w cathode current and cathode illumination.

The photomultipliers response to light input by delivering the charge to the anode. This charge may be allowed to flow through a resistor R_L or to charge a capacitor. The corresponding voltage signal then provides a measure of the input optical signal. If individual pulses need to be examined then it is important to ensure that the response time of the ext. circuitary is lessthan that of the pulse rise time. This usually implies a low value for the R_L.

Traditionally, Photomultipliers have been relatively bulky device with cathode diameter of 25 mm or more. Smallest one is 8 mm or so.

- Dark current-

$$i_n = aAT^2 \exp\left(-\frac{e\phi}{KT}\right)$$

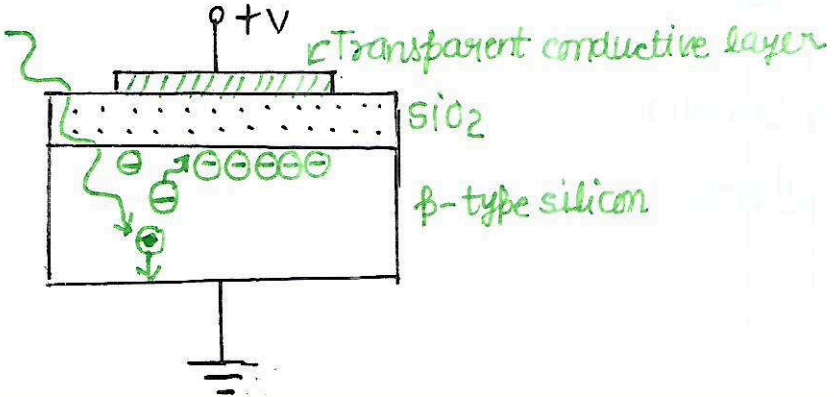
- Shot noise-

$$\Delta i = \left(2ie\Delta f\right)^{1/2}$$

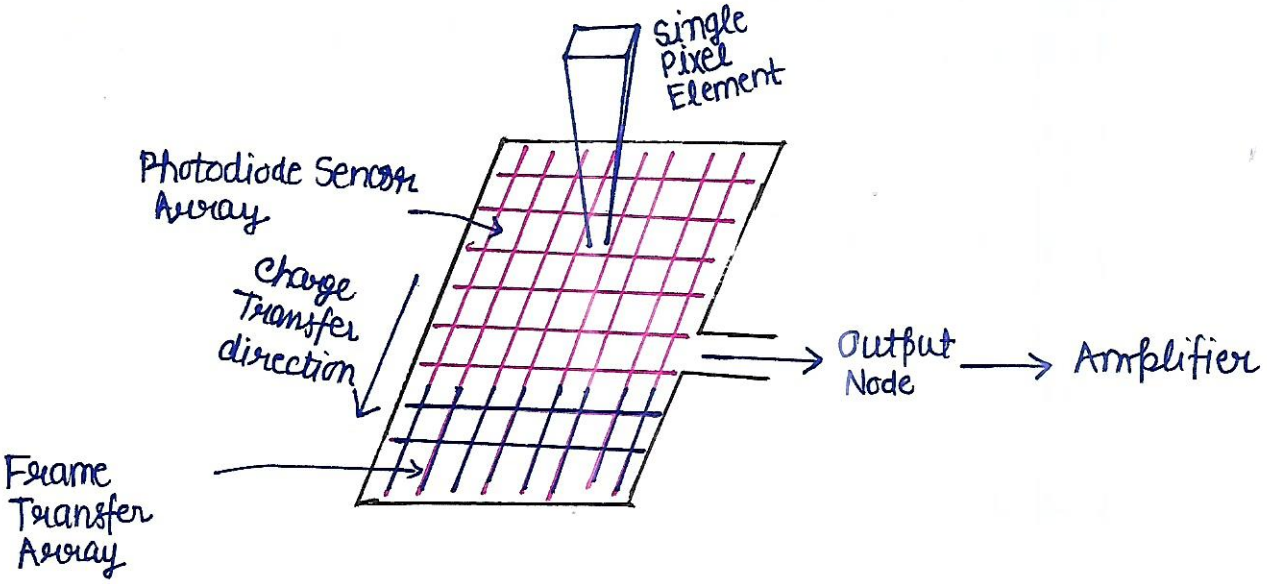
CCD-

We have seen how an optical image may be converted into an electrical signal by using a videocon-type device. An alternative approach is to use an array of discrete detectors. The image may be focused directly onto a 2D array, or alternatively the image may be scanned a 1D array.

An alternative approach was proposed by the researchers of Bell Lab in 1970s. This involved using a charge coupled device (CCD structure). The basic building block of this is the MOS (metal-oxide-semiconductor) capacitor. This is formed by growing a layer of SiO_2, on a p-type silicon substate, a metal electrode is then evaporated on top of the oxide layer.



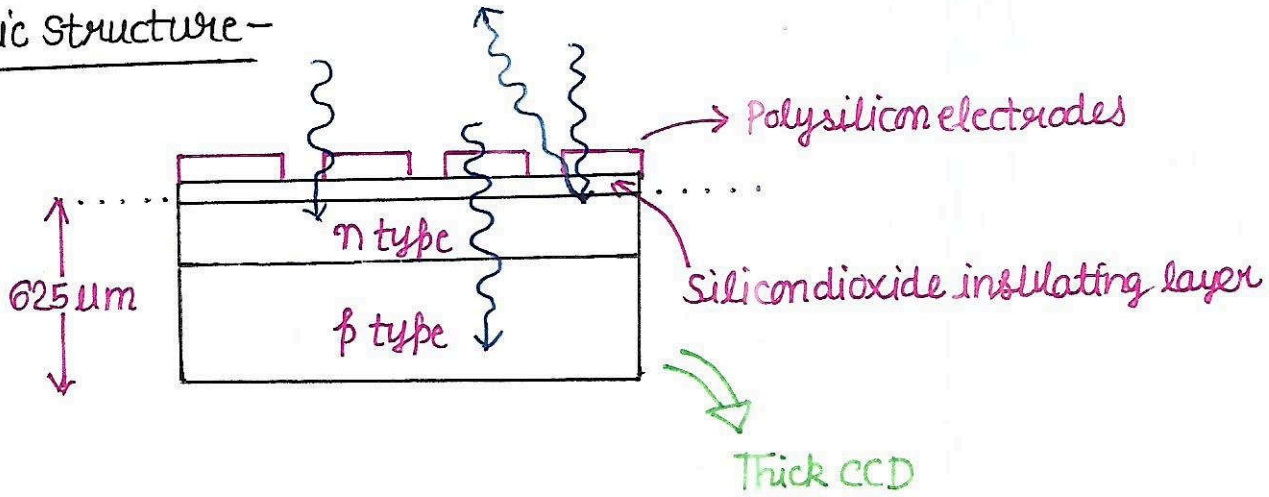
The metal electrode is known as the gate and is biased positively with respect to the silicon. Photogenerated e⁻ - hole pair within the silicon will be separated, with the electrons being attracted to the surface of the silicon under the gate, where they will remain whilst the gate voltage is +ve. The e⁻ are effectively trapped within a potential well formed under the gate contact. The amount of trapped charge will be proportional to the total integral light flux falling onto the device during the measurement period.



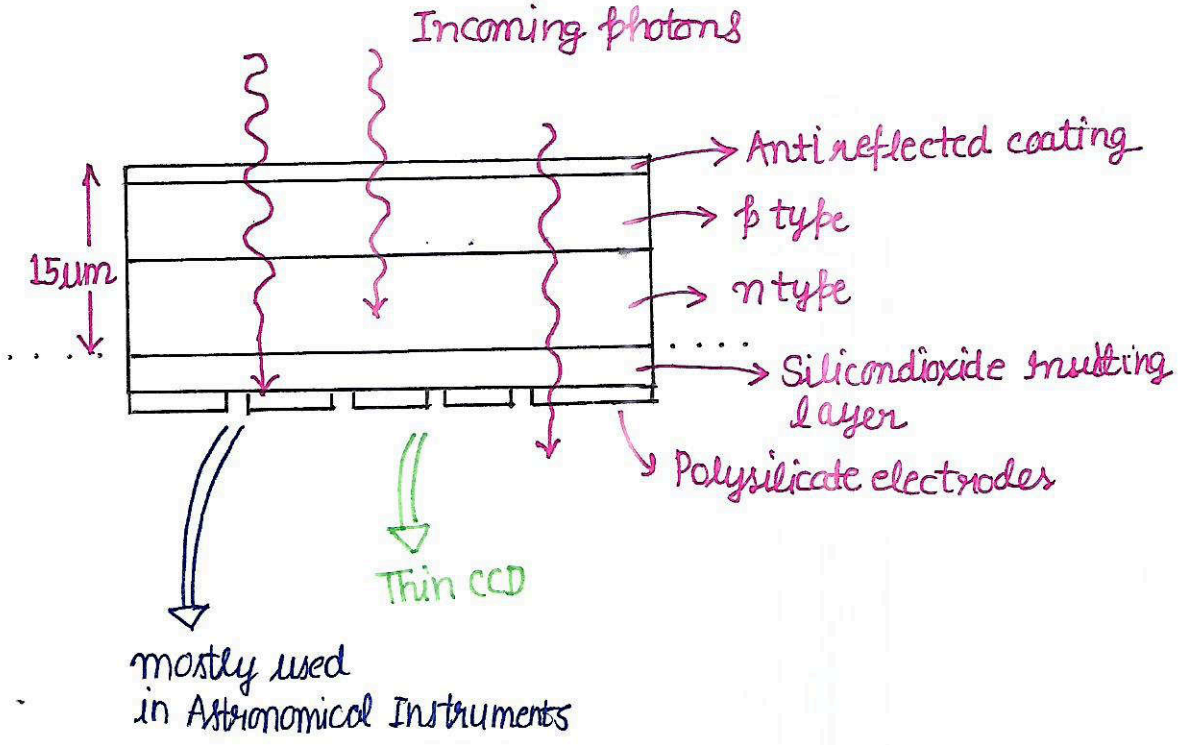
- The Photoelectric effect - turning photons into charge
- Replaces photographic Film and PMT
- Consists of a thin silicon wafer devided an array of thousands of tiny light sensitive squares. Each square corresponds to an individual pixel in the final image.
- Each photosite has tiny capacitors that holds charge.
- Light enters the array, and will liberate electrons from the silicon atom.
- Turns light into electrons - Photoelectric Effect - Analog device.
- Output - The voltage from each photosite - depends on number of photons that penetrated the silicon surface on each photosite.
  - → Output Voltage converted to a digital signal (electronics).
  - → Final output is an array of numbers.
- Instead of taking a picture - record an image.

- most astronomical instruments today use CCDs.
  - #Astronomy
  - #Photometry
  - #Imaging
  - #Spectroscopy
- Developed by Bell's Laboratory
- CCDs are manufactured on silicon wafers using the same technique used to manufacture computer chips. Scintific CCDs are very big, only a few can be fitted on wafer. This is one reason that they are so costly. Also the wafers are very fragile.

- Basic Structure-



- Thinned Back side Illuminated CCD-



Photoconductive detectors-

An e⁻ may be raised from the valence band to a conduction band in a semiconductor where the energy gap is Eg by the absorption of a photon of frequency ν provided that

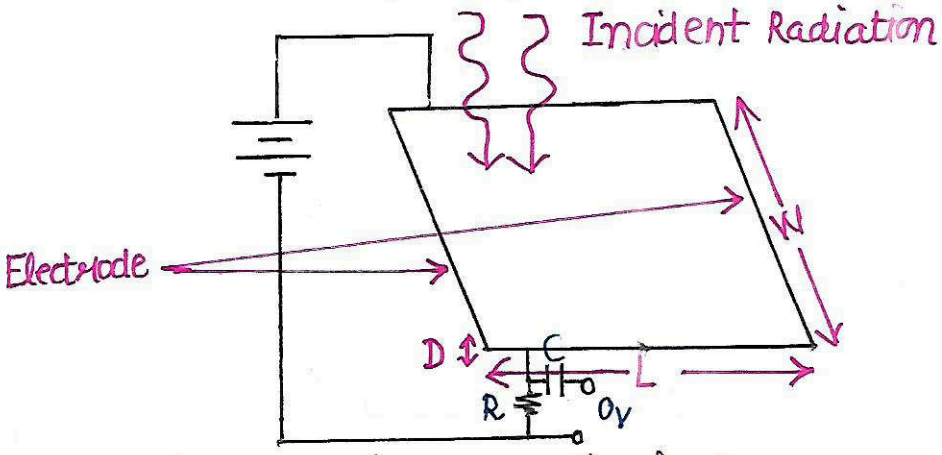
$$h\nu >> Eg$$

in terms of wavelength

$$\lambda \leq \frac{h\nu}{Eg}$$

$$\text{Bandgap wavelength } \lambda g = \frac{h\nu}{Eg}$$

As long as the e⁻ remain in the conduction band, the conductivity of the semiconductor will be increased. This is the phenomena of Photoconductivity



It is basic mechanism operative in photoconductive detectors. An external potential across the electrodes is usually provided by the circuit.

- Changes in conductivity of the detector.
- Increase flow of current which will increase the potential across the load Resistor R_L.

Hots only

reflection cofficient

$$\eta = \left(\frac{n-1}{n+1}\right)^2$$

$$I(0) = I_0(1-\eta)$$

This may then be detected using a high impedance voltmeter. If we are only interested in time varying part of the incident Radiation, then a blocking capacitor C may be inserted in the output line to remove any d.c component.

The optimum size of R_L in a particular situation is determined by the fractional change in the resistance of the photodetector when under max. illumination.

We suppose that the radiation falling normally onto the slab is monochromatic and of Irradiance I_0. Not all the incident energy will be available to generate e^- within the semiconductor.

→ Some will reflected
→ Some will pass the semiconductor without any absorption.

Reflection cofficient -

$$\mathcal{R} = \left(\frac{n-1}{n+1}\right)^2$$

Irradiance inside the surface -

$$I(0) = I_0(1-\mathcal{R})$$

Now Irradiance at point a distance x into semiconductor

$$I(x) = I(0)e^{-\alpha x}$$

α → Absorption cofficient

Thus the fraction of incident irradiance which is actually absorbed

$$A_I = (1-\mathcal{R})\,\eta_{abs}$$

Where

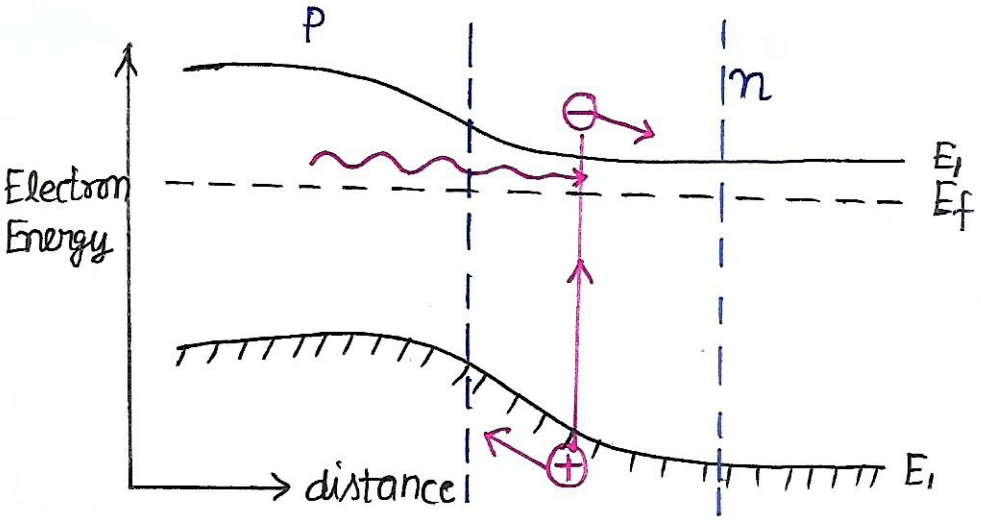
$$\eta_{abs} = 1 - e^{-\alpha D}$$

→ The total no. of e^- hole pairs generated within the slab per second is $\eta I_0 WL/h\nu$

Photovoltaic method of detection of light-

Photovoltaic is the direct conversion of light into electricity at the atomic level. Some material exhibits a property known as photoelectric effect that cause them to absorb photons of light and release electrons. Where the free electrons are captured, an electric current result than can be used as electricity.

The photovoltaic mode is zero biased. When a p-n junction is formed in a semiconductor material, a region deplated of mobile charge carrier is created which has a high internal $\vec{E}$ across it. If the e⁻ hole pair is generated by photon absorption within this region. The $\vec{E}$ sepates the e⁻ and hole as shown in figure.



We may detect the charge sepration. Firsty, if the device is left on open circuit, an externally measurable potential will appear b/w the p and n regions. This is the photovoltaic mode of operation.

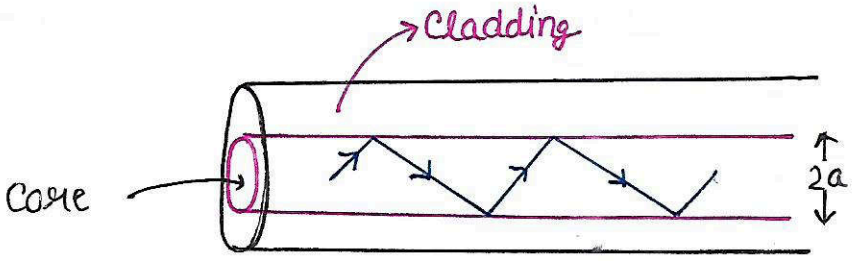
It has been realized that light with its carrier frequency of some 10^{14} Hz has a potential to be modulated at much higher frequency than either radio or microwave and thus opens up the possibilities of a single communication channel with extremely high information content.

difficulties aries-

(i) Need guided medium.

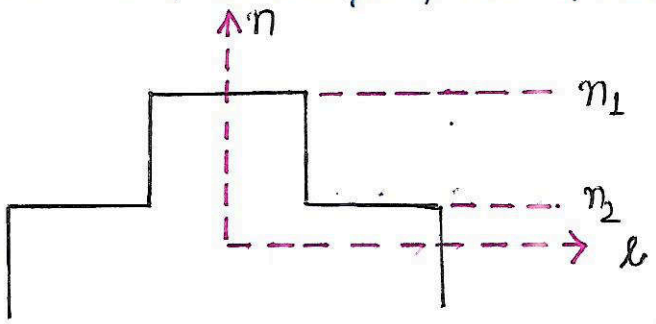
(ii) Atmospheric transmission limited.

Tyndall demonstrated that light could be guided within a water jet as a result of total internal reflection



Optical Fibre Waveguide-

In their simplest form fiber optics have a cylindrical geometry with a core region surrounded by a cladding. As In planer waveguide the refractive index of the core is greater than that of cladding. Such fibers are referred to as step index fibers because of the shape of the refractive index profile.



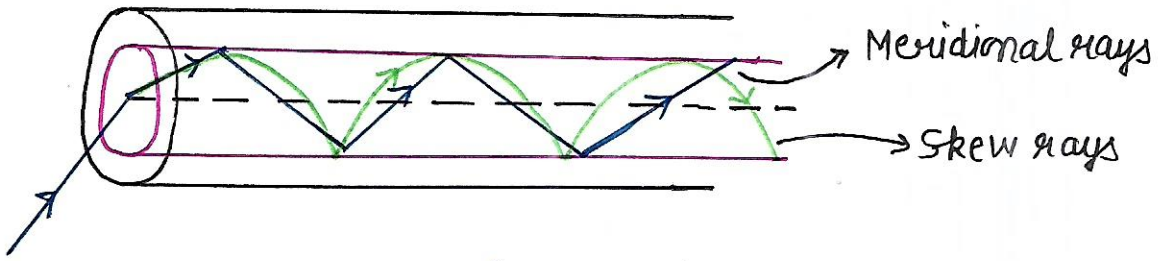
The resulting structure is often covered by an additional coating layer (usually of plastic) which serves to strengthen the fiber and to provide protection against chemical attack. As with the planer waveguide, radiation is restricted to travel down the guide in certain modes. With each mode corresponding to a distinct EMF distribution across the guide.

The equations describing the modes in circular waveguide are much more complicated then their planer waveguide counterparts and we make no attempt to derive them. However, we can take a number of concepts developing for planer waveguide and apply them.

We may devide the rays travelling within the fiber into two parts.

(a) Meridional Rays

(b) Skew Rays



As a planer waveguide the no. of modes in a step index circular waveguide is determined by the V parameter

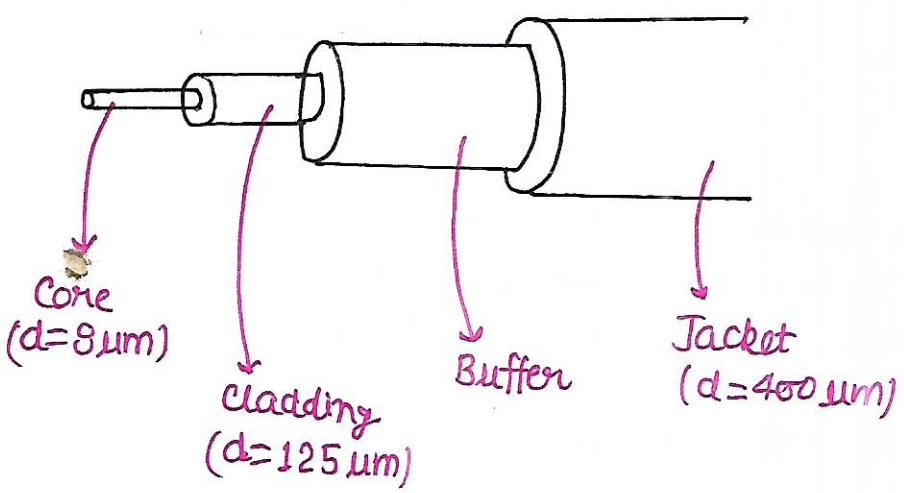
$$V = \frac{2\pi a}{\lambda_0}(n_1^2 - n_2^2)^{1/2}$$ → derived from planer dielectric waveguide

The fiber can support only one mode when $V < 2.504^4$ whilst when $V >> 1$ the number of modes N, than can propagate is given by

$$N \simeq V^2/2$$

Structure of fiber optics-

(II Semester notes - linked)



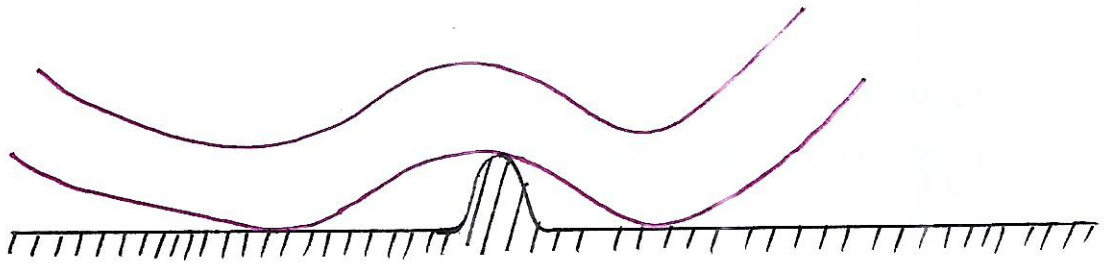
Losses in fibers-

(1) Bending Losses- In multimode fibers, the principal bending loss mechanism arises from mode coupling. From there we conclude that bends in a multimode fiber can lead to the possibility that energy in a particular mode is coupled into a different mode.
When the fiber is bent into an arc of a circle, the loss is found to be strongly dependant on the radius of curvature and a semiempirical expression that is sometimes used to describe the loss is

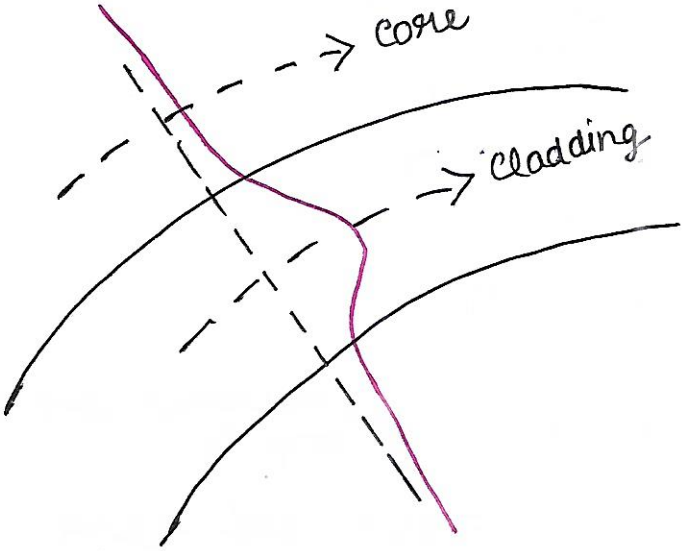
$$\alpha_B = Ce^{-R/R_c}$$

- α_B: absorption cofficient
- R: bend radius
- C: constant depends on the fiber parameter

(2) Intrinstic fiber losses- In this, we examine the losses in silica-based fiber that depend on the fiber material.



Small kinks or microbands can be formed when the fiber is pressed against a surface is not perfectly smooth.



This figure is illustration of the mechanism of radiation loss in fiber at bends. To maintain a plane waveform a part of the mode may have to travel at velocities greater than that of light in the cladding. Since this is not possible, this portion of the mode must be radiated away.

It is useful to consider the unit commonly used to characterize loss in fibers. This is $dB\,Km^{-1}$ and defined as follows: If a beam of optical power P_i is launched into one end of a fiber and if, after a length L of fiber, the power remaining in this fiber is P_f then the loss is given by

$$\text{Attenuation (loss)} = \frac{10\log_{10}(P_i/P_f)}{L} \; dB\,Km^{-1}$$

Losses intrinstic to silica fibers have two main sources;

(a) Scattering Losses

(b) Absorption Losses

(a) Scattering Losses-

Light propagate in fibers that the material is homogenous. Silica is an amorphous material and thus suffers from structural disorder. The same basic molecular units are present throughout the material but these are connected together in an essentially random way. This results in a fluctation in the refractive index through the material with each irregularity acting as a point scattering center. The scale of the fluctuation is of the order of $\lambda/10$ or loss, and the scattering is known as Rayleigh scattering and characterized by an absorption cofficient that varies as λ^{-4}. In details the absorption cofficient due to Rayleigh Scattering α_R can be written

$$\alpha_R = \frac{8\pi^3}{3\lambda^4} n^8 p^2 \beta K T_B$$

n: Refractive index; p: Photoelastic cofficient; β: Isothermal compressibility at temp T_B.

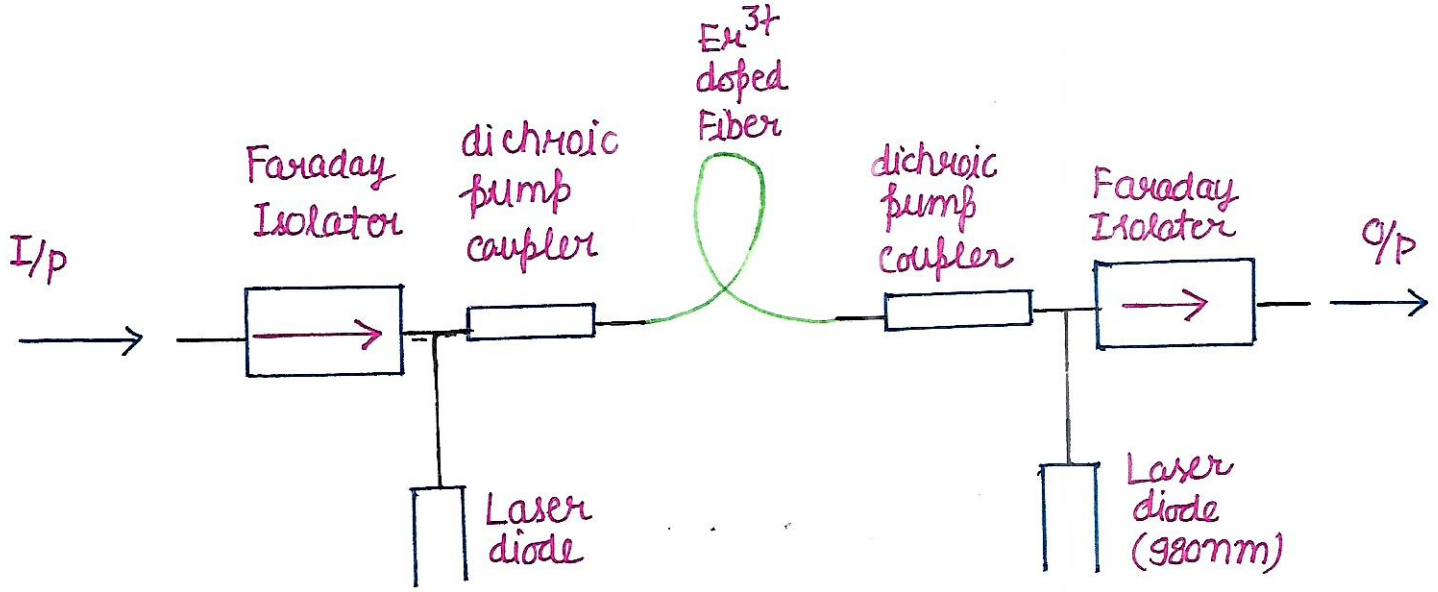
(b) Absorption Losses-

Absorption losses in the visible and IR regions arises mainly from the presence of impurities, trace of transition metal ions (Fe^{3+}, Cu^{2+}) or hydroxyl ions (OH^-).

Optical Fiber Amplifiers-

Optical Fiber amplifier is one of the key instruments for modern optical communication systems. A rare earth doped optical fiber amplifier, in which optical gain is provided by stimulated emission from rare earth elements, is the most widely used type of Amplifier.

Erbium, Praseodymium
Yetterium, Neodymium



Characteristics of fiber optics-

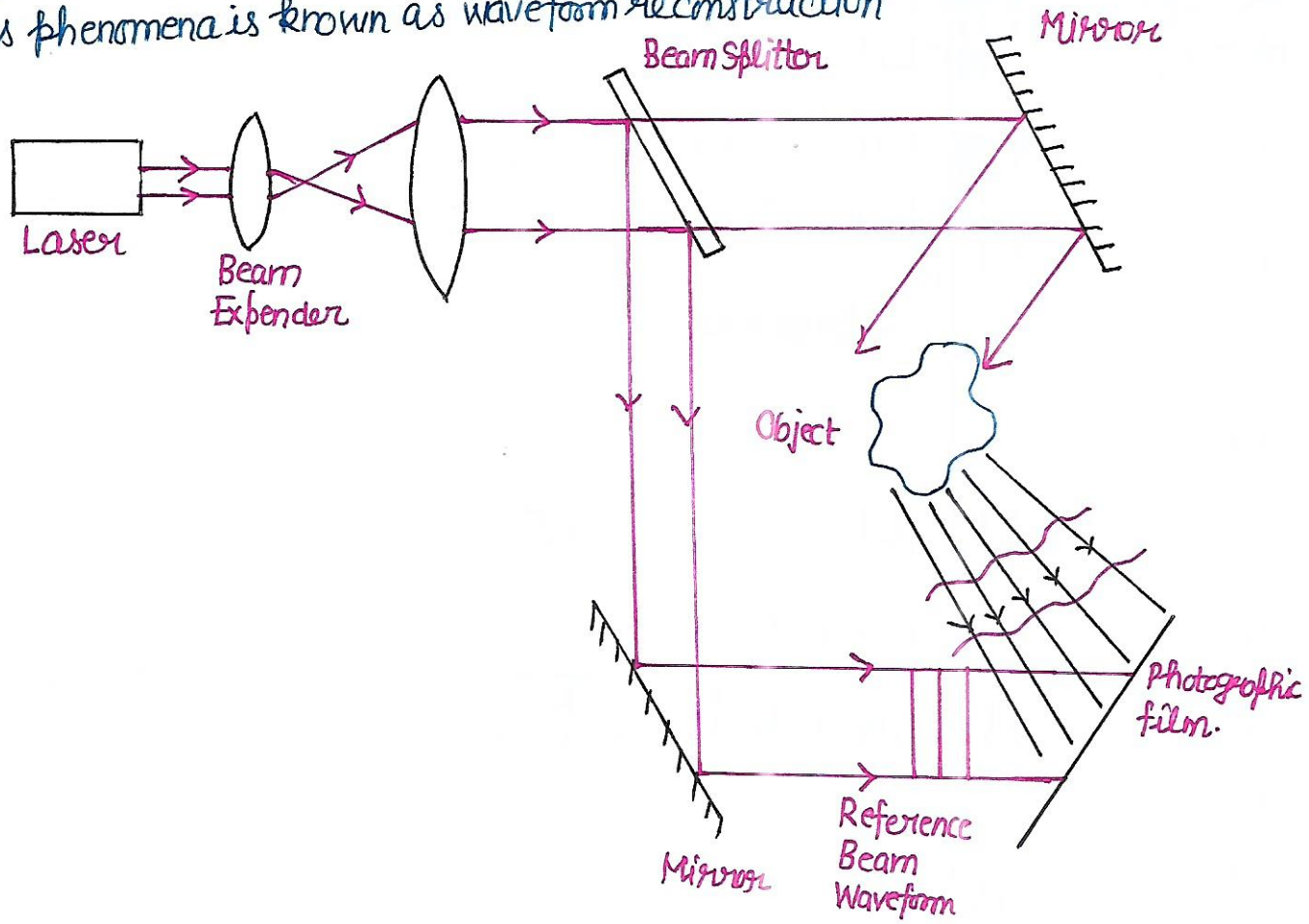
- Attenuation
- Chromatic dispersion
- Polarization dispersion
- Signal to noise ratio
- Self phase modulation and cross phase modulation
- Four wave mixing
- Raman and Brillion Scattering

UNIT-4

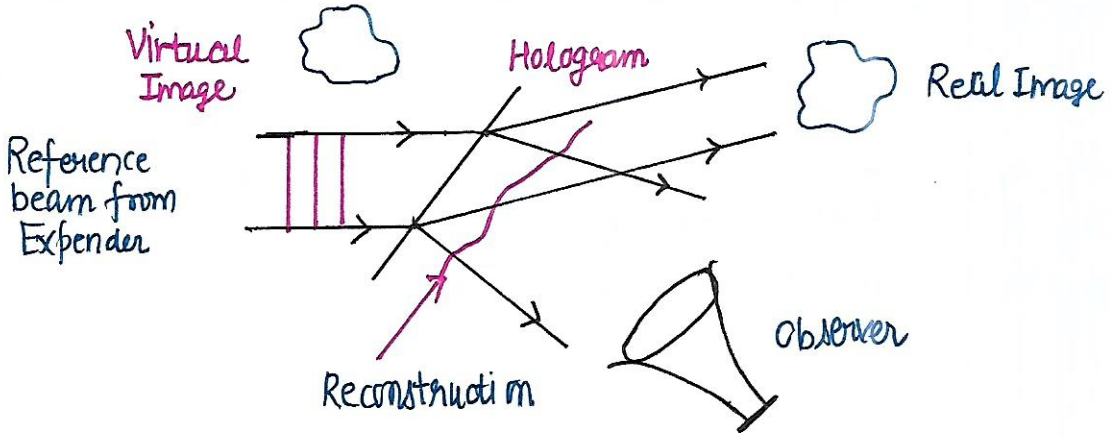
HOLOGRAPHY

Basic Principle, construction and Reconstruction-

Holography was developed prior to laser. The first holography was recorded by Gabor in 1948. The requirement of holography for light with a high degree of spatial and temporal coherence has closely linked with the development of holography. Holography is a method of recording information from a 3D object in such a way that a 3D image may subsequently be reconstructed. This phenomena is known as waveform reconstruction



Because of high degree of mutual coherence of two sets of (object) → wave produce an interference pattern on plate, which is recorded in the photographic emulsion and forms a hologram.



The photographic plate is now processed and illuminated with only the reference beam present. Most of the light from the reference beam passes straight through the hologram.

Constructive interference $\lambda = d\sin\theta$

↳ Local fridge spacing

Thus the construction of interference of these differacted wave reconstructed the original waveform from the object to an observer the waveform appear to be coming from the object itself.

The $\vec{E}$ of the waveform reflected from the object in the (x,y) plane at time t by

$$\vec{E}_{ob} = U_o(x,y)e^{-i\omega t}$$

↳ Amplitude

Irradiance

$$I(x,y) = |U_o + U_R|^2 = (U_o + U_R)(U_o^* + U_R^*)$$

The sum of Individual Irradiance

$$\boxed{I(x,y) = (U_o U_o^* + U_R U_R^*) + (U_o U_R^* + U_o^* U_R)}$$

#Application of Holography-

Undoubtebly, the full potential of the holographic technique is still to be realized although a number of application are now firmly estable. We shall discuss one established and one potential application namely holography Interferometry and computermemories.

- Holographic Interferometry-

A variation of the tecchnique is real time technique of holographic interferometry, in which the interference fringes are viewed in real time. A holograph is the subject that recorded as above.

Techniques –

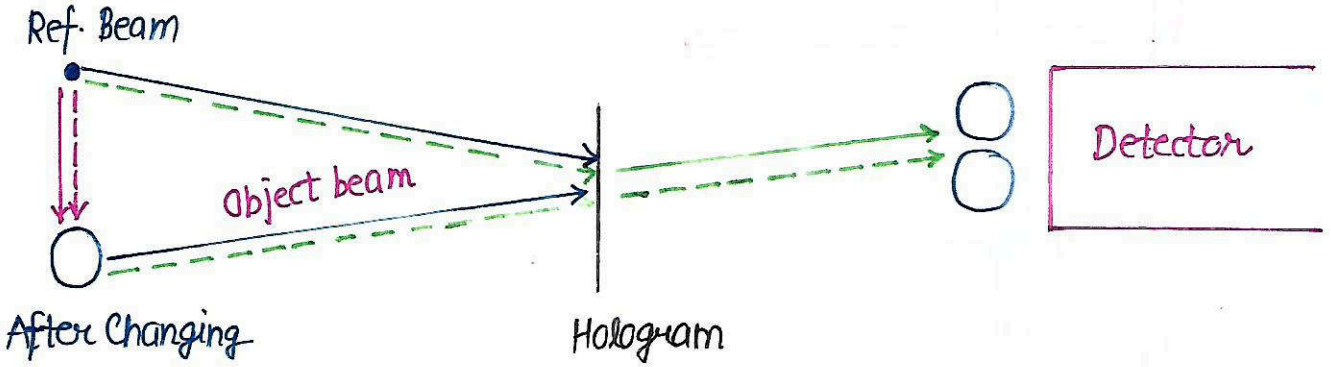
(i) Double exposure holographic interferometry – displacement of object is measured by counting fringes.

(ii) Real time holographic interferometry – time b/w two records.

(iii) Time-avarage holographic interferometry – Examine the spatial characteristic of low amplitude viberations of an object.

* Object move continuesly.

Holographic interferometry is a way to perform interferometry measures. It is a way to trace changes in object without mechanically touching the measured objects.



Applications –

→ Nondestructive testing such as detection of cracks

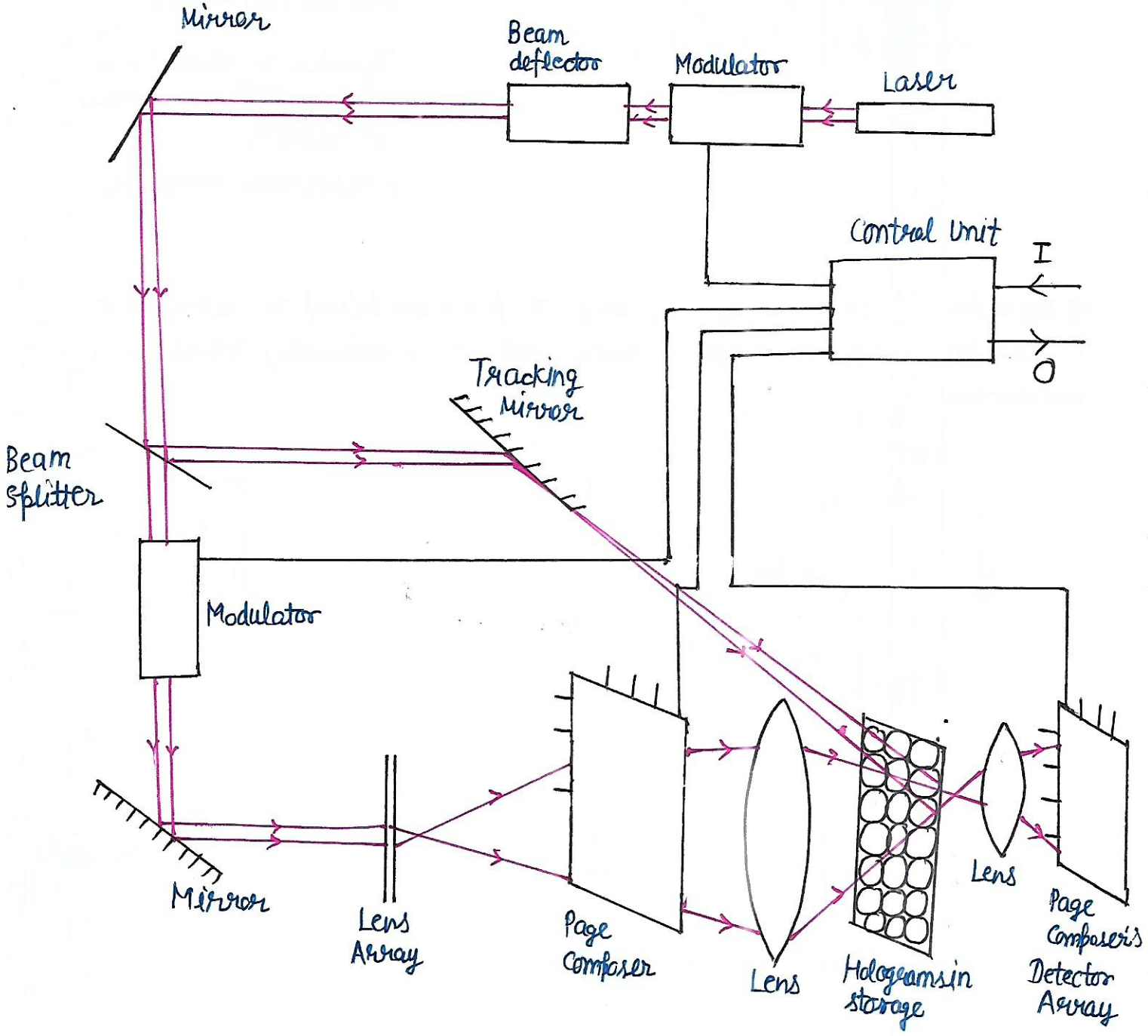
→ Analysis of viberating surfaces such as detecting of resonance of an aircraft tail.

→ Holographic contowing of objects.

• **Holographic computer memories–**

Holographic computer memories are being actively re-developed because they have a very high storage capacity - Theoretically up to 10^{10} bits mm^{-3} with rapid access. Though they are unlikely to replace established technologies such as magnetic disks, tape or optical disk storage, they could be used where very large total storage capacity is required. Holographic storage systems are also very interesting in that they are involved several of the opto electronic devices we have described, such as lasers, photodetectors, optical modulator or optical media.

A holographic memory records and read out a large number of bits simultaniously as we can appriciate by considering the following diagram.



The information to be stored in formed into a 2D array of bits by a device called a page composer. The page composer may be though of as an array of light valves which may be open or closed corresponding to ones, and zeros respectively. The array of up to perhaps 10^5-10^6 bits is then stored at one time in a particular location of the holographic memory. During recording, the light modulators allow maximum irradiance in both the signal and reference beam

Readout of the data is addressed when the hologram is only addressed only by the reference beam. The image is focused by a lens and projected onto the detector array such as a CCD array, which is now available with 2048 x 2048 pixels. The stored information is thus converted into electrical signals.

Advantages -

(i) The information about a particular array is distributed in a holographic fringe pattern accross the entire hologram.

(ii) The information is recovered essentially in parallel.

(iii) The holographic recording and construction is insensible to the exact position of the reference or reading beam on the hologram.

(iv) It is possible to record several holograms in different planes of one thick, sensitive material provided that different reference beam directions are used to record each hologram.

High Energy Applications of Laser -

For several years now lasers have been used to perform a large number of materials processing functions such as cutting, drilling, welding, marking, scribing and the surface treatment of a wide range of materials. These include materials such as diamond, ceramics, metals, wood, pliable materials and biological tissues. These technology process have often been successfully undertaken without the need to resort to relatively advances and expensive technologies.

Advantages -

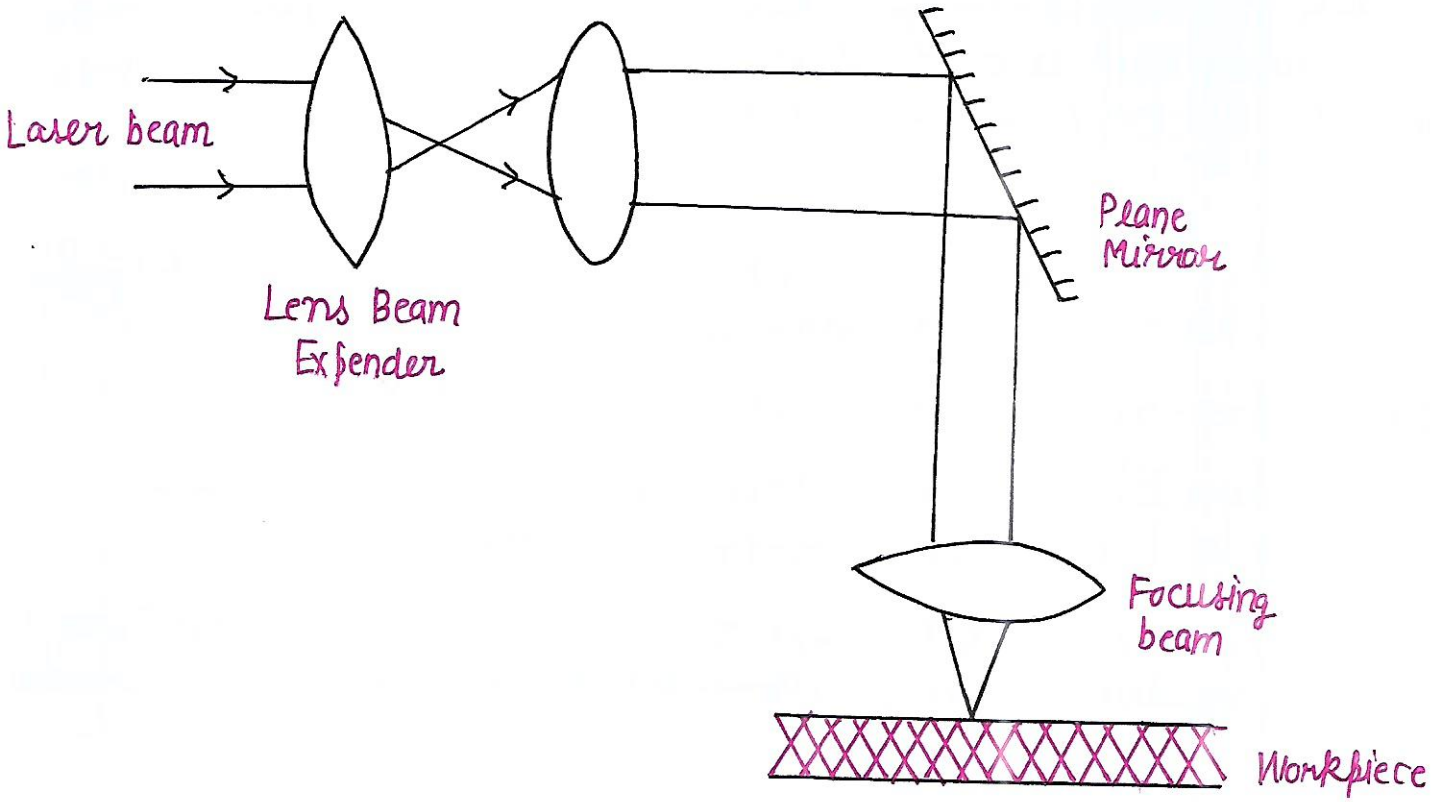
(i) Laser radiation is clean form of energy, in that no contaminating impurities need come into contact with the workpiece. Infact the working atmosphere can often be controlled to suit a particular task.

(ii) As we have seen laser beams, because of their high spatial coherence, may be focused into very small areas.

(iii) It is easy to control the irradiance.

(iv) The beam can be readily directed into relatively inaccessible locations.

The figure shows a basic beam delivery system.

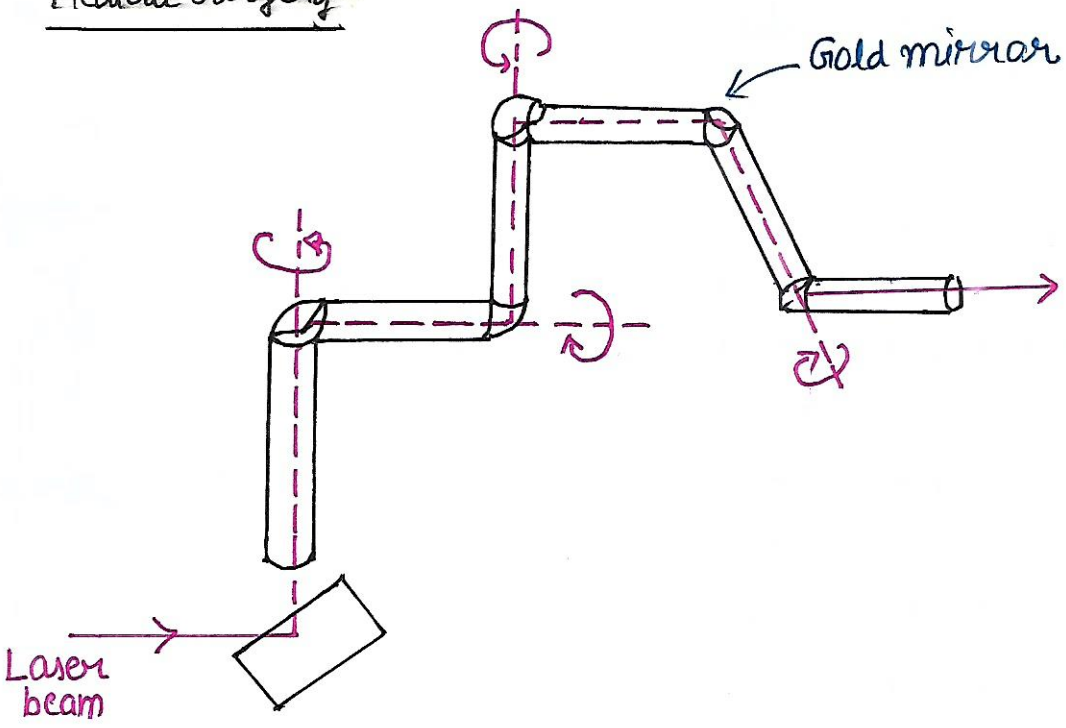


→ Not possible to put workpiece horizontally hence a system layout is designed.

→ Designed for inaccessible locations.

→ Beam delivery system use optical fibers

Medical Surgery-



It is articulated beam delivery system for use with CO_2 Laser in surgery.

This is for very careful control of the focus position on the workpiece, which may require manipulation of beam by an operator or machine vision system.

Industrial Application-

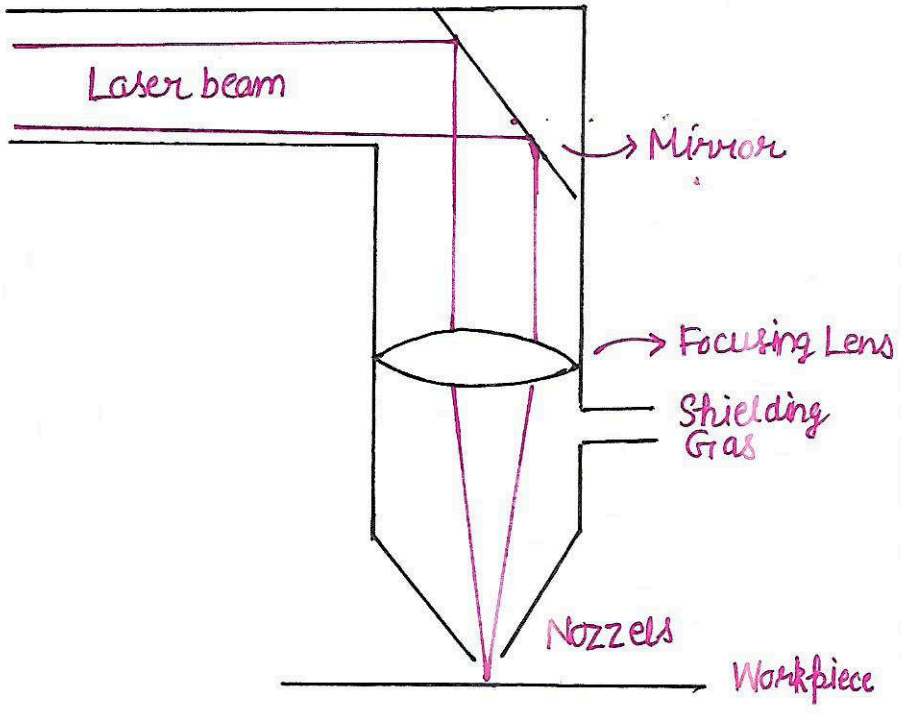
A detailed theoritical analysis of the interaction between a laser beam and a material surface is extremely difficult to carry out. The material parameters which must be taken into account including the reflection, thermal conductivity, Heat capacity, Latent Heat of fusion, Vaporization, melting point, boiling point, and absorption of laser radiation.

Nevertheless the theory enables orders of magnitude calculations to be undertaken to estimate quantities such as the temperature rise and rate of removal of material for given laser power inputs.

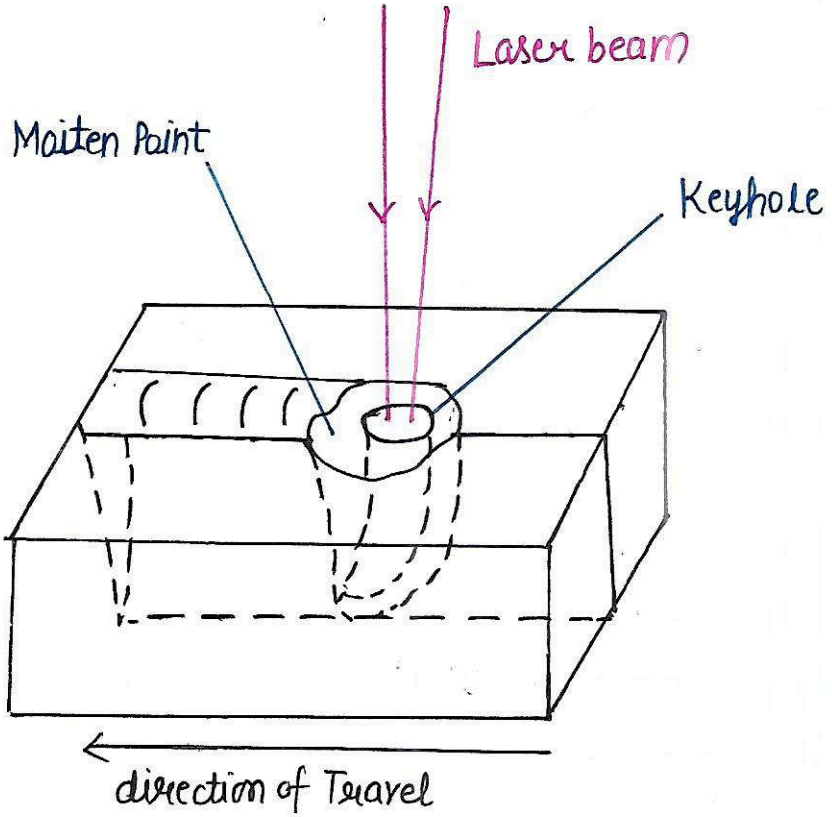
Laser welding has to compete with many well established such as soldering, Arc welding, Resistance welding, e- beam welding, Laser welding has no. of Advantages -

(i) Minimum Heat Input
(ii) Clean Technology
(iii) No physical contact with external components.
(iv) Weld dissimilar metals.
(v) Automated process.
(vi) Faster Weld rate.
(vii) High quality of resulting welds.

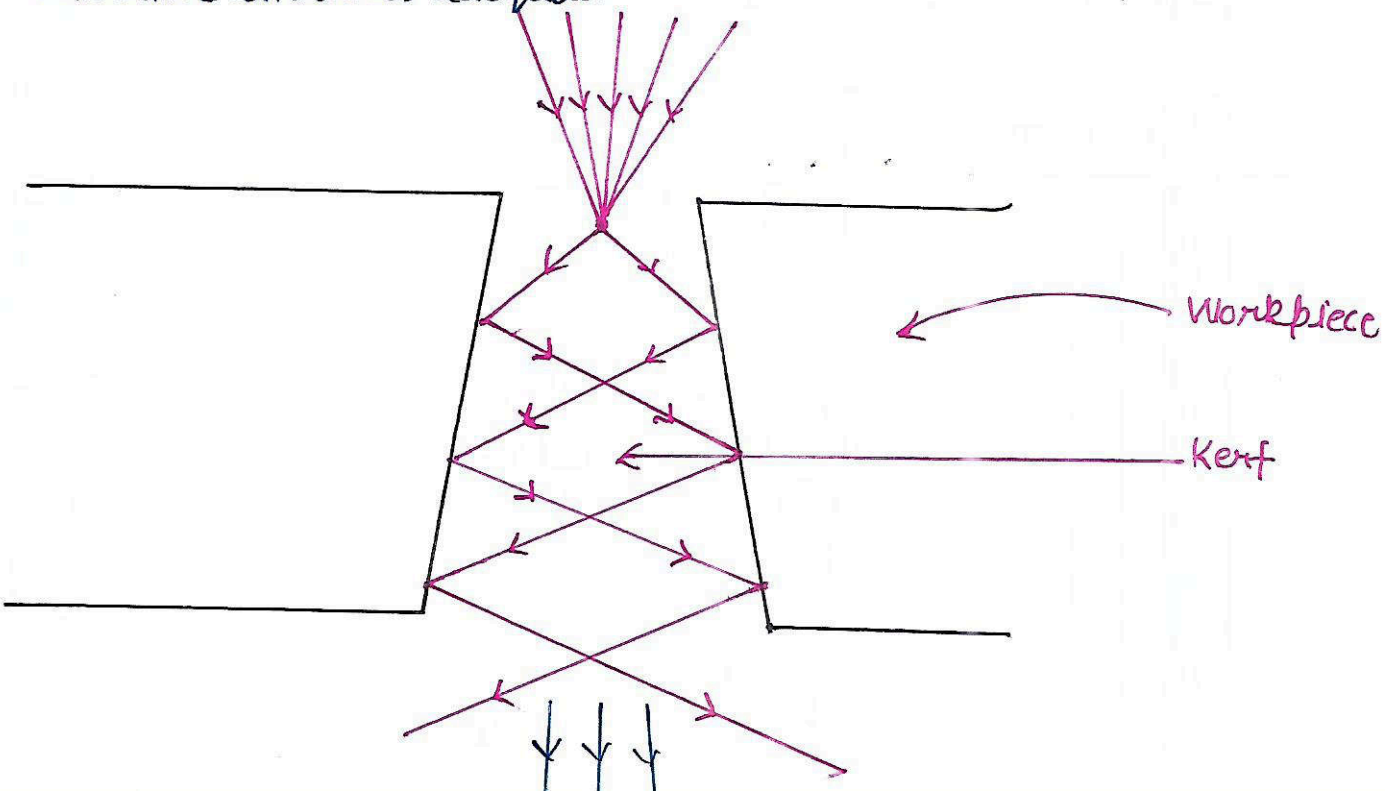
Welding is normally carried out using a shielding inert gas such as Ar or He.



When using multikilowatt CO_2 or high energy pulse mode lasers the welding process becomes rather more complicated. In these situations, when the laser beam first strikes the surface significantly amount of material may be vapourized forming a small hole known as a keyhole.



Laser cutting system are frequently essentially numerically method controlled system or robots. In this case of cutting sheet metal, for e.g. a two axis numerical control is adequate.



This was the sketch of Kerf product of Laser beam. The laser beam is channelled through the material by multiple reflections from the wall producing a waveguiding effect. A similar effect occurs in Laser drilling.

Medical Applications-

One of the most rapidly developing areas of laser applications is perhaps in medical. Many medical applications arise directly from the laser materials processing capabilities.

e.g.

CO_2 Laser ⟶ General Surgry.

Ruby Laser ⟶ Detached Retinas

One of the most commonest application of medical is use of CO_2 lasers in general surgery. The 10.6 μm emission wavelength is strongly absorbed by water molecules present in tissues, and the subsequently evaporation of the water leads to the physical removal of tissues.

A resent introduced Er:YAG Laser emits at wavelength of 2.94 μm, which is also strongly absorbed by water molecule, and the laser is consequently particularly useful for precise, localized tissues removal or ablation.

One of the earliest applications in ophthalmology was in treatment of detached retinas using light from a ruby Laser.

The hole drilling capabilities of lasers have been used to drill holes (fistulas) of about 300 μm diameter through the sclera to reduce the increased intraocular pressure in the eye resulting from glucoma.

Laser can also be used in the treatment of cancer, particularly in otherwise inaccessible parts of the body, such as the larynx and neck of the womb. Indeed Laser radiation has been used very successfully for several years in the treatment of early stages of cervical cancer.

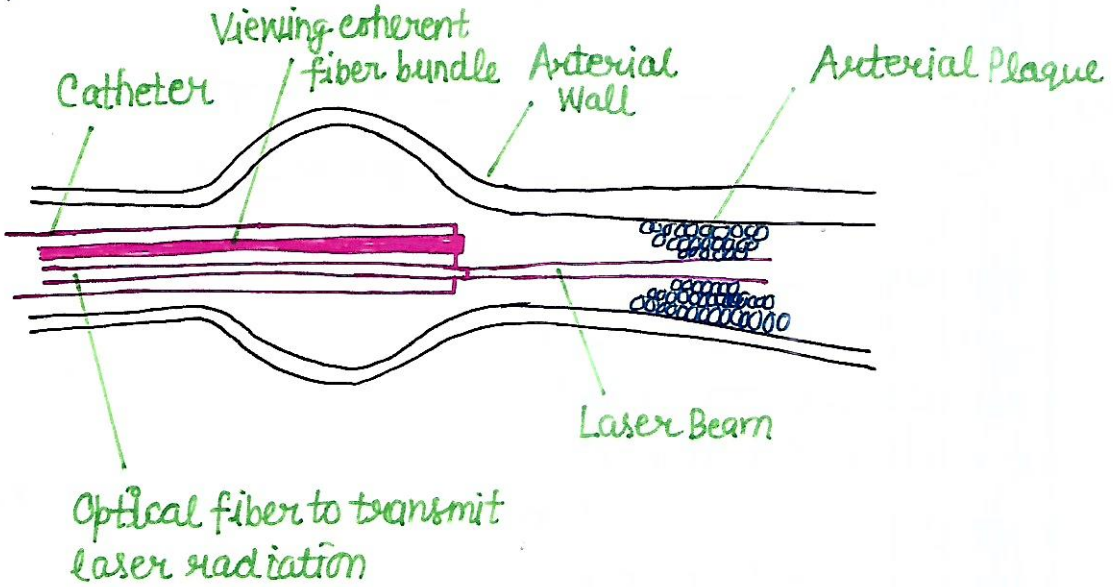
Laser Induced Nuclear Fission-

For many years, research has been directly towards a system for producing controlled their thermonuclear reactions to generate energy. Nuclear Fusion of light elements occurs within a very high temperature plasma such as exist in the sun.

A typical fusion reaction is

$$^{3}_{1}H + ^{2}_{1}H \longrightarrow ^{4}_{2}He + ^{1}_{0}n + 14\,MeV$$

This reaction yields a helium atom and an energetic neutron. This Figure shows the removal of arterial plaque using laser radiation transmitted down a fine optical fiber inserted into the artery. A viewing fiber bundle is also included in the tube inserted into the artery.



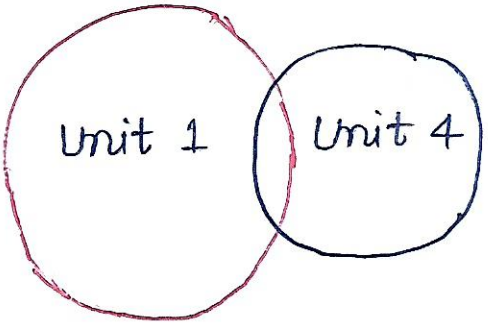
Important Questions

Environmental Physics

- Greenhouse Effect ✓
- Ozone Depletion on Earth ✓
- Atmospheric Layers ✓
- Air Pollution ✓
- Solar and Terrestrical Spectra ✓
- Water Pollution ✓
- Global Climate Models ✓
- Safe Waste disposal ✓

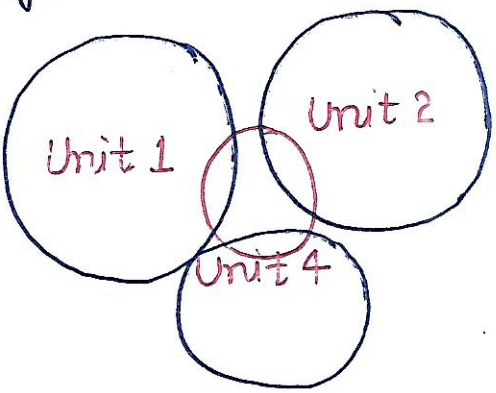
Computational Physics

1. Newton's Forward Interpolation formula for equal intervals, question ✓
1. Simpson's 3/8 Rule, question ✓
1. Simpson's 1/3 Rule, question ✓
2. Determine Eigenvalues and Eigenvectors ✓
1. Least square fitting method ✓
3. Flowchart concept and Advantages ✓
2. Guassian Elimination ✓
4. C Language (Input, Output Statement) ✓
1. Runge-Kutta method, question ✓
1. Trapezoidal Method, question ✓
4. C Language (Loop, Flowchart and matrix multiplication) ✓
4. Fortran 90 (Loop, Flowchart, matrix multiplication ✓
2. Guass Jacobi Method ✓
4. C Language (Arrays) ✓



Particle Physics

2. Parity Conversion ✓
2. CP Invariance ✓
2. Time Reversal ✓
2. Charge Conjugation ✓
2. Isospin Conservation ✓
2. CPT Theorem ✓
3. Fermi Theory of β decay ✓
1. Properties of elementary particles ✓
3. Weak Interaction ✓
X. Tau-Theta Puzzel ✓
1. Funamental particles ✓
4. Flavour and colour quantum number of quarks. ✓
4. Strangeness and Quark composition of mesons and Baryons ✓
4. Higgs Boson ✓
2. Reactions allowed or not X
4. Classification of Hadron and quark model. ✓
2. Global Guage Invariance. ✓



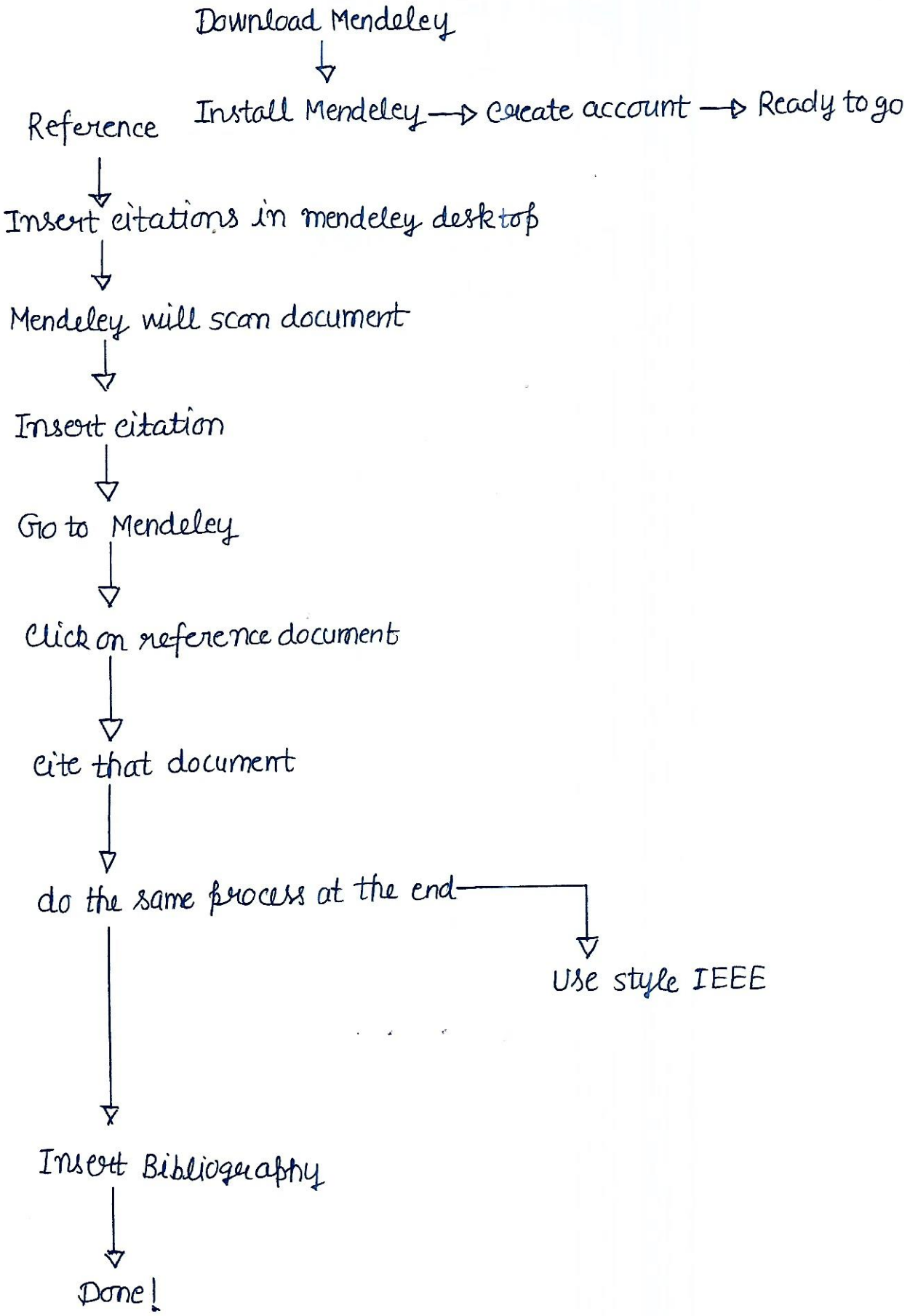
Electronics B

1. Modulation (Single Side band modulation) and Features ✓
1. Frequency modulation by reactanace modulator ✓
1. Demodulation ✓
1. Envalope tube modulator ✓
1. Foster Seely Phase modulator ✓
1. Super heterodyne receiver ✓
2. Transmission Line (TL) Eqn ✓
2. Linear Anteena ✓
3. Electes- Larmor Theory ✓
3. Chapmann's Theory of Layer formation ✓
3. Appleton-Hartee Theory ✓
4. Basic Principle transmission and receiver ✓
4. Colour TV system and Black & White TV system ✓
1. Plate modulated Class C amplifier ✗
2. Standing wave ratio and reflection cofficient ✓
4. Azimuth and Range measurement ✓
3. Skip distance and Maximum unstable frequency ✗
2. Quarter wavelength and Half wave length lines ✗
2. Hertz Anteena ✓
2. Marconi Anteena ✓
2. Yagi Anteena ✓
2. Loop Anteena ✓
2. Lens Anteena ✓
4. Duplexers ✓

LASER Physics

1. Electro-optic Effect ✓
1. Acousto-optic Modulation with diagram ✓
2. Photovoltaic and Photoconductive method of detection of Light ✓
3. Fibre Optics and its Basic characteristics ✓
3. Radiation and Core cladding Losses ✓
4. Principle of Holography and use in computer memories ✓
3. Absorption Scattering ✓
3. Optical Fibre Amplifications ✓
3. Step and graded Index Fibres ✓
1. Raman Nath and Bragg Regims ✓
2. PIN diode and Avalanche Photodiode ✓
4. Applications of Holography ✓
1. Magneto-Optic Effect ✓
4. Application of Laser in Medical Science and Industry ✓
2. Optical directional couplers ✓
2. CCD ✓
2. Photomultipliers ✓
4. Laser Interferometer
2. LCD display ✓

→ Install Mendeley in windows :



- Install Linux version:

Install Linux version

↓

Add Libre office plugin

↓

Then same procedure as Windows

CENTURY OF QUANTUM MECHANICS STILL GOING ON

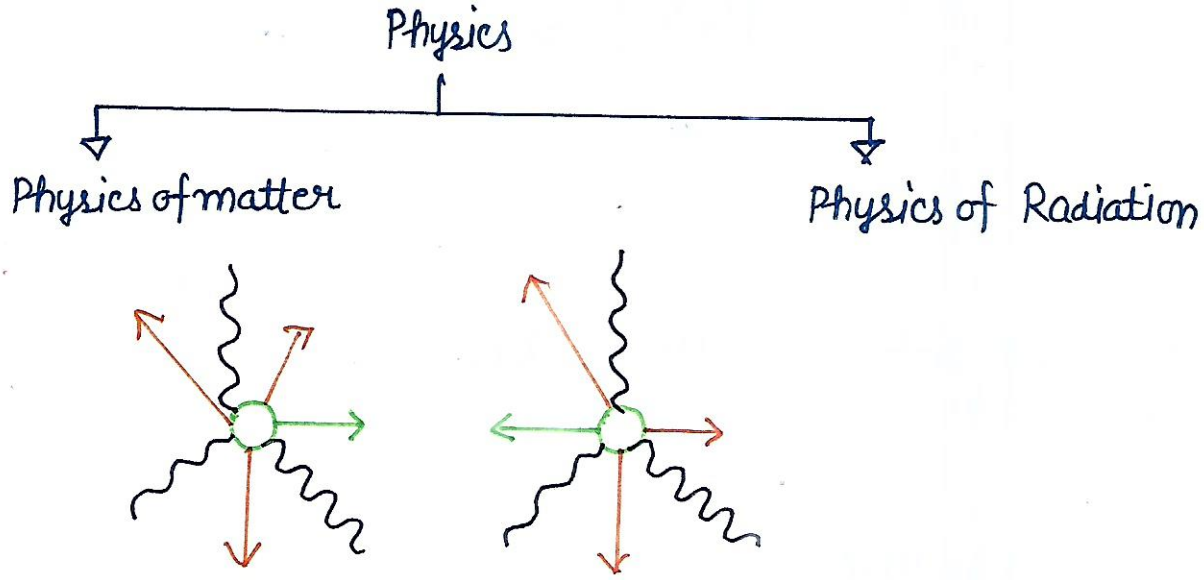
Webinar Notes

• Birth of Modern Physics

By Prof. Lokhanathan

→ Prism Experiment (In his badroom).

→ Apple falling story that leads to Gravitational Law.



- Thermodynamics is the only physical theory universal content concerning which I am convienced that, within the framework of the applicability of its basic concepts, it will never be overthrown.

• Kirchoff's Theorem -

- Energy is function of temperature (Frequency).

$$\lambda max = \frac{b}{T}$$

- Photons

↓

Problem → Wave & Particle duality

Einstein energy relation $v = \frac{\left(\frac{1}{h}\right) m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{v_0}{\sqrt{1-\frac{v^2}{c^2}}}$

$$v = \frac{c}{\sqrt{1-\frac{v^2}{c^2}}}$$

* Phase Wave that accompanying.

⇩

Birth of modern Physics.

• Evolution of Quantum Mechanics

By Prof. Ajoy Ghatak

- Ever man could see, wanted to know about light.
- Snell's Law.

⇩

Opticks by Newton

- Light can propagate through vaccum.
- Scattering nature of light
- Hygens wave theory.

- Thomas Young Experiment -

Light + Light produce darkness.

- Light is neither light wave or particle.

↓

ψ

↓

Schrodinger Eqn

↑

↓

No derivation

• Quantum Cosmology

By Prof. Pankaj Joshi

- Quantum Mechanics has limitations.

⇩

- Relativistic Quantum Mechanics

⇩

Relativity of Einstein Special Theory

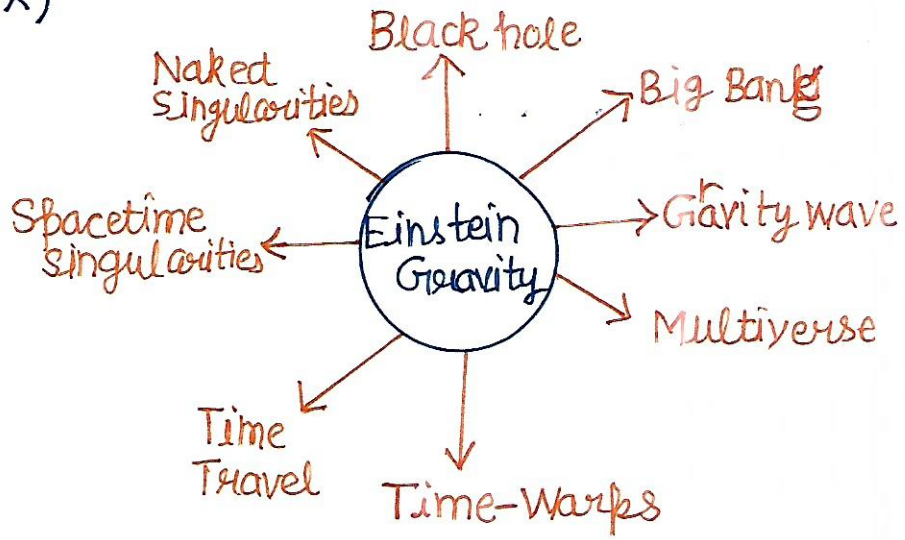
- We have Quantum Field Theory but as general relativity.
- There is a great problem of Quantum Gravity.
- Newton gravity isn't enough.

Quantum Cosmos

⇩

Quantum cosmology Framework

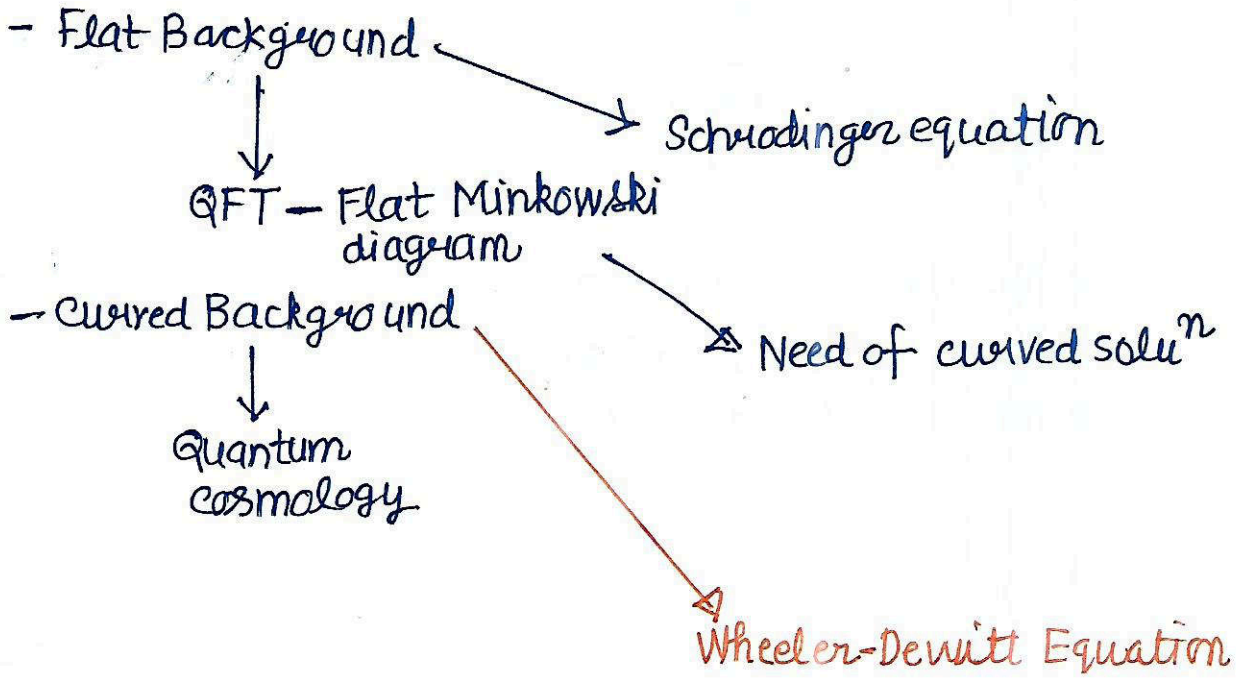
- Can we unify fundamental forces in a single theory of physics?
- Grand Unified Theory
- Gravity (X)



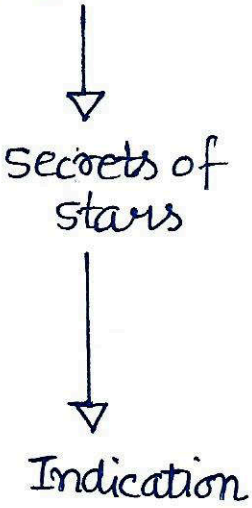
- Local geometery is Flat but spacetime is curved.



- String Theory
- Loop Quantum Gravity:
- Hawking Radiation from Blackholes.
- Quantization in a curved background.

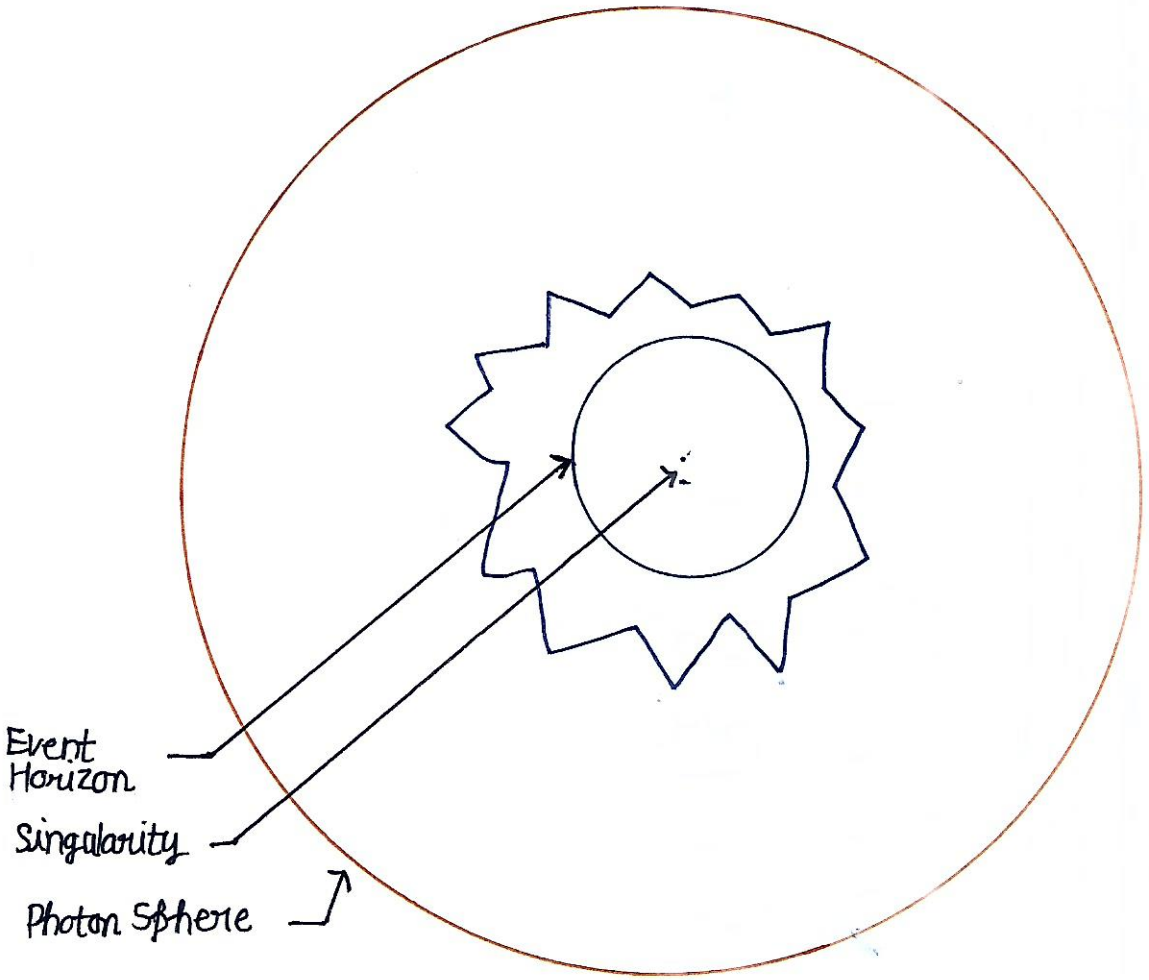


- What happens when a massive star dies ?

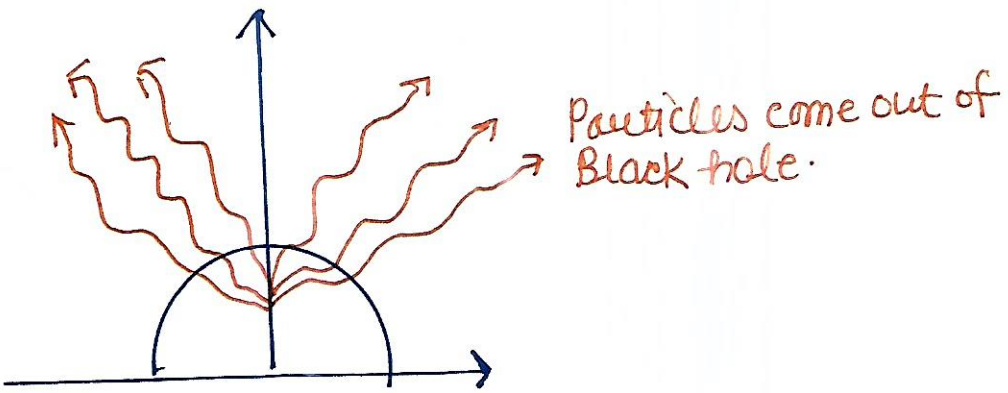


- Chandrashekher's Thesis
- Gravitational collapse

Black Hole



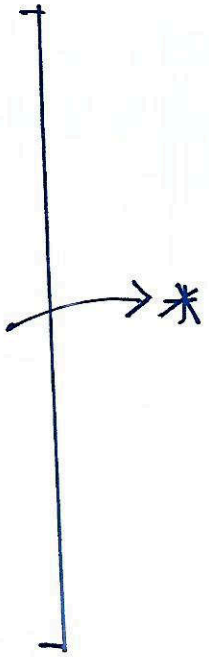
- Black hole and naked singularities final state develop as the collapse outcome.
  - PDE is more complicated than ODE.
  - Sometimes stars collapse create Blackhole, sometimes singularities.

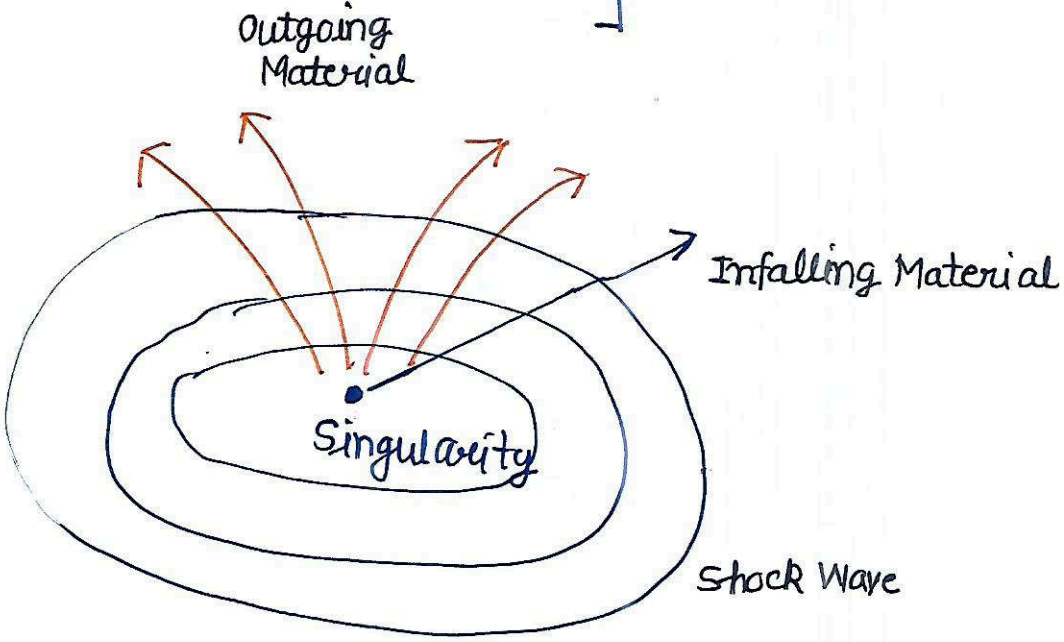


Heavy Weight star → Black hole

Bantum Weight star → Neutron star

Feather Weight star → White Dwarf





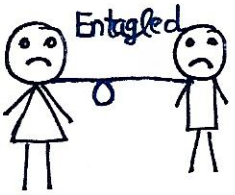
- Quantum gravity signal
- Paradoxs

The causality or Teleological Paradox

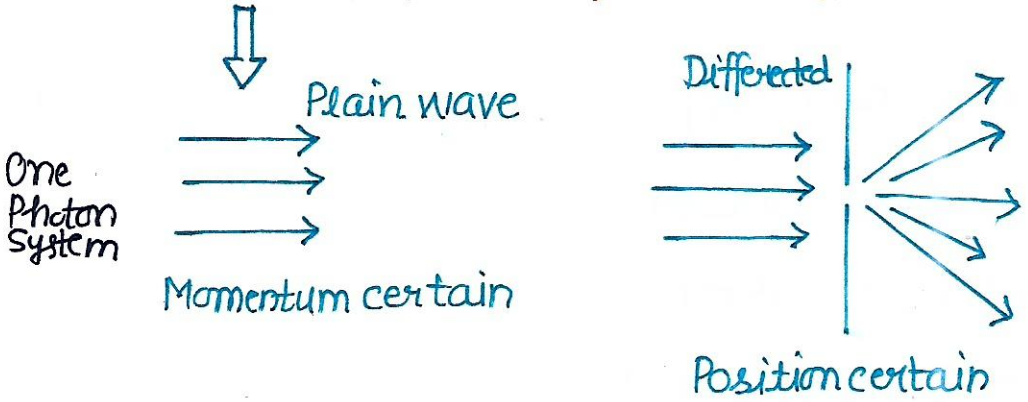
The Information Paradox

Central Singularity and Stability Issue

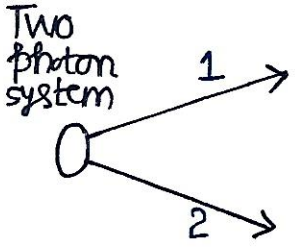
• Quantum Entaglement



- Einstein Objected to this kind of phenomenon.
- EPR Paradox (A. Eistein, B. Podolsky)



$$\Delta x \Delta p \geq \frac{\hbar}{2}$$



$$\psi(x_1, x_2) = \int e^{iKx_1} e^{iK(x_2 - a)} dK$$

$$\psi(x_1, x_2) = \int e(x_1 - x)\, \delta(x_2 - x - a)\, dx$$

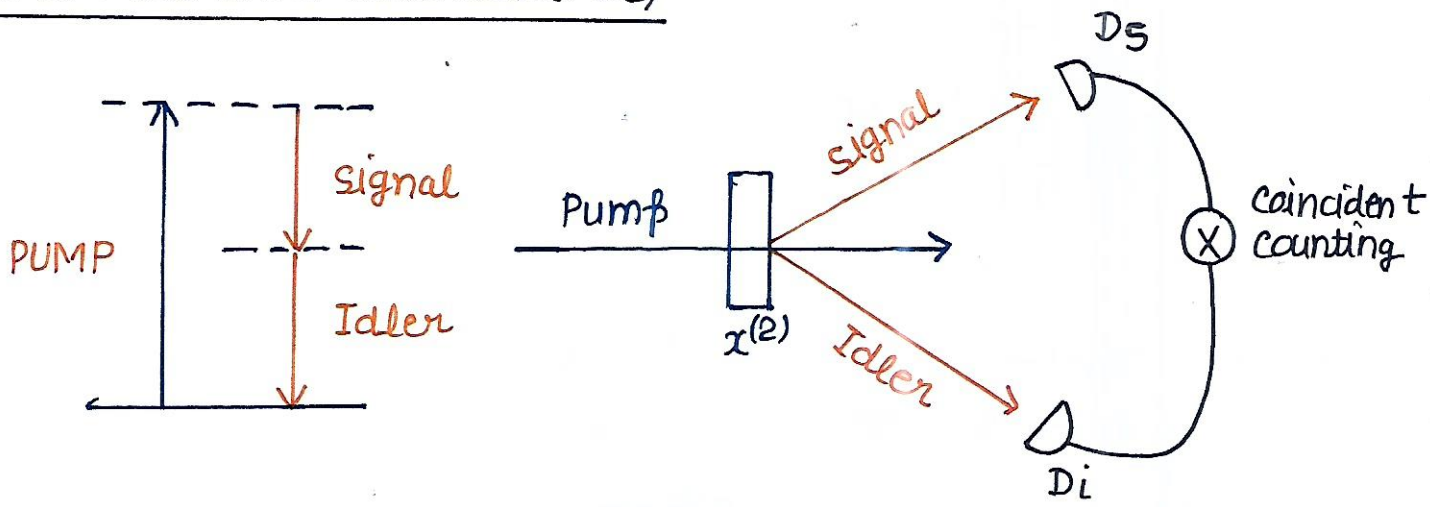
$$\Delta x^{(1)}_{cond} \Delta p^{(1)}_{cond} < \frac{\hbar}{2}$$

Non Local correlation??

EPR Questions :

- Is quantum mechanics incomplete?
- Does it require additional "hidden variable" to explain measurement result.

• Parametric down-conversion (PDC) –



Two-photon field $|\psi_{tp}\rangle \neq |\psi\rangle_s \otimes |\psi\rangle_i$

$q_p = q_s + q_i$ Conservation of momentum

$\omega_p = \omega_s + \omega_i$ Conservation of energy

$I_p = I_s + I_i$ Conservation of Orbital angular momentum

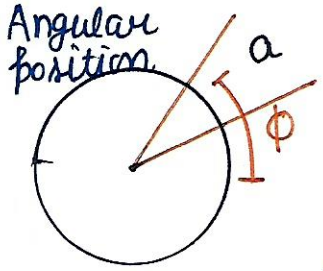
$$\frac{J_z}{W} = \frac{\iint \rho d\rho d\phi (P \times (E \times B))_z}{C \iint \rho d\rho d\phi (E \times B)_z} = \frac{\hbar I}{\hbar \omega}$$

Other method ! Four wave mixing

$$\Delta x^{(1)}_{cond}\ \Delta p^{(1)}_{cond} < \frac{\hbar}{2}$$

$$\Delta t^{(1)}_{cond}\ \Delta E^{(1)}_{cond} < \frac{\hbar}{2}$$

$$\Delta \phi^{(1)}_{cond}\ \Delta L^{(1)}_{cond} < \frac{\hbar}{2}$$



$$A_i = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} d\phi\, \psi(\phi) \exp(-iI\phi)$$

$$\psi(\phi) = \frac{1}{\sqrt{2\pi}} \sum_{-\infty}^{+\infty} A_i \exp(iI\phi)$$

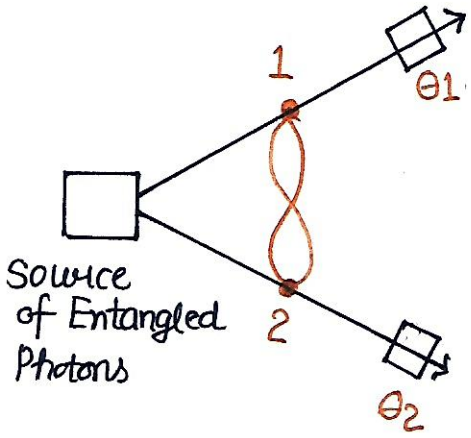
Entanglement in Polarization } two dimensional entaglement

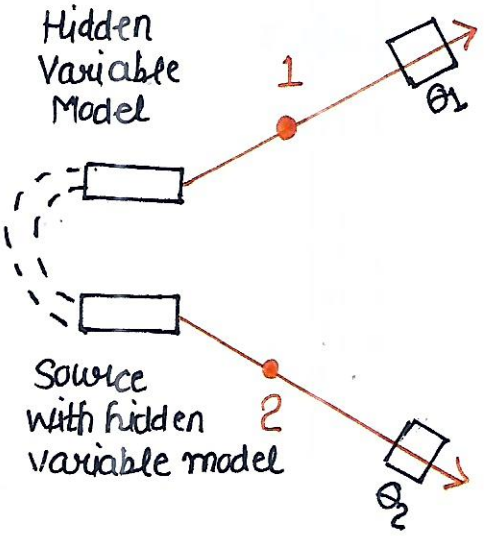
$$|\psi\rangle = \frac{1}{\sqrt{2}}[|H_S\rangle|H_i\rangle + |V_S\rangle|V_i\rangle]$$

$$|45\rangle = \frac{1}{\sqrt{2}}[|H\rangle + |V\rangle]$$

$$|-45\rangle = \frac{1}{\sqrt{2}}[|H\rangle - |V\rangle]$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}[|45_S\rangle|45_i\rangle + |-45_S\rangle|-45_i\rangle]$$





- Signal → Horizontal Verticle
  Idler → Verticle Horizontal

$$|\psi\rangle = |H_S\rangle|H_i\rangle + |V_S\rangle|V_i\rangle$$

- can two independent classical source using suchan arrangement
  -- Yes
  -- Emit Simultaniously
  -- Same Polarization
  -- 50% of the time both emit vertical polarization.

⇩

Is this entanglement

⇩

No

$|\psi\rangle = |45_s\rangle|45_i\rangle + |-45_s\rangle|-45_i\rangle$

⇩

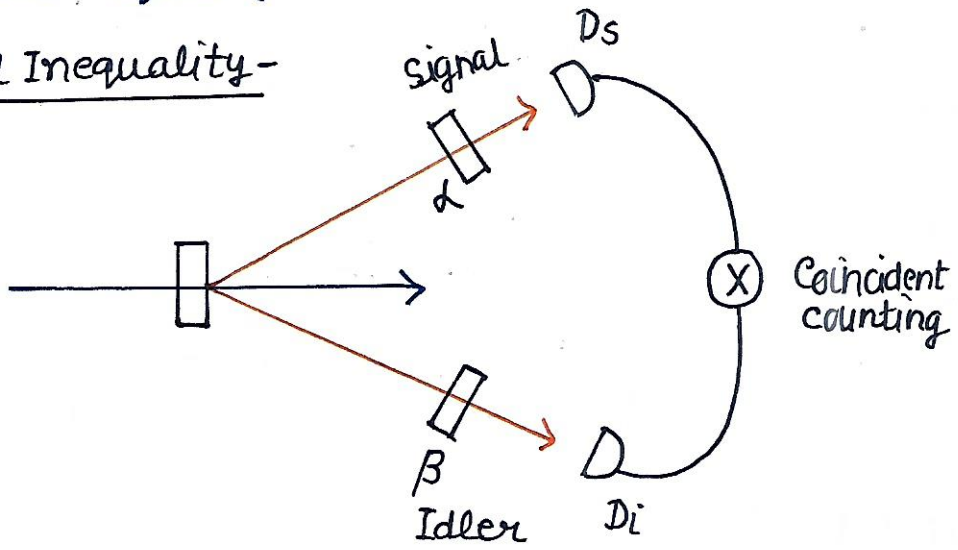
Is this entanglement ?

⇩

Of course, No

- Entanglement is always simultanious correlation into seperate conjugation basis.
- Hidden variable quantum mechanics by David Bohm.
- Bell Inequality

• **Bell Inequality -**



$$S = E(a,b) - E(a,b') + E(a',b) + E(a',b')$$

$$E(\alpha,\beta) = \frac{N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp) - N(\alpha,\beta_\perp) - N(\alpha_\perp,\beta_\perp)}{N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp) + N(\alpha,\beta_\perp) + N(\alpha_\perp,\beta_\perp)}$$

$\alpha = -45^\circ$ $\quad \beta = -22.5^\circ$

$\alpha' = 0^\circ$ $\quad \beta' = 22.5^\circ$

$\alpha_\perp = 45^\circ$ $\quad \beta_\perp = 67.5^\circ$

$\alpha'_\perp = 90^\circ$ $\quad \beta'_\perp = 112.5^\circ$

$|S| \leq 2$ For hidden variable theories

$|S| \leq 2\sqrt{2}$ For quantum correlation

- Applications of Quantum Entanglement-

- Hong-Ou Mandel Effect
- Quantum Lithography
- Quantum computation
  - Two qubit state
    - Polarization
    - Time bin
    - Position
    - Angle
- Quantum Cryptography

Alice

Bob

$|\psi\rangle = |H\rangle_{Alice}|H\rangle_{Bob} + |V\rangle_{Alice}|V\rangle_{Bob}$

$|\psi\rangle = |D\rangle_{Alice}|D\rangle_{Bob} + |A\rangle_{Alice}|A\rangle_{Bob}$

- Quantum key distribution
- Quantum Teleportation
  - -- over 143 Kms
- Quantum Information
- Quantum remote sensing
- Quantum Metrology

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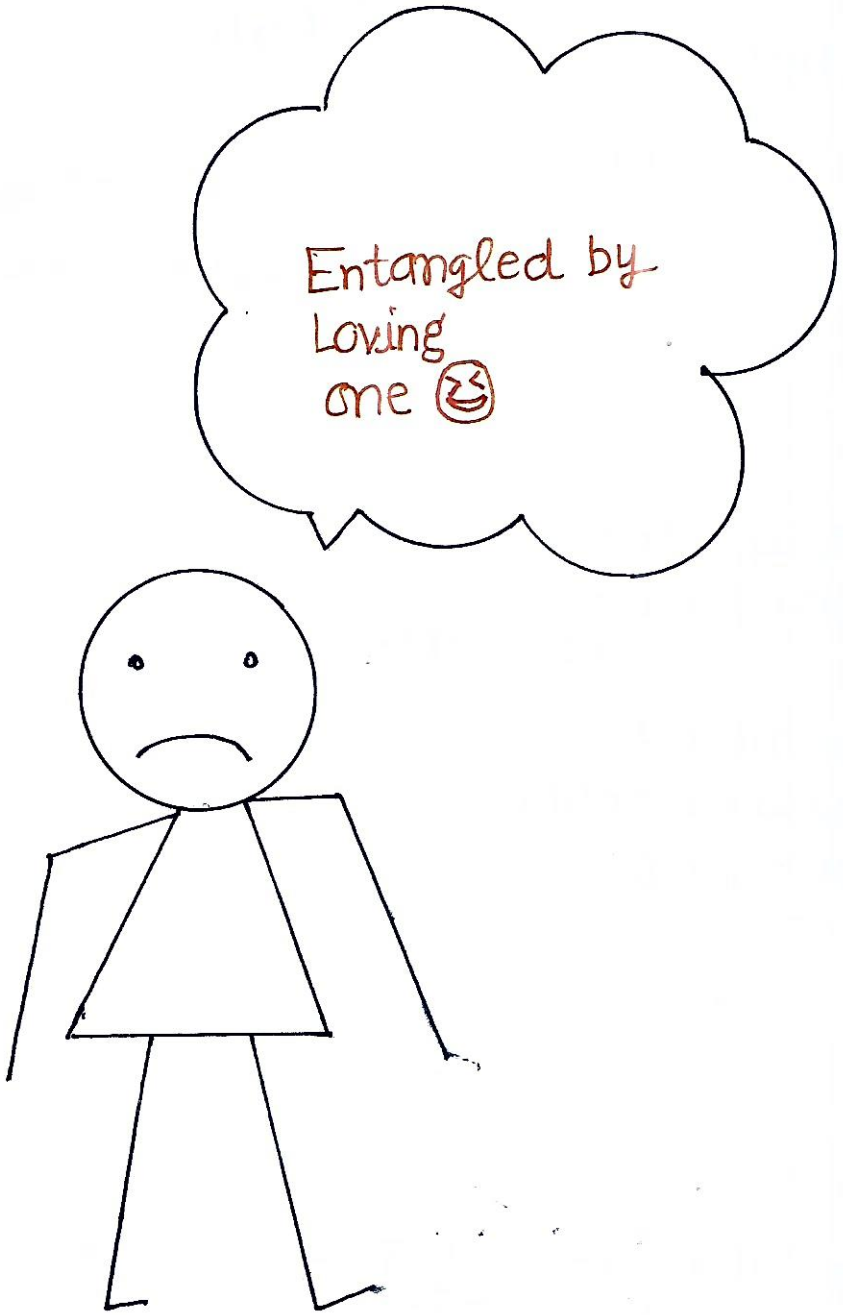
- Application of Quantum Mechanics Machine Learning methods to Materials Design

Stone Age 3300BC → Bronze Age 1200BC → Iron Age 600BC → Present Age → Future

- Materials have played an important role in the evolution of human civilization.

→ Machine Learning


Entangled by
Loving
one

• Uncertainty Principle

- Darwin's Origin of species
- Gobel Impossible Theorem
- Understanding genetics codes
- Quantum Revolution
- Unanswered question of Einstein
- S. Hawking

Wave particle duality



$\Delta x = \infty \qquad \Delta p_x = 0$

•

•

$\Delta x = 0 \qquad \Delta p_x = \infty$

$$\Delta x = (\langle x^2 \rangle - \langle x \rangle^2)^{1/2}$$

$$\Delta p_x = (\langle p_x^2 \rangle - \langle p_x \rangle^2)^{1/2}$$

$$\Delta x \, \Delta p_x \geq \frac{\hbar}{2}$$

$$\Delta E \, \Delta t \geq \frac{\hbar}{2}$$

ELECTRODYNAMICS

- UNIT 1
  BASIC ELECTROSTATICS
- UNIT 2
  ANALYSIS OF $\vec{E}$ FIELD
- UNIT 3
  $\vec{E}$ and $\vec{B}$ FIELD
- UNIT 4
  IN DEPTH SOLUTION FOR MAXWELL EQUATIONS

Alessandro Volta → Volt produced first large electric current
George Simon Ohm → Ohm experimented with resistance
Michael Faraday → discovered EMI
James Maxwell
Henry Lorentz
Richard Feynman

• Scope -

- Electrical Force and QM Effect
- Concepts of field, flux and circulation
- Vector calculas
- Electrostatics and Guass Law
- Application of Guass Law and Electric Shielding.

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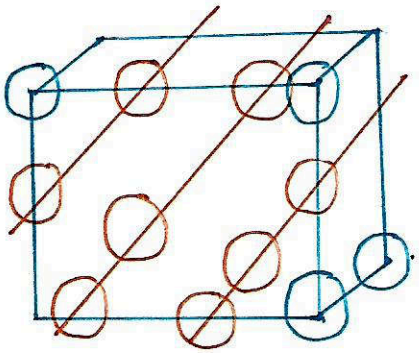
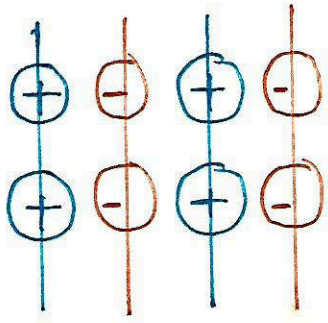
• Electrical Forces -

- 1% more protons than electrons in human body.
- So perfect is the balance that you don't feel any force from a person next to you.

Net $\vec{E}$ force = 0





$F \propto \frac{1}{r^2}$

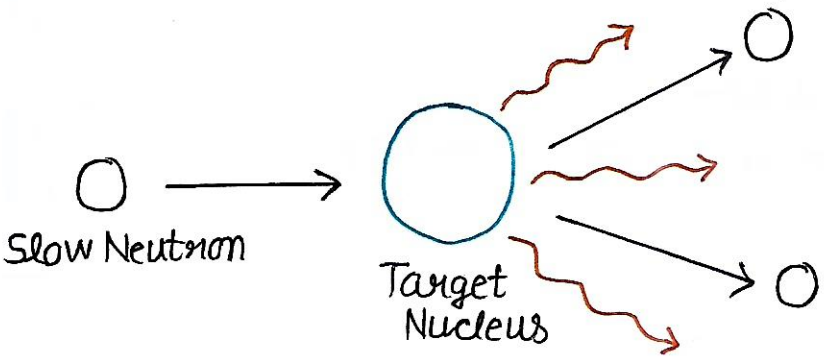
Chemical Force = Electric Force

If this Electrical force is so terrific, why don't the protons and e- get on the top of each other?

$$\sigma_x \sigma_p \geq \frac{\hbar}{2}$$

- What holds the nuclear together?

  Nuclear Force ; Greater than Electrical Energy but falls off much more rapidly than $1/r^2$ (Yukawa Potential)



(in form of Electrical Energy)

- Material Structure and their properties ?

  - Electrical Force
  - Quantum-Mechanical Force

→ Mechanical, Electrical Properties

EM Force $F = q(E + v \times B)$

Relativistic Equation $\frac{d}{dt}\left[\frac{mv}{\sqrt{(1 - v^2/c^2)}\,1/2}\right] = F = q(\vec{E} + v \times \vec{B})$

- Superposition of Field –

$$\vec{E} = E_1 + E_2$$
$$\vec{B} = B_1 + B_2$$

- E and B Field produced by one charge-

$$E = \frac{-q}{4\pi\epsilon_0}\left[\frac{e_{r'}}{r'^2} + \frac{r'}{c}\frac{d}{dt}\left(\frac{e_{r'}}{r'^2}\right) + \frac{1}{c^2}\frac{d^2}{dt^2}e_{r'}\right]$$

$$B = -e_{r'} \times \frac{E}{c}$$

- Field- Any physical quantity which takes on different values at different places.

- Vector Field-

Transport phenomena.

E → Flux $\quad \oint_S E \cdot da = \frac{Q}{\epsilon_0}$

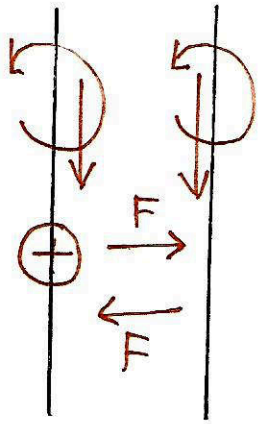
E → Circulation $\quad \oint_C E \cdot ds = -\frac{d}{dt}\int_S B \cdot da$

B → Flux $\quad \oint_S B \cdot da = 0 \Rightarrow$ Magnetic Monopole (X)

B → Circulation $\quad c^2 \oint_C B \cdot ds = \frac{d}{dt}\int_S E \cdot da + \frac{1}{\epsilon_0}\int_S J \cdot da$

$$F = -q(E + v \times B)$$

$$F = -q(E \neq v \times B)$$



What is a magnet?

→ Permanent currents in the atoms.

- The motion of the e⁻ in atomic orbits
- The spin of e⁻

- Magnetism is Relativistic Effect.

#Scalars and Vectors-

Field — Flux
Field — circulation

Dirac said, "I understand what an equation means if i have a way of figuring out the characteristics of its solution without actually solving it.".

$$A \cdot B = \text{Scalar} = A_x B_x + A_y B_y + A_z B_z$$

$$A \times B = \text{Vector}$$

$$(A \times B)_x = A_y B_z - A_z B_y$$

$$(A \times B)_y = A_z B_x - A_x B_z$$

$$(A \times B)_z = A_x B_y - A_y B_x$$

$$A \times A = 0$$

$$A \cdot (A \times B) = 0$$

$$A \cdot (B \times C) = (A \times B) \cdot C$$

$$A \times (B \times C) = B(A \cdot C) - C(A \cdot B)$$

- Equalities

$$\Delta f(x,y,z) = \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y + \frac{\partial f}{\partial z}\Delta z$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$

• Heat Flow-

h, A vector which points in the direction of flow and has a magnitude equal to the amount of thermal energy.

$$h = \frac{\Delta J}{\Delta a} ef$$

* Characteristics-

<Taylor series> $= f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 +$

↓

Without moving your object, you can determine the positions.

$\frac{\partial T}{\partial x}$ ← derivative of temp. with respect to position.

Prove $\left(\frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}, \frac{\partial T}{\partial z}\right)$ = Vector

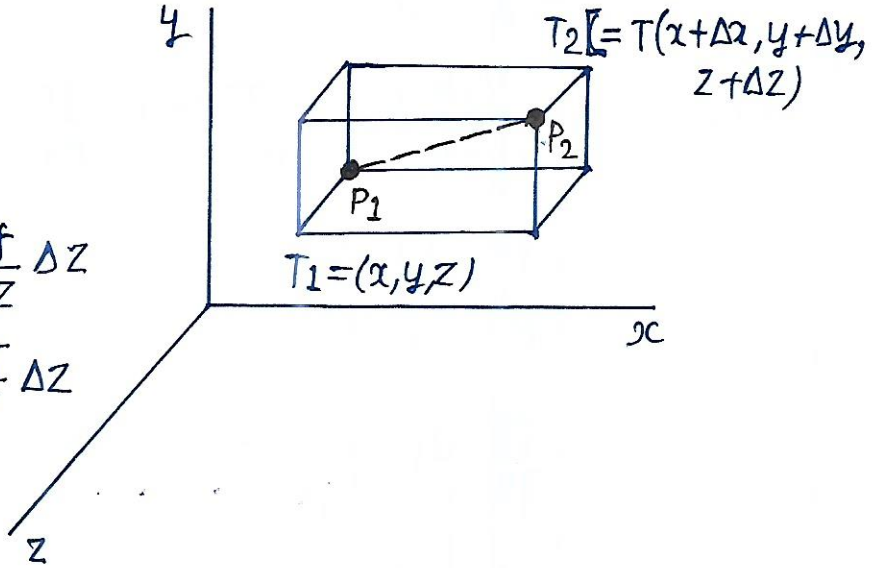
$\Delta R = (\Delta x, \Delta y, \Delta z)$

$\Delta T = T_2 - T_1$

$\Delta f(x,y,z) = \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y + \frac{\partial f}{\partial z}\Delta z$

$\Delta T(x,y,z) = \frac{\partial T}{\partial x}\Delta x + \frac{\partial T}{\partial y}\Delta y + \frac{\Delta T}{\partial z}\Delta z$

Where $\Delta x, \Delta y, \Delta z \longrightarrow 0$



$\Delta R = (\Delta x, \Delta y, \Delta z)$

ΔT = Scalar

$\left(\frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}, \frac{\partial T}{\partial z}\right) \longrightarrow$ Vector

∇T = gradient of T = del-T

$$\nabla T = \left(\frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}, \frac{\partial T}{\partial z}\right)$$

$$\Delta T = \nabla T \cdot \Delta R = \frac{\partial T}{\partial x}\Delta x + \frac{\partial T}{\partial y}\Delta y + \frac{\partial T}{\partial z}\Delta z$$

- The eqn says that the difference in temp. b/w two nearby points is the dot product of the gradient of T.
- Vector displacement b/w the points.

→ $x' = x\cos\theta + y\sin\theta$ $\qquad x = x'\cos\theta - y'\sin\theta$

$y' = -x\sin\theta + y\cos\theta$ $\qquad y = x'\sin\theta + y'\cos\theta$

$z' = z$ $\qquad z = z'$

$\Delta T(x,y,z) = \frac{\partial T}{\partial x}\Delta x$ ← only rotating w.r.to x

$P_1 = (x', y', z')$

$P_2 = (x'+\Delta x, y'+\Delta y, z'+\Delta z)$

$$\Delta T = \frac{\partial T}{\partial x'}\Delta x' + \frac{\partial T}{\partial y'}\Delta y' = \left(\frac{\partial T}{\partial x'}\cos\theta - \frac{\partial T}{\partial y'}\sin\theta\right)\Delta x$$

$\Delta x' = \Delta x\cos\theta$

$\Delta y' = -\Delta x\sin\theta$

$$\frac{\partial T}{\partial x} = \frac{\partial T}{\partial x'}\cos\theta - \frac{\partial T}{\partial y'}\sin\theta \quad \text{vs} \quad x = x'\cos\theta - y'\sin\theta$$

ΔT is definetely a vector field.

- <u>The Operator ∇ -</u>

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) = (\nabla_x, \nabla_y, \nabla_z)$$

* ∇ is a operator, it means nothing in itself.
* $T\nabla \neq \nabla T$ ← Order is important.

* $\boxed{\Delta T = \nabla T \cdot \Delta R}$

$$\nabla \cdot h = \nabla_x h_x + \nabla_y h_y + \nabla_z h_z = \frac{\partial h_x}{\partial x} + \frac{\partial h_y}{\partial y} + \frac{\partial h_z}{\partial z}$$

$\nabla' \cdot h = \nabla \cdot h \Rightarrow$ Scalar field

$\nabla \cdot h \Rightarrow$ divergence of h.

$\nabla \times h \Rightarrow$ curl of h

$$(\nabla \times h)_z = \nabla_x h_y - \nabla_y h_x = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$$

• Maxwell's Equations (In vector form)–

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$ ← ρ: Charge density, ϵ_0: Flux

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \cdot B = 0$$

$$c^2 \nabla \times B = \frac{\partial E}{\partial t} + \frac{j}{\epsilon_0}$$

$\rho \rightarrow$ Electric charge density
$j \rightarrow$ Electric current density

• Diff^n Eq^n of Heat Flow–

$$J = K(T_2 - T_1)\frac{A}{d}$$

$$\Delta J = -K \Delta T \frac{\Delta A}{\Delta S}$$

$$\frac{\Delta J}{\Delta A} = -K \frac{\Delta T}{\Delta S}$$

$$-K \nabla T = h$$

- $\nabla \times \nabla T = 0$

  curl (grad T) = 0

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$

- $\nabla \cdot (\nabla \times h) = 0$

  $A \cdot (A \times B) = 0$

  div(curl h) = 0

$$\vec{\nabla} \times \vec{E} = 0$$

$$\vec{E} = \vec{\nabla} \phi$$

- Theorem 1 –

  If $\nabla \times A = 0$

  $A = \nabla \psi$

- Theorem 2 –

  If $\nabla \cdot D = 0$

  $D = \nabla \times C$

$$\nabla \cdot \vec{B} = 0$$

$$\vec{B} = \nabla \times \vec{A}$$

- The Laplacian ∇^2 –

$$D \nabla^2 c = \frac{\partial c}{\partial t}$$

$$\nabla \cdot (\nabla T) = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

$$= \nabla \cdot \nabla T$$

$$= (\nabla \cdot \nabla) T$$

$$= \nabla^2 T$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

- $\nabla \times (\nabla \times h) = \nabla(\nabla \cdot h) - \nabla^2 h$

→ Gradient–

The gradient can be interpreted as the direction and rate of fastest increase.

→ Divergence–

Divergence measures the change in density of a fluid flowing according to a given vector field.

→ curl–

The curl is a vector operator that describes the infinitesimal rotation of a vector field in 3D Euclidean space

Euclidean space is the fundamental space of classical geometry. Originally it was the 3D space of Eulidean geometry.

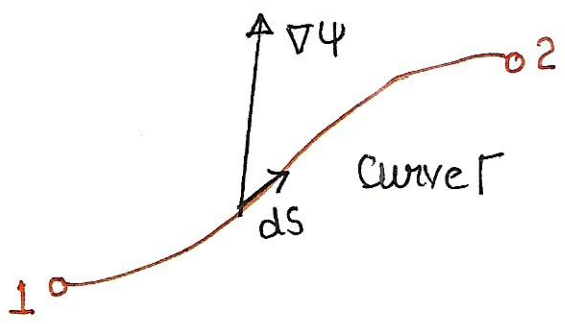
Guass Theorem, flow and circulation–

characteristics of vector Field → Flux, Circulation } Best describe by divergence and curl.

Since the gradient represents the rate of change of a field quantity, if we integrate it.

$$\Psi(2) - \Psi(1) = \int_1^2 (\nabla \Psi)\, ds$$

Along Γ



$$\int_1^2 \rho\, ds = \sum \rho_1 \Delta s_1$$

(ρ: Value of Function; Δs_1: Length of its segment)

$$\int_1^2 \nabla\psi\, ds = \sum [\nabla\psi]\Delta s$$

$$= \sum [\nabla\psi]\Delta s$$

$$\Delta T = \nabla T \cdot \Delta R = \frac{\partial T}{\partial x}\Delta x + \frac{\partial T}{\partial y}\Delta y + \frac{\partial T}{\partial z}\Delta z$$

$$\Downarrow$$

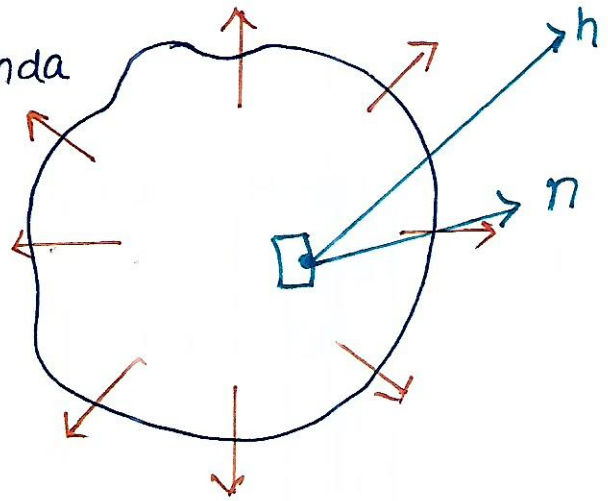
$$\Delta\psi = \nabla\psi \cdot \Delta s$$

$$\Delta\psi_1 = \psi(a) - \psi(1) = (\nabla\psi)_1 \Delta s_1$$
$$\Delta\psi_2 = \psi(b) - \psi(a) = (\nabla\psi)_2 \Delta s_2$$
$$\Delta\psi_3 = \psi(c) - \psi(b) = (\nabla\psi)_3 \Delta s_3$$
$$\vdots$$
$$\Delta\psi_n = \psi(2) - \psi(z) = (\nabla\psi)_n \cdot \Delta s_n$$

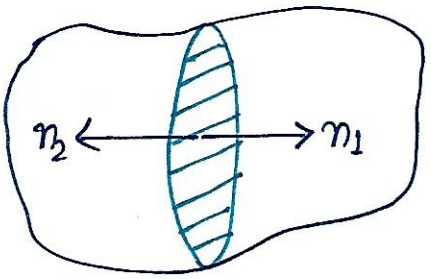
$$\boxed{\Delta\psi_n = \psi(2) - \psi(1) = \sum (\nabla\psi)_n \cdot \Delta s_i}$$

Total heat flow outward through $S = \int_S h \cdot n\, da$

Flux of E through $S = \int_S E \cdot n\, da$



$$S = \int_S h \cdot n\, da = -\frac{dQ}{dt}$$ ← Internal heat energy



$$n_1 = -n_2$$

$$\text{Flux through } S_1 = \int_{Sa} c \cdot n\, da + \int_{Sab} c \cdot n_1\, da$$

$$\text{Flux through } S_2 = \int_{Sb} c \cdot n\, da + \int_{Sab} c\, n_2\, da$$

$$+ \qquad +$$

$$\text{Flux } (S_1+S_2)S = \int_S c \cdot n\, da$$

- Flux through cube- Flux through $S_1 = -\, C_x(1)\,\Delta y\,\Delta z$, $S_2 = C_x(2)\,\Delta y\,\Delta z$

$$\int c \cdot n\, da = \left(\frac{\partial C_x}{\partial x} + \frac{\partial C_y}{\partial y} + \frac{\partial C_z}{\partial z}\right)\Delta x\,\Delta y\,\Delta z = (\nabla \cdot C)\Delta V$$

$C_x(2) = C_x(1) + \frac{\partial C_x}{\partial x}\Delta x$

[∵ Flux out of S_1 and $S_2 = \frac{\partial C_x}{\partial x}\Delta x\Delta y\Delta z$
Flux out of S_3 and $S_4 = \frac{\partial C_y}{\partial y}\Delta x\Delta y\Delta z$
Flux out of S_5 and $S_6 = \frac{\partial C_z}{\partial z}\Delta x\Delta y\Delta z$]

The divergence of a vector at the point P is the flux per unit volume, in the neighbourhood of P.

$$\nabla \vec{C} = \frac{\int C n\, da}{\Delta V}$$

- Heat Conduction-

$$\text{Heat out} = \int h \cdot n\, da = \nabla h\, \Delta V = -\frac{d}{dt}(q\,\Delta V) = -\frac{dq}{dt}\Delta V$$

$$-\frac{dq}{dt} = \nabla \cdot h$$

- **Equation of Radial Heat Flow-**

Uniform of emission at point of interest

$$\int_S h \cdot n\,da = h \cdot 4\pi R^2 = W$$

$$h = \frac{W}{4\pi R^2} \Rightarrow h = \frac{W}{4\pi R^2} e_r$$

$$\boxed{E = \frac{Q}{4\pi\epsilon_0 R^2} e_r}$$

- **Heat diffusion Equation-**

$$h = -K\nabla T$$

↑ Thermal conductivity

$$-\frac{dq}{dt} = \nabla \cdot h \Rightarrow \boxed{\frac{dq}{dt} = K\nabla^2 T}$$

$T \propto C_V$

heat content per unit volume

$$\Delta q = C_V \Delta T$$

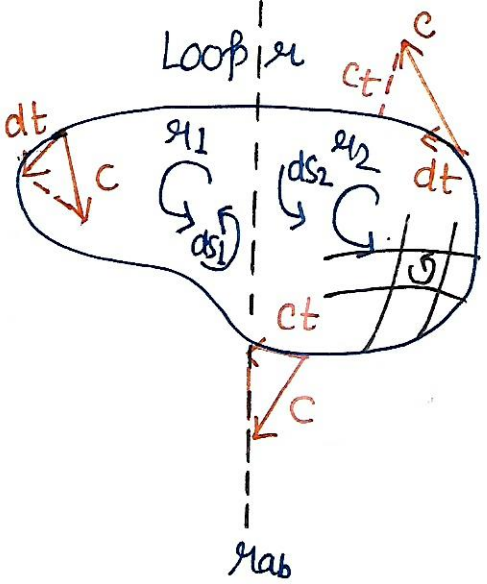
$$\boxed{\frac{dq}{dt} = C_V \frac{dT}{dt}}$$

$$\frac{dT}{dt} = \frac{K}{C_V}\nabla^2 T = D\nabla^2 T$$

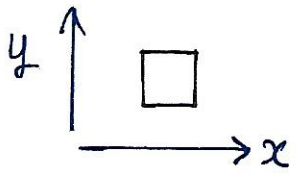
↑ diffusion constant

- **The circulation of Vector field-**

$$\oint_\Gamma C_t\,ds = \oint_\Gamma C \cdot ds$$



- The circulation around a loop is the sum of the circulations around two partial loops.



$$\oint_C ds = C_x(1)\Delta x + C_y(2)\Delta y - C_x(3)\Delta x - C_y(4)\Delta y$$

square

$$\oint c\, ds = -[C_x(1) - C_x(3)]\Delta x + [C_y(2) - C_y(4)]\Delta y$$

$$\oint c\, ds = \left(-\frac{\partial C_x}{\partial y} + \frac{\partial C_y}{\partial x}\right)\Delta x \Delta y = (\nabla \times C)\,\Delta a$$

$$\boxed{\oint c\, ds = (\nabla \times C)\Delta a = (\nabla \times C)\, n \Delta a}$$

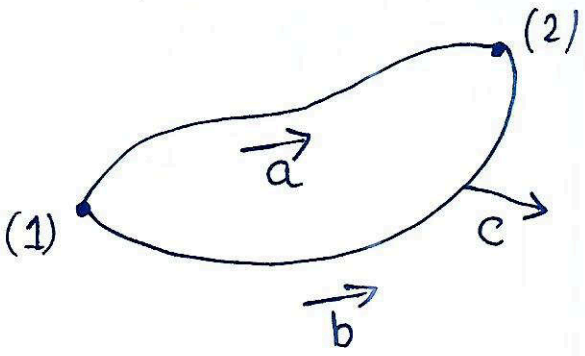
- Stoke's Theorem-

The circulation of any vector C around any loop Γ is equal to the surface integral of the curl of C normal to the surface which is bounded by Γ.

$$\oint C\, ds = \int_S (\nabla \times C) \cdot n\, da$$

• Curl Free and divergence free field –

$$\nabla \times C = 0$$



$$\oint_\Gamma C\,ds = \int_S (\nabla \times C)\, n \cdot da = 0$$

$$\oint_\Gamma C\,ds = a\int_1^2 C\,ds + b\int_1^2 C\,ds = 0$$

$$a\int_1^2 C\,ds = -b\int_2^1 C\,ds = b\int_1^2 C\,ds \Longleftarrow \text{It is path independent.}$$

Electrostatics and Flux of Electric potential –

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$

$$\nabla \times E = -\frac{\partial E}{\partial t}$$

$$\nabla \cdot B = 0$$

$$c^2 \nabla \times B = \frac{\partial E}{\partial t} + \frac{j}{\epsilon_0}$$

• Can be very complicated

Statics

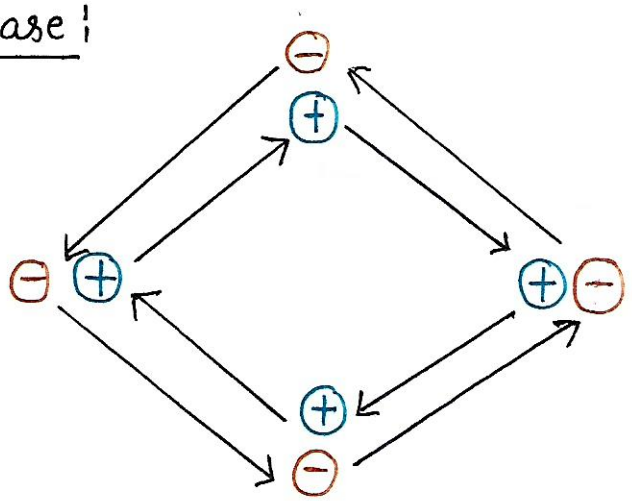
↓ Electrostatics

• Charge are stationary

↓ Magnetostatics

• Current does n't change as a function of time.

- The static case :



Electrostatics (zero curl and a given divergence) ← Coulomb's Law, Initial position of charges.

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot E = -\frac{\partial B}{\partial t} = 0$$ ← charges are statics.

Magnetostatics (Zero divergence and a given curl)

$$c^2 \nabla \times B = \frac{\partial B}{\partial t} + \frac{j}{\epsilon_0} = \frac{j}{\epsilon_0} \Rightarrow \frac{j}{\epsilon_0 c^2}$$

$$\nabla \cdot B = 0$$

- E and B field are not interconnected, which means that electricity and magnetism are distinct phenomena so long as charges and currents are static.

Electrostatics ⟶ Focus on divergence

↓

Magnetostatics ⟶ Focus on curl

↓

Electromagnetism

- Columb's Law-



$$F_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r_{12}^2} e_{12} = -F_2$$

There is a force between two charges at rest, which is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

$$\frac{1}{4\pi\epsilon_0} = \frac{\mu_0 c_0^2}{4\pi} = 10^{-7} c_0^2 = 9 \times 10^9 \ Nm^2/c^2 \text{ or } Vm/c$$ ⟵ Prefector

- E- Field-

The E field is force per unit charge instead of a physical parameter as a function of space and time.

$$E_1 = \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_{12}^2} e_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_{12}^3} r_{12} = (E_x, E_y, E_z)$$

$$E_x(x_1, y_1, z_1) = \frac{q_2}{4\pi\epsilon_0} \cdot \frac{x_1 - x_2}{[(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2]^{3/2}}$$

- Superposition-

The force (or E field) on any charge is the vector sum of the columb force from each other charges

$$E(1) = \sum_j \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_{1j}^2} e_{1j}$$

Often it is convient to ignore the fact that the charge come in package like e^-, p^+, and think of them as being spread out in a distribution.

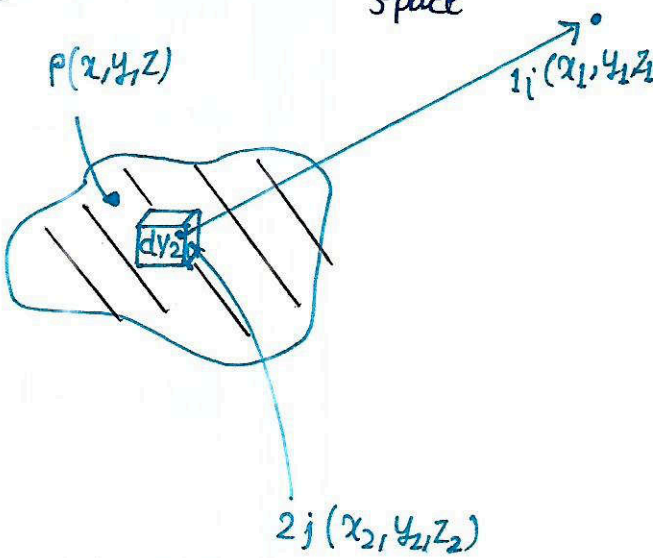
charge distribution (Charge density)

$$\rho(x, y, z)$$

$$\Delta q = \rho(2) \Delta V_2$$

$$\boxed{\Delta q_2} \quad \Delta V_2$$

$$E(1) = \sum_j \frac{1}{4\pi\varepsilon_0} \frac{\rho(2)\Delta V_2}{r_{1j}^2} e_{1j} = \frac{1}{4\pi\varepsilon_0} \int_{\text{all space}} \frac{\rho(2)\, e_{12}\, dV_2}{r_{12}^2} = \frac{1}{4\pi\varepsilon_0} \int_{\text{all space}} \frac{\rho(2)\, r_{12}\, dV_2}{r_{12}^2}$$



- Potential work and Energy –

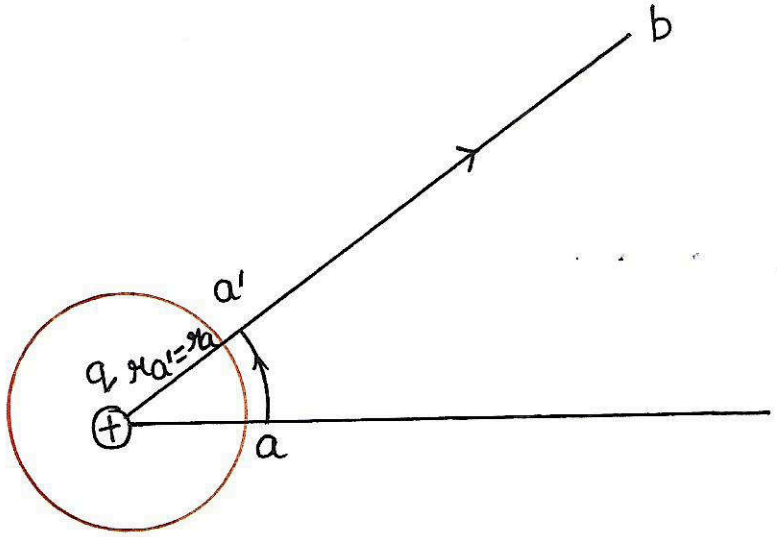
Work done against E forces in carrying a charge along some path from point a to b.

$$W = -\int_a^b F \cdot ds$$

- Electric Potential –

Work done against Electric force in carrying one unit of charge along some path from point a to b.

$$W = -\int_a^b E \cdot ds$$



$$W = -\int_a^b E\, ds$$

$$W = -\frac{q}{4\pi\varepsilon_0} \int_a^b \frac{dr}{r^2}$$

$$W = -\frac{q}{4\pi\varepsilon_0} \left(\frac{1}{r_a} - \frac{1}{r_b} \right)$$

- W depends only on the endpoints, and it can be represented as the difference between the two numbers.

$$W = -\int_a^b E \cdot ds$$

$$W = -\frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

$$W = \phi(b) - \phi(a)$$ ← Scalar Field

$$\phi_P(P) = -\int_{P_0}^{P} E \cdot ds$$

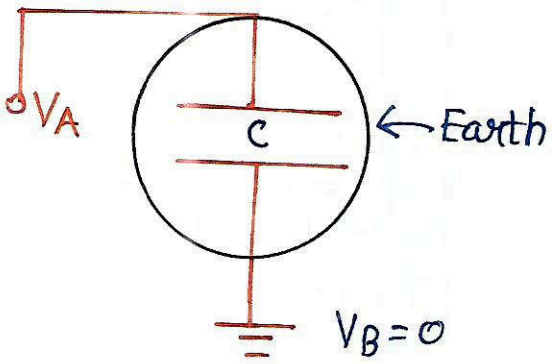
$$\phi(x,y,z) = -\frac{q}{4\pi\epsilon_0}\cdot\frac{1}{r}$$

- Superposition principle for potential

$$\phi(P) = -\int_{P_0}^{P} E\,ds = -\int_{P_0}^{P}\sum_j E_j\,ds$$

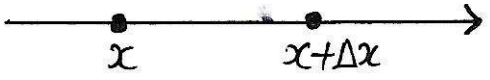
$$\phi(P) = -\sum_j\left(-\int_{P_0}^{P} E_j\,ds\right)$$

$$\phi(P) = \sum_j \phi_j(P)$$



$$\Delta V = \frac{\Delta Q}{C} \to 0$$

$$\Delta V = 0$$



$$\Delta W = \phi(x+\Delta x, y, z) - \phi(x,y,z)$$

$$\Delta W = \frac{\partial \phi}{\partial x}\Delta x$$

$$\Delta W = -\int E \cdot ds = -E_x \Delta x$$

$$E_x = \frac{\partial \phi}{\partial x}$$

$$E_x = -\nabla\phi$$

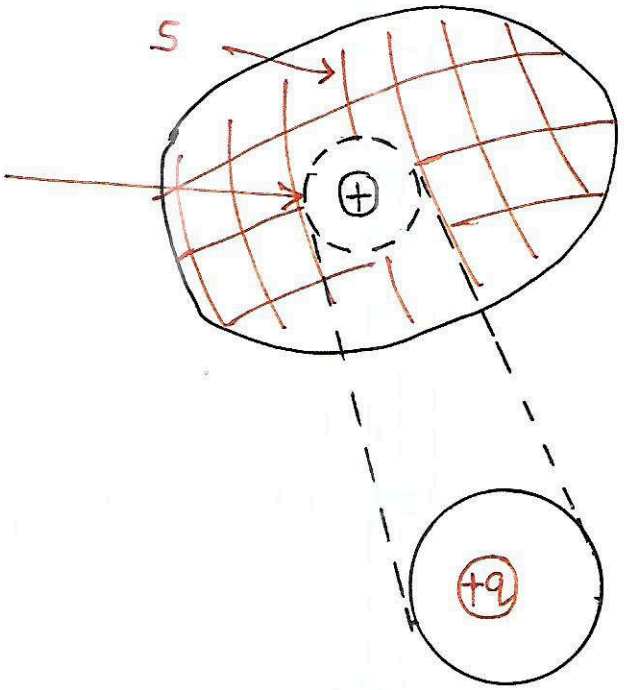
- The work done around a closed path is zero.

$$\oint E\,ds = 0$$

$$\nabla \times E = 0$$

Flux through $S' = E_n \cdot A$

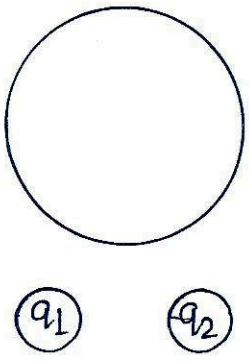
$$\Rightarrow \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}(4\pi r^2) = \frac{q}{\epsilon_0}$$



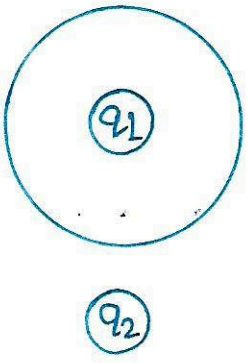
$$\int_S E_n\,da = \begin{cases} 0 \; ; \; q \text{ outside } S \\ q/\epsilon_0 \; ; \; q \text{ inside } S \end{cases}$$

- Guass's Law | The divergence of E-

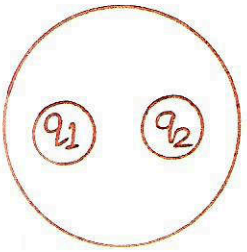
$$\int_S E_n\,da = \int_S (E_{1n} + E_{2n})\,da = \int_S E_{1n}\,da + \int_S E_{2n}\,da$$



Flux = 0

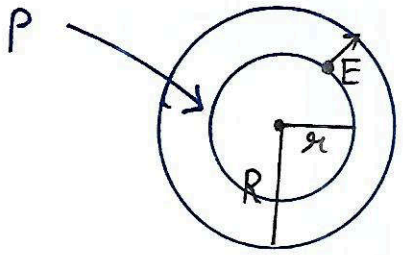


Flux = q_1/ϵ_0



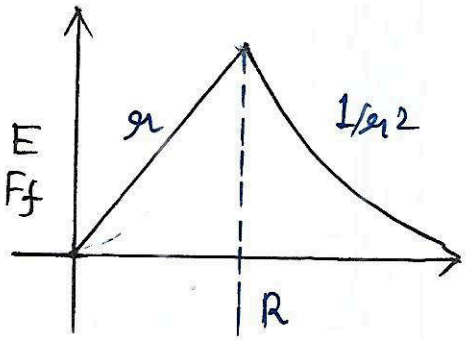
Flux = $(q_1 + q_2)/\epsilon_0$

$$\Rightarrow \frac{\rho \frac{4}{3}\pi r^3}{\epsilon_0}$$



$$E \cdot 4\pi r^2 = \frac{\rho \frac{4}{3}\pi r^3}{\epsilon_0}$$

$$E = \frac{\rho r}{3\epsilon_0}$$



$$\int_S E_n da = \text{Sum of charge Inside}/\epsilon_0 = \frac{\sum_{\text{inside}} q_i}{\epsilon_0}$$

$$\nabla \cdot E\, dV = \frac{\rho\, dV}{\epsilon_0}$$

$$\Downarrow$$

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$

$$\nabla \times E = 0$$

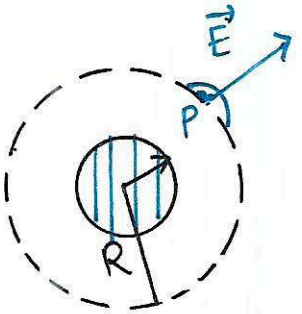
} Columb force

Electrostatics

- **Field of a sphere of charge-**

$$\int_S E_n da = E \cdot 4\pi R^2 = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi\epsilon_0 R^2}$$



→ It is same as that for a point charge Q
(efficient proof for Newton's problem of gravitation).

Electrostatic Field and Shielding -

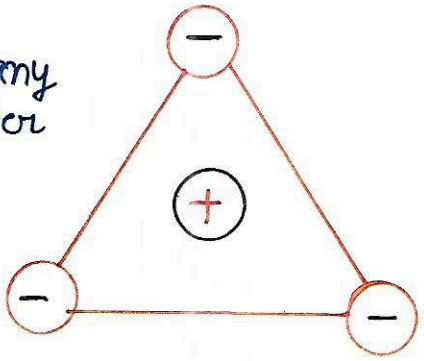
- Two laws of electrostatics -
  - The Flux of E from a volume is proportional to the charge inside. (Guass Law)
  - The circulation of E is zero. (E is a gradient)

$$\nabla \times E = 0$$



  - With Guass Law only (for conductors) + Arguments of symmetry (or E is a gradient of potential).

- There is no points of stable equilibrium in any electrostatic field - except right on top of another charge.

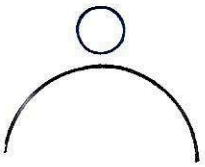


- What is Equilibrium ?

$$\sum F = 0$$



Stable

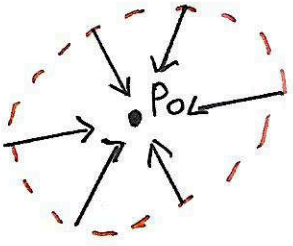


Unstable

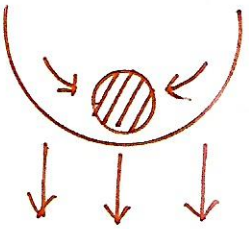
$$\frac{dU}{dr} = 0$$

$$\frac{d^2U}{dr^2} > 0$$

- Dillemma -



- Static Equilibrium

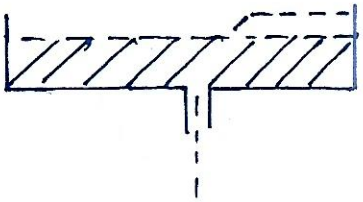


- Dynamic Equilibrium



$\frac{dx}{dt} \neq 0$

- Steady state



- Stability of Atom-

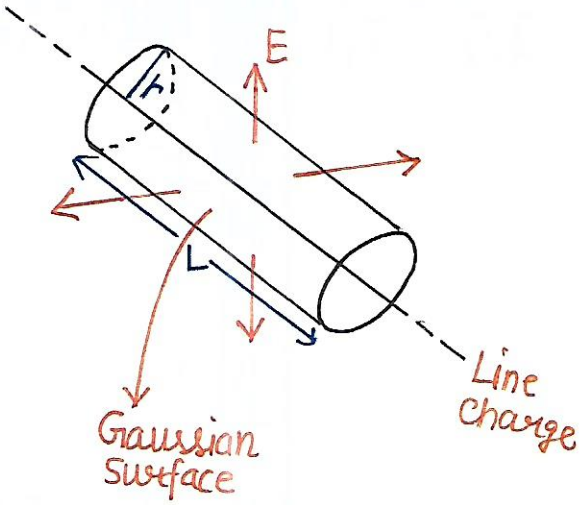
Problem- Energy radiation and collapse of electrons to the nucleus.
(Again Unstable)

Quantum Mechanics ⟶ ↓

Electrostatic Force + Uncertainty Principle

• The Field of a line charge-

- An Infinite line charge
  - Assumption- Field is radial, and has the same value anywhere equidistant from the line.



$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

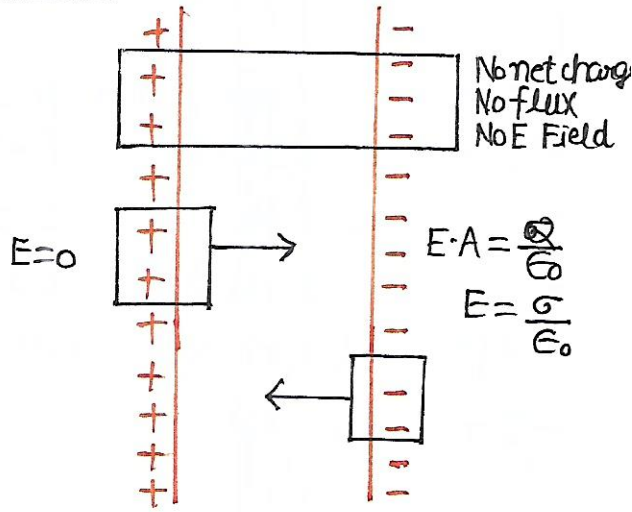
$$E \cdot 2\pi r \cdot L = \frac{Q}{\epsilon_0}$$

$$E \cdot 2\pi r = \left\{ \frac{Q}{L\epsilon_0} \rightarrow \lambda \right.$$

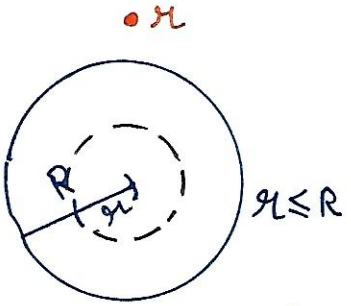
$$\boxed{E = \frac{\lambda}{2\pi r \epsilon_0}}$$

• Two sheets with equal and opposite charge densities-

$$E = \begin{cases} E = \frac{\sigma}{\epsilon_0} & \text{Between the sheets} \\ E = 0 & \text{outside} \end{cases}$$



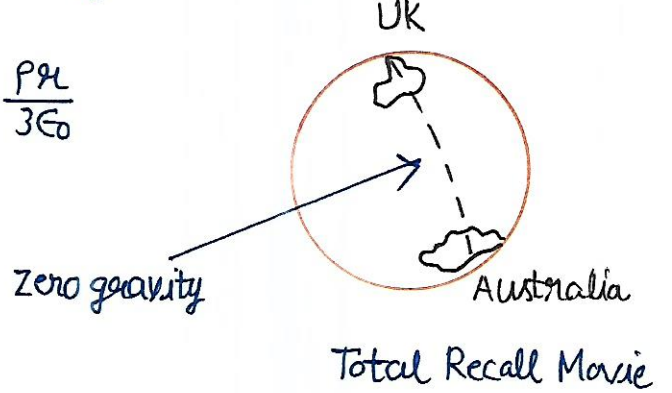
• Sphere of charge-



$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$4\pi r^2 \cdot E = \frac{\frac{4}{3}\pi r^3 \rho}{\epsilon_0}$$

$$E = \frac{\rho r}{3\epsilon_0}$$

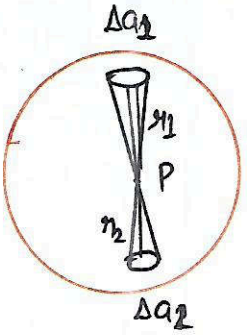


Total Recall Movie

- Is the field of a point charge exactly $1/r^2$?

$$\frac{\Delta q_2}{\Delta q_1} = \frac{r_2^2}{r_1^2} \quad \& \quad \frac{\Delta q_2}{\Delta q_1} = \frac{\Delta a_2}{\Delta a_1}$$

$$\frac{E_2}{E_1} = \frac{q_2/r_2^2}{q_1/r_1^2} = 1$$

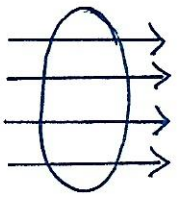


- The Validity of Guass Law-
  - Columb's Law is still valid, at least to some extent, at distances of the order of 10^{-13} cm
  - It fails at the distance of 10^{-14} cm.

$$1\text{Å} = 10^{-10}\text{ m}$$
$$1\text{Å} = 10^{-8}\text{ cm}$$

- Scope-
  - Electrical Force and Quantum Mechanical Effect.
  - Flux, circulation and Field
  - Vector calculus- Divergence, curl, Guass Theorem, Stoke's Theorem
  - Electrostatics and Guass Law.
  - Application of Guass Law.
  - Electric Shielding.

#Visulatization-



Visulatization ⟶ Derivation

Microscopic scale ⟶ Macroscale

#Scope-

- The Electric Field in various circumstances
- Electrostatic Energy
- Dielectrics

Equations of Electrostatic Potential-

$$\nabla \cdot E = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \nabla\phi = -\frac{\rho}{\epsilon_0} \longrightarrow \nabla^2\phi = -\frac{\rho}{\epsilon_0}$$

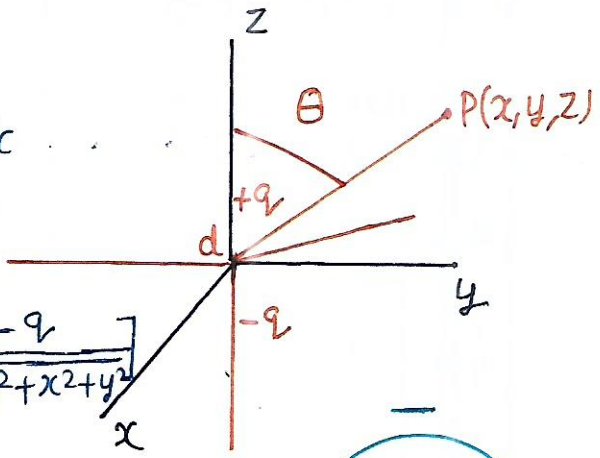
$$\nabla \times E = 0 \qquad E = -\nabla\phi$$

$$\phi(1) = \int \frac{\rho(2)\, dV_2}{4\pi\epsilon_0 r_{12}}$$

$$\nabla^2(\text{Something}) = (\text{Something Else})$$

The Electric Dipole-

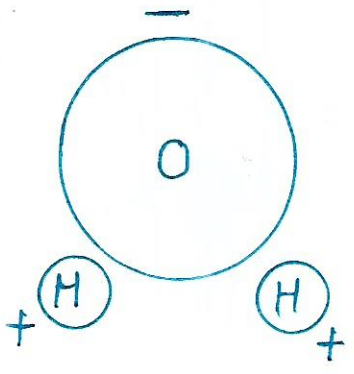
The electrostatic potential is mere superposition of electrostatic potential created by each charge



$$\phi(x,y,z) = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{\sqrt{[z-(d/2)^2]+x^2+y^2}} + \frac{-q}{\sqrt{(z+\frac{d}{2})^2+x^2+y^2}}\right]$$

Related Applications:

- Dipole moment Antenna
- Atomic dipole
- Polar molecules



2. Electrodynamics : Analysis of $\vec{E}$ Field

First Order Approximation-

$$\left(z-\frac{d}{2}\right)^2 \approx z^2 - zd$$

$$\left[z-\left(\frac{d}{2}\right)\right]^2 + x^2+y^2 \approx x^2+y^2+z^2-zd$$

$$= r^2 - zd \quad [\because x^2+y^2+z^2 = r^2]$$

$$= r^2\left(1-\frac{zd}{r^2}\right)$$

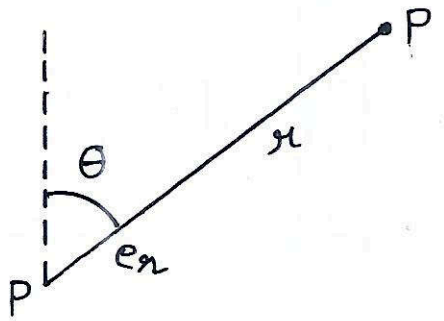
$$\phi(x,y,z) = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r}\left(1-\frac{zd}{r^2}\right)^{-1/2} - \frac{q}{r}\left(1+\frac{zd}{r^2}\right)^{-1/2}\right]$$

$$\phi(x,y,z) = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r}\left(1+\frac{1}{2}\frac{zd}{r^2}\right) - \frac{q}{r}\left(1-\frac{1}{2}\frac{zd}{r^2}\right)\right]$$

$$(1+x)^n \approx 1+nx$$

$$\phi(x,y,z) = \frac{1}{4\pi\epsilon_0}\cdot\frac{z}{r^3}\,qd$$

$$\phi(x,y,z) = \frac{1}{4\pi\epsilon_0}\cdot\frac{P\cos\theta}{r^2}$$



The Potential of a dipole will die out faster than of a single charge.

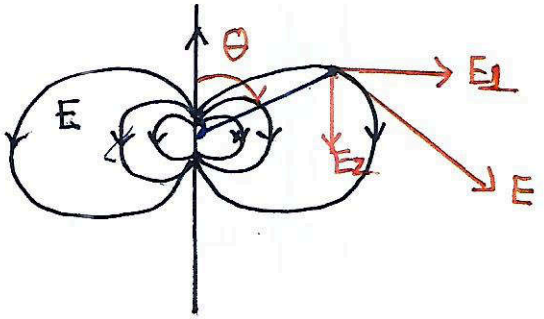
- The Potential of a dipole decreases as $1/r^2$ for a given direction from the axis whereas for a point charge it goes as $1/r$.

$$\phi(x,y,z) = \frac{1}{4\pi\epsilon_0}\frac{P\cdot e_r}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{P\cdot r}{r^3}$$

The Electric field of a dipole-

$$E_z = \frac{-\partial\phi}{\partial z}$$

$$E_z = \frac{P}{4\pi\epsilon_0}\frac{3\cos^2\theta-1}{r^3}$$



$$E_x = \frac{-\partial \phi}{\partial x}$$

$$E_x = \frac{P}{4\pi\epsilon_0} \frac{3zx}{r^5}$$

$$E_y = \frac{-\partial \phi}{\partial y}$$

$$E_y = \frac{P}{4\pi\epsilon_0} \frac{3zy}{r^5}$$

$$E_1 = \sqrt{xE^2 + E_y^2}$$

$$E_1 = \frac{P}{4\pi\epsilon_0} \frac{3\cos\theta \sin\theta}{r^3}$$

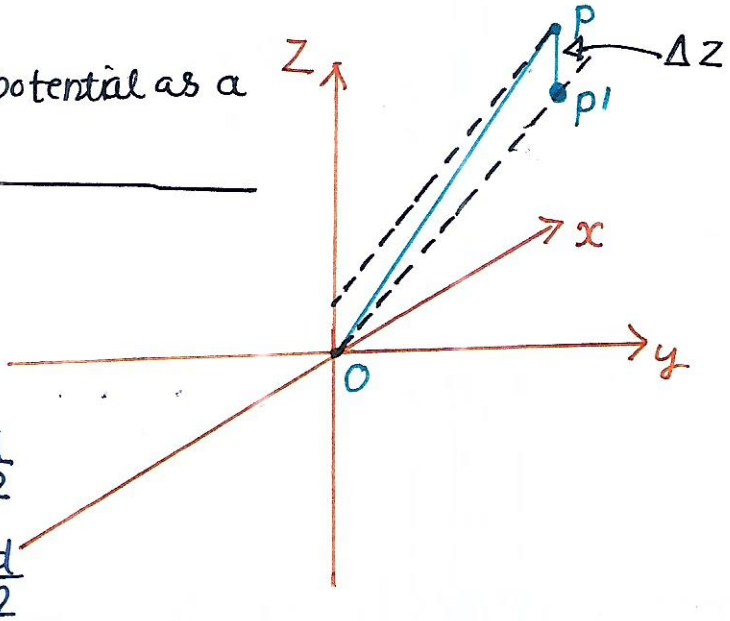
The dipole Potential as a gradient-

$$\phi(x,y,z) = \frac{1}{4\pi\epsilon_0} \frac{P \cdot e_r}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{P \cdot r}{r^3} = \frac{1}{4\pi\epsilon_0} P \nabla\left(\frac{1}{r}\right)$$

$$\nabla\left(\frac{1}{r}\right) = -\frac{r}{r^3} = -\frac{e_r}{r^2}$$

$$U = -\vec{E} \cdot \vec{P}$$

Physical reason for the dipole potential as a gradient-



$$\phi_0 = \frac{q}{r}$$

$$\phi_+ = \frac{-\partial \phi_0}{\partial z} \Delta z$$

$$\phi_+ = \phi_0 + \Delta\phi_+ = \frac{q}{r} - \frac{\partial}{\partial z}\left(\frac{q}{r}\right)\frac{d}{2}$$

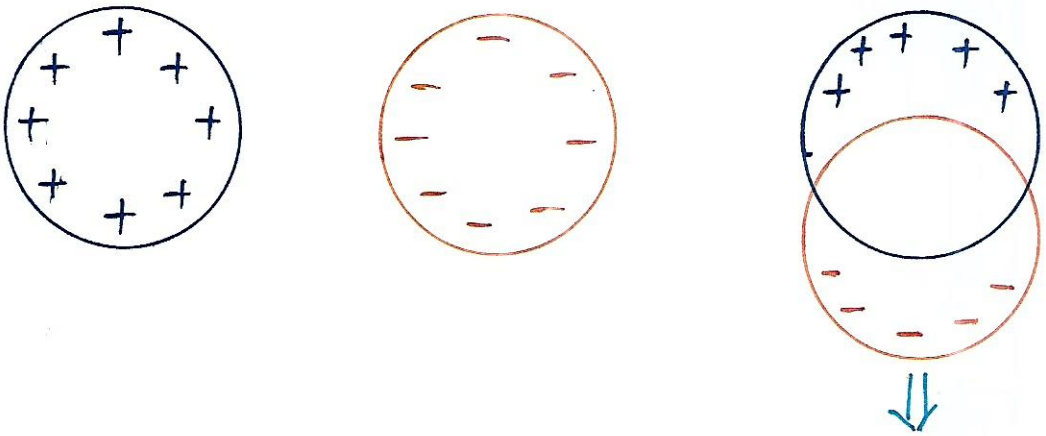
$$\phi_- = \phi_0 + \Delta\phi_- = \frac{-q}{r} + \frac{\partial}{\partial z}\left(\frac{-q}{r}\right)\frac{d}{2}$$

$$\phi = \phi_+ + \phi_-$$

$$\phi = -p \nabla \phi_0$$

The dot product b/w the dipole moment and the gradient of the potential of a unit point charge.

Non Uniform distribution of surface charge-

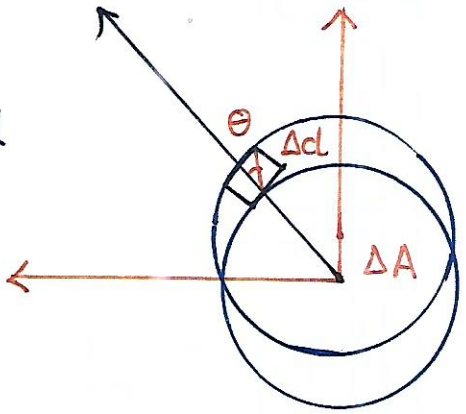


Electric Dipole

$$\Delta Q = \Delta A \cdot \Delta d \cos\theta \cdot \rho$$

$$\Delta Q = \Delta A \sigma \qquad \therefore \sigma_0 = \rho \Delta d$$

$$\sigma = \rho \Delta d \cdot \cos\theta = \sigma_0 \cos\theta$$

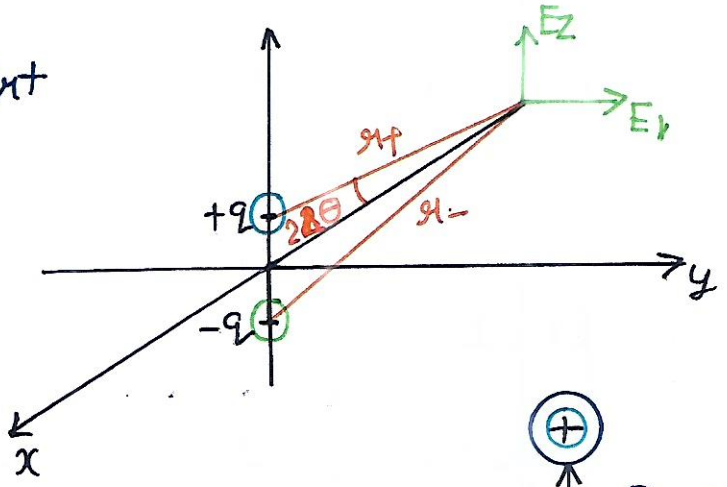


Dipole moment and electric field inside the sphere-

$$E_{r+} = \frac{\frac{4\pi r_+^3 \rho}{3}}{4\pi\epsilon_0} \frac{e_{r+}}{r_+^2} = \frac{\rho r_+}{3\epsilon_0} e_{r+}$$

$$E_{r-} = \frac{-\rho r_-}{3\epsilon_0} e_{r-}$$

$$E = E_{r+} + E_{r-} = -\rho \Delta d / 3\epsilon_0$$



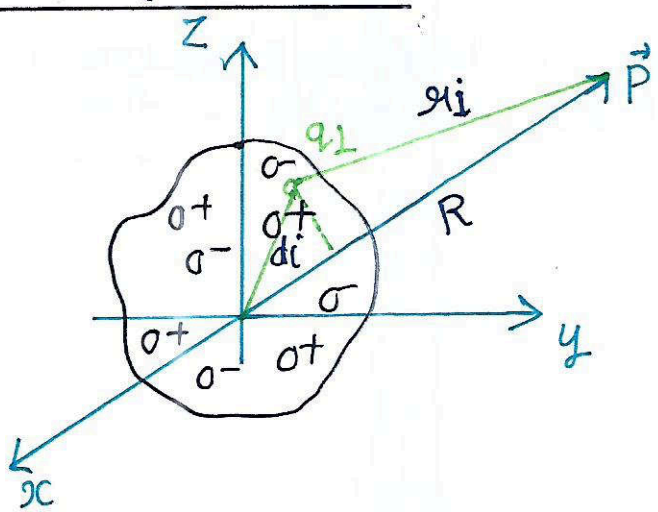


$P = q\Delta d$

- No matter where you put your point, you have a uniform Z field pointing from north to south.
- We do not have any r defendance.

The dipole approximation for an arbitrary distribution-

$$\phi = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$ ← q_i charge, r_i ← distance



$$\phi = \frac{1}{4\pi\epsilon_0}\cdot\frac{1}{R}\sum_i q_i = \frac{Q}{4\pi\epsilon_0 R} = 0$$

$$r_i \cong R - d_i \cdot e_r$$

$$\frac{1}{r_i} \cong \frac{1}{R}\left(1 + \frac{d_i \cdot e_r}{R}\right)$$

$$\phi = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{R} + \sum_i q_i \frac{d_i \cdot e_r}{R^2}\right)$$

$$\phi = \frac{1}{4\pi\epsilon_0}\frac{p \cdot e_r}{R^2}$$

This is precisely dipole moment. It is generalization of our earlier definition and reduces to it for a special case of two point charge.

- In the space outside the conductor the field is just like that of two point charges, whereas inside the conductor it is zero.
- Thus we can compute the field by computing the field due to q and an imagination point $-q$ at a suitable point.
- $-q$ is image charge.

Force on a point charge-

$$F = \frac{1}{4\pi\epsilon_0}\frac{(q)^2}{(2a)^2}$$

High Voltage breakdown-

$$\phi_1 = \frac{1}{4\pi\epsilon_0}\frac{Q}{a}, \quad \phi_2 = \frac{1}{4\pi\epsilon_0}\frac{q}{b}$$

$$\frac{Q}{a} = \frac{q}{b} \Rightarrow \frac{E_a}{E_b} = \frac{Q/a^2}{q/b^2} = \frac{b}{a} << 1$$

Finding an Electrostatic Field-

- Solving problems of conductors.
- Charge distribution — direct method
  Indirect method.

$$\zeta = x+iy$$

$$F(\zeta) = \zeta^2$$

$$F(\zeta) = 1/\zeta^3$$

$$F(\zeta) = \zeta \log \zeta$$

$$F(\zeta) = \zeta^2 = (x+iy)^2 = x^2 - y^2 + 2ixy$$
$$= U(x,y) + iV(x,y)$$

$$U(x,y) = x^2 - y^2$$

$$V(x,y) = 2xy$$

$$\frac{\partial U}{\partial x} = \frac{\partial V}{\partial y} \qquad \frac{\partial V}{\partial x} = -\frac{\partial U}{\partial y}$$

$$\Rightarrow \quad \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0$$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

Proof?

$$f(x+iy) = u(x,y) + iV(x,y)$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial z}\cdot\frac{\partial z}{\partial x} = \frac{\partial f}{\partial z} \qquad \frac{\partial f}{\partial y} = \frac{\partial f}{\partial z}\frac{\partial z}{\partial y} = i\frac{\partial f}{\partial z}$$

$$\frac{\partial u}{\partial x} + i\frac{\partial V}{\partial x} = \frac{\partial f}{\partial z} \qquad \frac{\partial u}{\partial y} + i\frac{\partial V}{\partial y} = i\frac{\partial f}{\partial z}$$

$$-\frac{\partial V}{\partial x} + i\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y} + i\frac{\partial V}{\partial y}$$

$$\Downarrow$$

$$\frac{\partial V}{\partial x} + \frac{\partial u}{\partial y} = 0$$

$$\frac{\partial u}{\partial x} - \frac{\partial V}{\partial y} = 0$$

Intersting insights into colliodal particle-

$$D^2 = \frac{\varepsilon_0 KT}{2n_0 q^2}$$

$$D = \sqrt{\frac{\varepsilon_0 KT}{2n_0 q^2}}$$

Energy and Force associated with Capacitors-

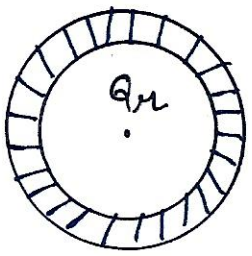
- Potential energy of system or earth.
- Quantum Mechanics potential energy is more convenient.
- Law of conservation of energy.

$$U = \sum_{\substack{\text{all} \\ \text{pairs}}} \frac{q_i q_j}{4\pi\epsilon_0 r_{ij}}$$

$$\text{Total Energy} = P + K$$

- Linear sum of energies.

- Energy Evaluation-



$$dU = \frac{Q_r dQ}{4\pi\epsilon_0 r}$$

$$Q_r = \rho \cdot \frac{4}{3}\pi r^3 \Rightarrow \qquad dU = \frac{4\pi\rho^2 r^4 dr}{3\epsilon_0}$$

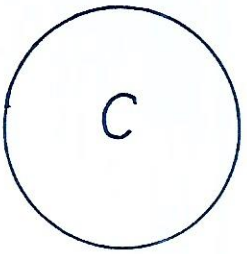
$$dQ = \rho\, 4\pi r^2 dr$$

$$U = \int_0^a \frac{4\pi\rho^2 r^4 dr}{3\epsilon_0} = \frac{4\pi\rho^2 a^5}{15\epsilon_0}$$

$$\boxed{U = \frac{3}{5} \cdot \frac{Q^2}{4\pi\epsilon_0 a}}$$

• Energy of Capacitor–

– Capacitor has been charged by transferring charge from one plate to other plate in small increments dQ.



– Energy

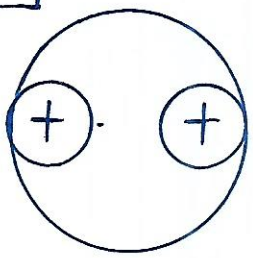
$$dU = V\,dQ = \frac{Q\,dQ}{C} \quad \left(\because V = \frac{Q}{C}\right) \text{ and } CV = Q$$

$$U = \int_0^Q \frac{Q\,dQ}{C} = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$$

$$C_{\text{cond. sphere}} = 4\pi\epsilon_0 a$$

$$\boxed{U_{\text{conducting sphere}} = \frac{1}{2}\cdot\frac{Q^2}{4\pi\epsilon_0 a}}$$

$$U_{\text{shell}} = \frac{1}{2}\cdot\frac{Q^2}{4\pi\epsilon_0 a}$$ [Need less energy to charge]



$$U_{\text{sphere}} = \frac{3}{5}\frac{Q^2}{4\pi\epsilon_0 a}$$

• Force b/w the plates of the capacitor–

Virtual work $\Delta W = F\Delta Z$

$$U = \frac{1}{2}\cdot\frac{Q^2}{C}$$

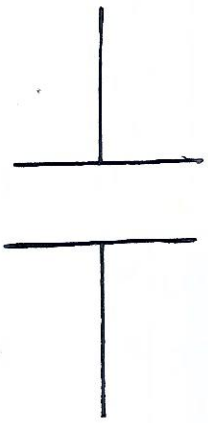
$$\Delta U = \frac{1}{2}Q^2\,\Delta\left(\frac{1}{C}\right)$$

$$F\Delta Z = \frac{Q^2}{2}\Delta\left(\frac{1}{C}\right) = \frac{Q^2}{2C^2}\Delta C$$

$$C = \epsilon_0\frac{A}{d}$$

$$\frac{1}{C} = \frac{d}{\epsilon_0 A}$$

$$\Delta\left(\frac{1}{C}\right) = \frac{\Delta d}{\epsilon_0 A} = \Delta Z$$



Parallel Plate capacitor

$$F\Delta Z = \frac{Q^2}{2}\Delta\left(\frac{1}{C}\right)$$

$$\boxed{F = \frac{Q^2}{2\epsilon_0 A}}$$

- **Torque on variable capacitor-**

$$\Delta W = \tau \Delta\theta$$ ← Small angular momentum

$$C = \epsilon_0 \frac{(n-1)}{d}\left(1 - \frac{\theta}{\pi}\right)$$

$$\tau\Delta\theta = \frac{Q^2}{2C^2}\Delta C$$

$$\tau = \frac{Q^2 d}{2\epsilon_0 (n-1) A} \cdot \frac{1}{(1-\theta/\pi)^2}$$

- Torque acts to decrease θ.

$$Q = \sigma/A$$
$$E_0 = \sigma/\epsilon_0$$
$$F = \frac{Q^2}{2\epsilon_0 A} = \frac{1}{2} Q \frac{\sigma}{\epsilon_0}$$

$$\boxed{F = \frac{1}{2} Q E_0}$$

- The avarage field acting on the surface is $E_0/2$.

- **Electrostatic Energy of an Ionic crystal-**

- The sum of the potential energies of all the pairs of the ions.

$$U_1 = \frac{2e^2}{a}\left(-1 + \frac{1}{2} + \frac{1}{3} + \dots\right) = 1.386\frac{e^2}{a}$$

$$U_2 = \frac{4e^2}{a}\left(-1 + \frac{2}{\sqrt{2}} - \frac{2}{\sqrt{5}} + \frac{2}{\sqrt{10}} + \dots\right) \Rightarrow$$

$$U = \sum U_1 = -1.747\frac{e^2}{a} = -8.94\,eV$$

- Columb force is responsible for Ionic Bonding.

- The subject that tries to understand bulk matter behaviour in terms of laws of atomic behaviour is called solid-state physics.

- Energy in Electrostatic Energy -

$$U = \frac{1}{2} \int_{\text{all space}} \frac{\rho(1)\rho(2)}{4\pi\epsilon_0 r_{12}} dV_1 dV_2$$

(pair of charges elements counted twice)

$$\int \frac{\rho(2)}{4\pi\epsilon_0 r_{12}} dV_2 = \phi(1)$$

$$U = \frac{1}{2} \int \rho\phi \, dV$$

Interpretation $U = \frac{1}{2}[q_1\phi(1) + q_2\phi(2)]$

- Electrostatic energy can be located at a point where we have an electrostatic field.

$$U = \frac{\epsilon_0}{2} \int E \cdot E \cdot dV$$

$$u = \frac{\epsilon_0}{2} E \cdot E = \frac{\epsilon_0 E^2}{2}$$

- Derivation of potential energy in terms of E Field -

$$U = \frac{1}{2} \int \rho\phi \, dV$$

$$\rho = -\epsilon_0 \nabla^2 \phi$$

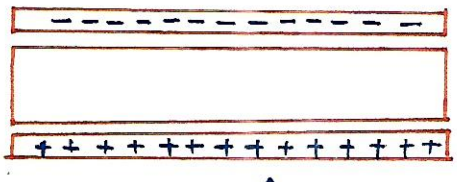
$$U = -\frac{\epsilon_0}{2} \int \phi \nabla^2 \phi \, dV$$

$$\int_{\text{Vol}} \nabla(\phi \nabla\phi) dV = \int_{\text{surface}} (\phi \nabla\phi) \cdot n da$$

$$U = \frac{\varepsilon_0}{2}\int_{\text{all space}} (\nabla\phi)\cdot(\nabla\phi)\,dV = \frac{\varepsilon_0}{2}\int_{\text{all space}} E\cdot E\,dV$$

Dielectric constants and susceptibility-

- Peculier property of matter under the influence of $\vec{E}$ field.



$$C = \frac{\varepsilon_0 A}{d}$$

$$Q = CV$$

- Faraday's Experiment

- We must conclude that C is increased then V is smaller.. Inside the capacitor, the electric field is reduced eventhough Q remain unchanged.

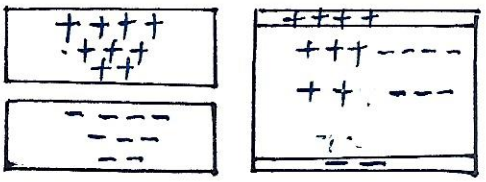
$$\int E\cdot dA = \frac{q}{\epsilon_0}$$

- The flux of $\vec{E}$ field is directy related to enclosed charge.

• Early model of dielectric-

- The phenomena of the dielectric constant increased by is explained by the Effect of charges which would be induced in each sphere.
- It was assumed that each of the atoms of a material was a perfect conductor, but insulated from the others.
- Dielectric, insulting or a very poor conductor of electric current. When dielectric are placed in an $\vec{E}$ field, practically no current flows in them because, unlikely metals, they have no loosly bound or free e⁻ that may drift through the material

Non Uniform Dielectric-



- If P is not constant, we can find a charge density in the volume, because more charge come into one side of a small volume element than leave it on the others.

$$\sigma_{Pol} = P \cdot n$$

-Total charge displaced out of any volume V by polarization $-\Delta Q_{Pol}$

$$\Delta Q_{Pol} = -\int_S P \cdot n \, da$$

$$+$$

$$\Delta Q_{Pol} = \int_V \rho_{Pol} \, dV$$

$$\Downarrow$$

$$\boxed{\int_V \rho_{Pol} dV = -\int_S P \cdot n \, da}$$

$$\int_V \rho_{Pol} \, dV = \sigma_{Pol} \Delta A = -\int_S P \cdot n \, da = P \Delta A$$

→ No volume polarization

→ only surface polarization

→ $\boxed{\sigma_{Pol} = P}$

- Guass mathematical theorem:

$$\int_S E \, dA = \int_V \nabla \cdot \vec{E} \, dV$$

$$\int_V \rho_{Pol} \, dV = -\int_S P \cdot n \, da = -\int_V \nabla \cdot P \, dV$$

$$\Downarrow$$

$$\boxed{\rho_{Pol} = -\nabla \cdot P}$$

$\rightarrow \nabla \cdot E = \frac{\rho}{\epsilon_0}$

$$\nabla \cdot E = \frac{\rho_{free} + \rho_{Pol}}{\epsilon_0} = \frac{\rho_{free} - \nabla \cdot P}{\epsilon_0}$$

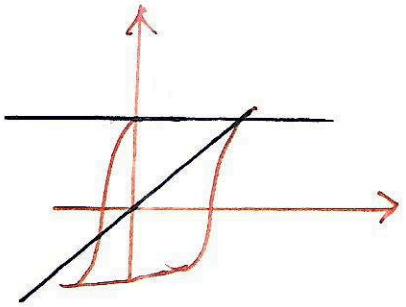
$$\boxed{\nabla \cdot \left(E + \frac{P}{\epsilon_0} \right) = \frac{\rho_{free}}{\epsilon_0}}$$

$\rightarrow \nabla \times E = 0$

- Using $P = \chi \epsilon_0 E$

$$\nabla \left(E + \frac{P}{\epsilon_0} \right) = \nabla \left[(1+\chi) E \right] = \nabla (KE) = \frac{\rho_{free}}{\epsilon_0}$$

- ρ_{free} is known
- P is proportional to E.

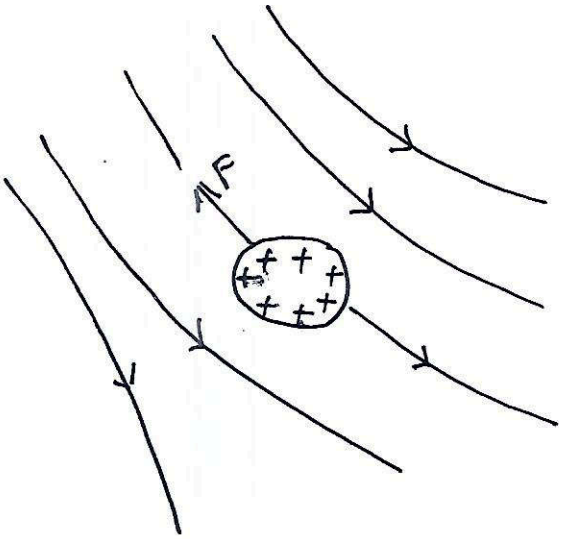


→ P is not proportional to E in ferroelectric materials.

- Why does a charged object pick up pieces of dielectric?

→ There are polarization charges of both sign and there is a net attraction.

→ Force is proportional to the gradient to the square of the E Field.



#Electronic Polarization-

- **Molecular dipole-**



→ Limited motion of charged particles
→ Polarization

$$K-1 = \frac{P}{\varepsilon_0 E}$$

* Polarge molecules e.g H_2O
* Nonpolar molecules e.g. O_2

- **Electronic Polarization-** The displacement of the electron distribution produces induced dipole moment, which is proportional to the $\vec{E}$ Field.

$$m\frac{d^2x}{dt^2} + m\omega_0^2 x = q \cdot E$$

↑ K.E.

↑ Restoring force

Modelling Heat flow and Membranes :

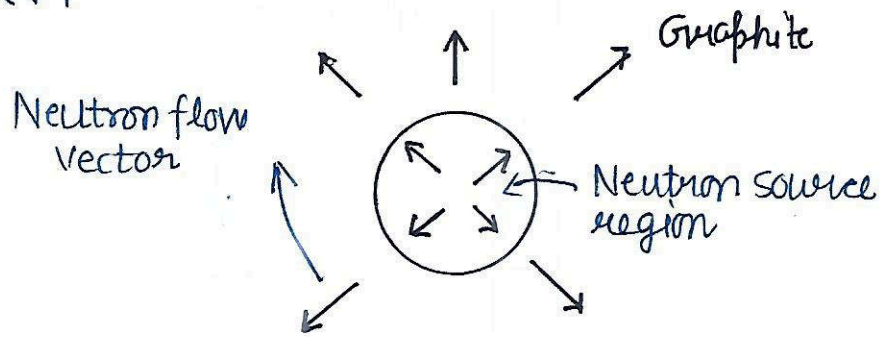
- The eqn for many different physical situations have exactly same appearance.

$$\nabla \cdot (KE) = \frac{\rho_{free}}{\epsilon_0} \qquad E = -\nabla\phi$$

$$\nabla \times E = 0 \quad \Rightarrow \quad \nabla \cdot (K\nabla\phi) = -\frac{\rho_{free}}{\epsilon_0}$$

- It is necessary to have an eqn, which describes how the heat flows at various places.

$$h = -K\nabla T$$



- $J_x = D\frac{\partial N}{\partial x}$

$$J = -D\Delta N$$

Similar ↕ ↑ Density of Neutron

$$h = -K\Delta T$$

$$\rightarrow \nabla \cdot J = \nabla(-D\Delta N) = S - \frac{\partial N}{\partial t}^{0} = S \Rightarrow \nabla(D\nabla N) = -S$$

The Magnetic Field-

- The force on an electric charge depends not only on where it is, but also on how fast it is moving.
- Electric force : Independent to motion of charge.
- Magnetic force : Depend on velocity of charge

Lorentz Force or Electromagnetic force

$$\boxed{F = q(\vec{E} + v \times \vec{B})}$$

↑ E & B Fields

* Electric current: e^- or other charges in motion with a net flow or drift.
* Current Density: The amount of charge passing per unit area.

The amount $\Delta q = j \cdot n \Delta S$

- j: current density
- n: Unit vector normal to ΔS.

⇒ current density: The current density is related to the avarage velocity of the charge

$$\Delta q = \rho v \cdot n \Delta S \Delta t$$

$$\Downarrow$$

$$j = \rho v = N q v$$

N: Number of charge per unit volume.

current- The total charge passing per unit time through any surface S.

$$I = \int_S j \cdot n \, ds$$

$$I = \int_S j \cdot n \cdot ds = -\frac{d}{dt}(Q_{inside}) \Longrightarrow \nabla \cdot j = -\frac{\partial \rho}{\partial t}$$

Gauss' Theorem

$$\int \vec{j} \cdot \vec{n} \, ds = \int (\nabla \cdot j) \, dv = -\frac{dQ}{dt}$$

$$= -\frac{d}{dt} \int \rho \, dv$$

$$= -\int \frac{\partial \rho}{\partial t} \, dv$$

$$\boxed{\nabla \cdot j = -\frac{\partial \rho}{\partial t}}$$

The magnetic force on a current-

The total magnetic force ΔF on the volume ΔV is the sum of the individu charges.

$$\Delta F = (N\Delta V)(q\vec{v} \times \vec{B})$$

$$\Delta F = j \times B \Delta V \qquad [\because j = Nqv]$$

$$\Delta F = j \times BA\Delta L$$

$$\Delta F = I \times B \Delta L$$

The $\vec{B}$ Field of steady currents | Ampere's Law-

- We have seen that there is a force on the wire in the presence of a $\vec{B}$ Field produced, say, by a magnet.
- We might expect that there should be a force on the source of $\vec{B}$ field i.e. on the magnet when there is a current in the wire.

⇩

- The wire itself generate a magnetic field.
- Moving charges produce a magnetic field.

⇩

- Given a current, what magnetic field does it make?

Magnetostatics-

- No charges moving.

$$\nabla \cdot B = 0$$

$$c^2 \nabla \times \vec{B} = \frac{j}{\epsilon_0}$$

A special kind of dynamics situation with large number of charges in motion, where we can approximate by a steady flow of charge.

- The study of steady currents.

• Since the divergence of the curl of any vector is zero.

$$\nabla \cdot j = 0$$

$$\nabla \cdot j = -\frac{\partial \rho}{\partial t} = 0$$

AMPERE'S LAW-

- The requirement that $\nabla \cdot j = 0$ means that we may only have charges which flow in paths that close back on themselves like circuits (Excluding charging and discharging of capacitors).

$$\oint_\Gamma B \cdot ds = \int_S (\nabla \times B) n \, ds \qquad \text{[By Stokes theorem]}$$

$$\oint_\Gamma B \cdot ds = \frac{1}{\epsilon_0 c^2} \int_S j \cdot n \, ds$$

$$\boxed{\oint_\Gamma B \cdot ds = \frac{I \text{ through } \Gamma}{\epsilon_0 c^2}}$$

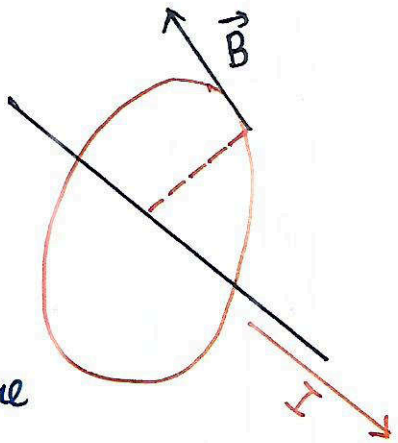
This law called Ampere Law. It plays same role in magnetostatics as that Gauss' Law played in electrostatics.

The $\vec{B}$ Field of a straight wire and of a solenoid-

$$\oint B \cdot ds = B \cdot 2\pi r$$

$$B \cdot 2\pi r = \frac{I}{\epsilon_0 c^2}$$

$$B = \frac{I}{4\pi\epsilon_0 c^2} \cdot \frac{2I}{r}$$



The magnitude of the $\vec{E}$ field will decrease as you go further and further from the wire and specifically $1/r$.

$$\boxed{B = \frac{1}{4\pi\epsilon_0 c^2} \frac{2I \times e_r}{r}}$$

Solenoid-

- Since the current produce a $\vec{B}$ field, it will exert a force on a nearby wire which is also carrying a current.
- If the wires are parallel, each is at right angle to the $\vec{B}$ field of other.
- When the currents are in the same direction, the wire attract; when the currents are moving in opposite directions, the wire repel.
- Suppose we have a long coil of wire wound in a tight spiral, which is called a solenoid.

* Experimental Observation- The field outside is very small compared to the field inside when a solenoid is very long compared with its diameter

$n = \frac{N}{L}$ → N → total no. of turns
L → total length of wire.

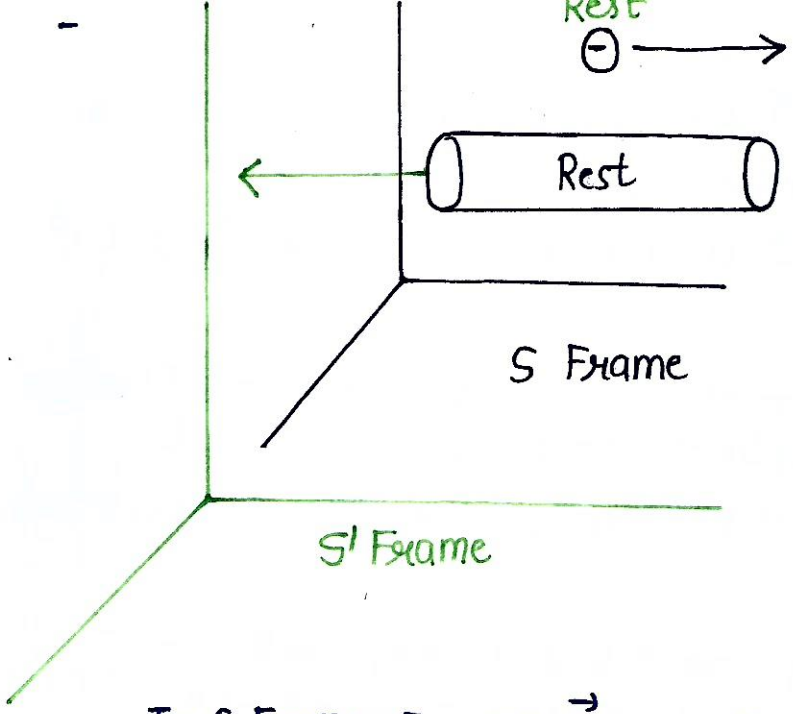
$$B_0 L = \frac{NI}{\epsilon_0 c^2} \Rightarrow \boxed{B_0 = \frac{nI}{\epsilon_0 c^2}}$$

Equivalence to a bar magnet to a solenoid-

- All magnetism is produced from current; in a permanent magnet there are permanent internal currents.
- In a bar magnet, large no. of e^- all spin in the same direction, and their total effect is equivalent to a current circulating on the surface to the bar.
- This is analogous to a uniform polarized dielectric equivalent to a distribution of charge on its surface.

Relativity of $\vec{B}$ and $\vec{E}$ Field-

Electromagnetic theory ⇨ Electricity and Magnetism



In S Frame, $F = qv_0 \times \vec{B}$ + $B = \frac{1}{4\pi\epsilon_0 c^2} \cdot \frac{2Ie_r}{r}$

$$F = \frac{1}{4\pi\epsilon_0 c^2} \cdot \frac{2IqV_0}{r}$$

$$F = \frac{1}{4\pi\epsilon_0 c^2} \cdot \frac{2\rho_- AV V_0}{r}$$

$$F = \frac{q}{2\pi\epsilon_0} \frac{\rho_- AV^2}{rc^2}$$

Let us assume $V = V_0$

In S' Frame;

$$v = 0$$
$$v \times \vec{B} = 0$$

Total charge $Q = \rho_0 L_0 A_0$

$$L = L_0\sqrt{1-\frac{v^2}{c^2}} = L_0\sqrt{1-\gamma}$$

$$\rho = \rho_0\sqrt{1-v^2/c^2}$$

Electric force acting on the particle in the S' Frame-

$$E' = \frac{\rho' A}{2\pi\epsilon_0 r} = \frac{\rho_+ A\frac{v^2}{c^2}}{2\pi\epsilon_0 r\sqrt{1-\frac{v^2}{c^2}}}$$

$$F' = \frac{F}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$F' = \frac{q}{2\pi\epsilon_0}\frac{\rho_+ A}{r}\frac{\frac{v^2}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\rho_+' = \frac{\rho_+}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\rho_-' = \frac{\rho_-}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\rho_-' = \rho_-\sqrt{1-\frac{v^2}{c^2}}$$

- F' is slightly larger than F. It is called relativistic electromagnetic theory.

$$\Delta t = \frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Downarrow$$

$$\boxed{\frac{\Delta p_y'}{\Delta p_y} = \frac{F'\Delta t'}{F\Delta t} = 1}$$

- Velocity of light is always constant.

The Vector Potential-

- Magnetostatic is when flow of charge is constant.

$$\nabla \cdot B = 0$$

$$c^2 \nabla \times B = \frac{j}{\epsilon_0}$$

- Procedure of finding $\vec{B}$ Field.

$$\nabla \cdot B = 0 \qquad \vec{E} = -\nabla\phi$$

$$B = \nabla \times A$$

$$B_x = (\nabla \times A)_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{A_z}$$

$$B_y = (\nabla \times A)_y = \frac{\partial A_x}{\partial Z} - \frac{\partial A_z}{\partial x}$$

$$B_z = (\nabla \times A)_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}$$

Scalar Potential $\phi' = \phi + c$

Now $B = \nabla \times A' = \nabla \times A$

$$\nabla \times A' - \nabla \times A = \nabla \times (A' - A) = 0$$

- Choosing the right vector potential-

$$A' - A = \nabla\psi$$

$$\downarrow$$

$$A' = A + \nabla\psi$$

$$\nabla \cdot A' = \nabla \cdot A + \nabla^2\psi$$

↑ Laplacian

$$\boxed{\nabla \cdot A = 0}$$

The Vector potential of known currents:

$$c^2 \nabla \times (\nabla \times A) = \frac{j}{\epsilon_0}$$

$$\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$$

$$\nabla^2 A = 0$$

Magnetostatic $\nabla^2 A = \frac{-j}{\epsilon_0 c^2}$ Electrostatic $\nabla \cdot \nabla \phi = -\frac{\rho}{\epsilon_0}$

$$\nabla^2 A_x = \frac{-j_x}{\epsilon_0 c^2} \qquad \nabla^2 A_y = \frac{-j_y}{\epsilon_0 c^2} \qquad \nabla^2 A_z = \frac{-j_z}{\epsilon_0 c^2}$$

$$\phi(1) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(2)\, dV_2}{r_{12}}$$

$$A_z(1) = \frac{1}{4\pi\epsilon_0 c^2} \int \frac{j_z(2)\, dV_2}{r_{12}}$$

$$A(1) = \frac{1}{4\pi\epsilon_0 c^2} \int \frac{j(2)\, dV_2}{r_{12}}$$

$\nabla(A) = 0$ so long as $\nabla \cdot j = 0$

Comparing Quantum Mechanics-

- Is field concept real?

The whole idea of the field is just an abstract. The value of magnetic field is not very definite.

False in general (True only for statics)	True always
$F=\frac{1}{4\pi\epsilon_0}\cdot\frac{q_1q_2}{r^2}$ (Columb's law)	$F=q(E+v\times\vec{B})$ [Coulamb] $\rightarrow \nabla\cdot E=\frac{\rho}{\epsilon_0}$ [Gauss]
$\nabla\times E=0$ $E(1)=\frac{1}{4\pi\epsilon_0}\cdot\frac{\rho(2)e_{12}}{r^2_{12}}dV_2$ For conductors $E=0, \phi=c$ $Q=CV$	$\nabla\times E=-\frac{\partial B}{\partial t}$ [Faraday] $E=-\nabla\phi-\frac{\partial B}{\partial t}$ In a conductor, E makes current
$c^2\nabla\times B=\frac{j}{\epsilon_0}$ (Ampere) $B(1)=\frac{1}{4\pi\epsilon_0c^2}\int\frac{j(2)\times e_{12}}{r^2_{12}}dV_2$	$\nabla\times A=B$ $\nabla\cdot B=0$ (Ampere) $c^2\nabla\times B=\frac{j}{\epsilon_0}+\frac{\partial E}{\partial t}$
$\nabla^2\phi=-\frac{\rho}{\epsilon_0}$ (Paission)	$c^2\nabla\cdot A+\frac{\partial\phi}{\partial t}=0$
$\phi(1)=\frac{1}{4\pi\epsilon_0}\int\frac{\rho(2)}{r_{12}}dV_2$ $A(1)=\frac{1}{4\pi\epsilon_0c^2}\int\frac{j(2)}{r_{12}}dV_2$	$\phi(1,t)=\frac{1}{4\pi\epsilon_0}\int\frac{\rho(2,t')dV_2}{r_{12}}$ $A(1,t)=\frac{1}{4\pi\epsilon_0c^2}\int\frac{j(2,t')}{r_{12}}dV_2$ $t'=t-\frac{r_{12}}{c}$
$U=\frac{1}{2}\int\rho\phi\,dV+\frac{1}{2}\int j\cdot A dv$	$U=\int\left(\frac{\epsilon_0}{2}E+\frac{\epsilon_0c^2}{2}B\cdot B\right)dv$

Running Motors -

* Motors

Electrical Energy

↓

Mechanical Energy

* Generators-

Mechanical Energy

⇩

Electrical Energy

• Galvanometer-

A sensitive instrument for electrical measurement where an exceedingly small current will make the coil turn, and for small angle the amount of rotation will be proportional to the current.
Voltmeter and ammeter work on the same principle.

• DC Motor-

- Polarities at one part of coil.
- Faraday in 1840

⇩

discovery of Induction Effect

- First Galvanometer

⇩

e^- push the e^- further down the wire for a long distance.

⇩

The beginning of Telegraph

↓

Transmission of information over a long distance.

- Relativity Effect of Induction

$$F = q(E + v \times \vec{B})$$

- Magnetic flux changes as function of time

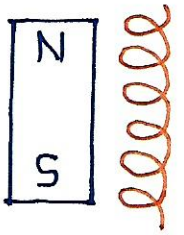
⇩

Flow of electric current

⇩

Induction Effect

- Some equivalant Picture-



- Electromotive Force-

  - emf
  - The net integration push on the electrons in the circuit

- Faraday's complete discovery -

  EMF can be generated in a wire in the 3 ways.

  1. By moving the wire.
  2. By moving the magnet near the wire.
  3. By changing a current in a nearby wire.

- Faraday Flux rule-

  The EMF is equal to the rate of change of the magnetic flux when the flux that passes through the loop is changing with t.

  The normal component of B integrated over the area of the loop.

• Phone transmitter or receiver-

- A coil of wire around one of the yokes will have the flux through it changed when a sound wave hits the diaphragm.

⇩

Emf in the coil

⇩

Electrical representation of sound

- Telephone designed by Alexander Graham Bell

⇩

carbon button microphone

⇩

It uses sound pressure to vary the electric current from the battery

• Transformers and Inductances-

- One of the most interesting features of Faraday's discoveries is that a current changing in one coil makes an emf in the second coil.

Transformer

⇩

A combination of two coils usually with an arrangement of iron sheets to guide the magnetic fields. It can transform one emf to another

Self Inductance

There is a changing flux not only through coil (b), but also through coil (a).

⇩

The varying current in coil (a) produces a varying $\vec{B}$ field inside itself

⇩

Flux of this field changing continuously

⇩

Self Inductance

- Lenz's Law-

The emf tries to oppose any magnetic flux change.

Self Inductance ⟷ Mechanical Inertia.

⇩

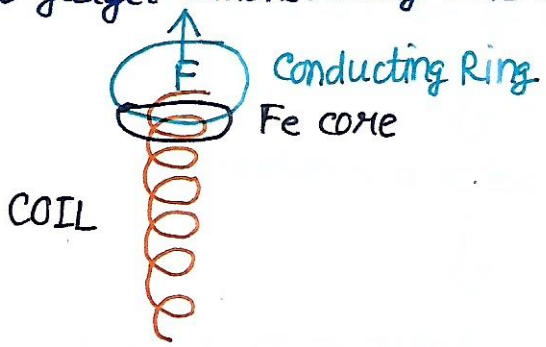
Large self Inductance

⇩

Dangerous

Eletromagnetic Ring Launcher

This is a gadget demonstrating Lenz's Rule in dramatic way.



⇩

Electromagnetic pulse generator

⇩

Magnetic pulse welding

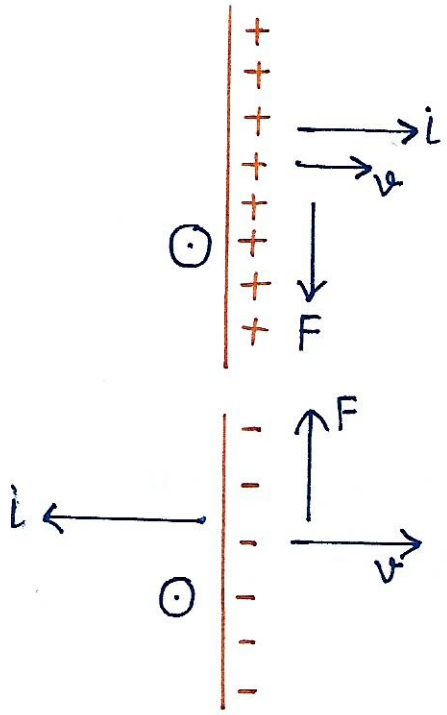
- Magnetic Leviation : Superconductors-

- If we have a sheet of a perfect conductor and put an electromagnet next to it, when we turn on the current in the magnet, eddy currents appear in the sheet, so that no magnetic flux enters.
- Since the eddy currents are opposing the fields, the magnets are repelled from the conductor.

• The Physics of Induction-

- Electromotive Force.
- Flux rule.

$$E = WB\frac{dL}{dt} = WBv$$



What happens if loop is stationary?

$$F = q(E + v \times \vec{B})$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

- Faraday's Law leads to the 2nd Law of maxwell.

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\oint_\Gamma E \cdot ds = \oint_S (\nabla \times E)\, n\, da = -\int_S \frac{\partial B}{\partial t}\, n\, da$$

$\vec{B}$ Flux

- The emf in the circuit is equal to the rate of change of the $\vec{B}$ flux through the circuit- applies whether the flux changes because the field changes or because the circuit moves (or both).

In depth solutions for Maxwell's Equations

- Two laws :

$v \times B$ ____ circuit moves

$\nabla \times E = \frac{-\partial B}{\partial t}$ ____ Field changes

- The part of the emf that comes from the E-field does not depend on the existance of a physical wire.
- Whereas the $v \times B$ part does depend on the existance of a physical wire.
- The flux rule must be applied to the circuit where material of the circuit remains the same. If it changes, then we return to basics.

$$F = q(E + v \times B)$$

$$\nabla \times \vec{E} = -\frac{\partial B}{\partial t}$$

- Particle accelerator by an induced E-field -

- The EMF generated by a changing $\vec{B}$ Field can exist without conductors. i.e. without wires.

⇩

Particle Accelerator

$$2\pi r E = \frac{\partial}{\partial t}(B_{av} \cdot \pi r^2)$$

⇩

$$E = \frac{r}{2} \cdot \frac{dB_{av}}{dt}$$

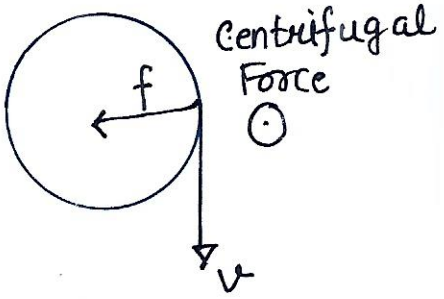
$$qE = \frac{dp}{dt}$$

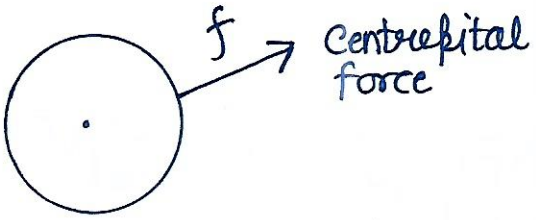
⇩

$$\int \frac{dp}{dt} = \frac{qr}{2} \int \frac{dB_{av}}{dt}$$

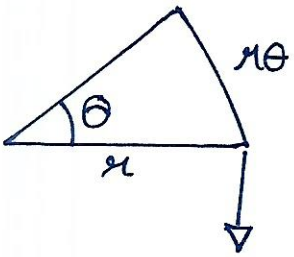
$$p = p_0 + \frac{qr}{2} \Delta B_{av}$$

p_0 of momentum of e⁻ star cut.





$$\left.\begin{aligned} \phi &= \phi_0 + \frac{qr}{2}\Delta B_{av} \\ qvB_{orbit} &= \phi\frac{v}{r} \end{aligned}\right\} \quad \Delta B_{av} = 2\Delta B_{orbit}$$



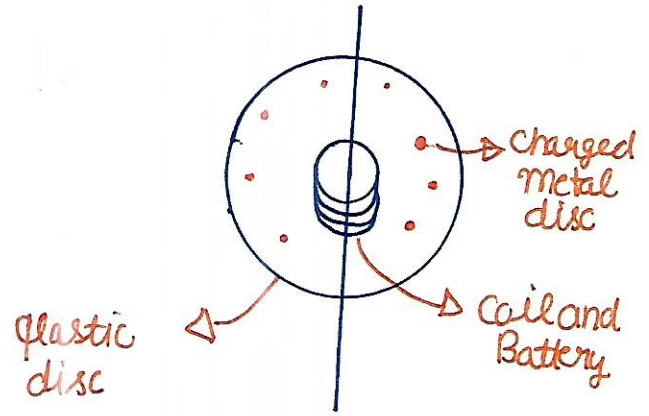
$$\Rightarrow \frac{d(r\theta)}{dt} \rightarrow r\frac{d\theta}{dt} \rightarrow r\omega$$

- The betatron is used to accelerate e^- to energies of tens of millions of volts, or even to hundred to millions of volts.

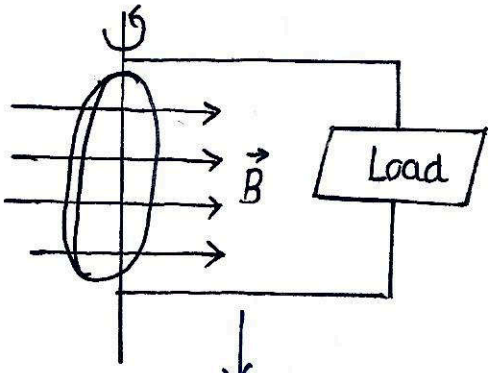
• Paradox –

- So in physics a paradox is only a confusion in our own understanding.

- Current is interupted
- Flux must go zero.
- $\vec{E}$ Field center to axis
- Start rotate → Paradox
  - Rotating
  - Not Rotating



- Alternating current generator

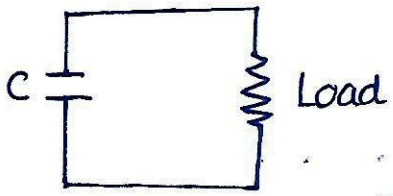


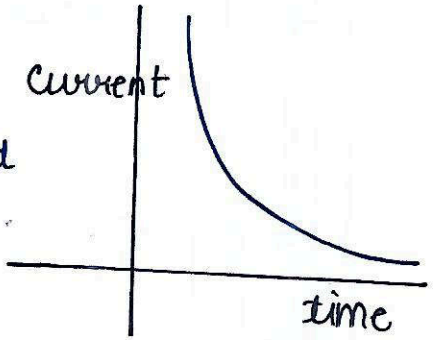
$$EMF = -\frac{d}{dt}(Flux) = -\frac{d}{dt}(BS\cos\omega t)$$

$$E = BS\omega \sin\omega t$$

$$V = BS\omega\sin\omega t = V_0 \sin\omega t$$

Alternating Voltage





$$I = \frac{E}{R}$$

$$I = \frac{V_0}{R}\sin\omega t$$

• Maxwell Equations -

1. $\nabla \cdot E = \frac{\rho}{\epsilon_0}$

Flux of E through a closed surface = $\frac{\text{Charge Inside}}{\epsilon_0}$

2. $\nabla \times E = \frac{-\partial E}{\partial t}$

Line integral of E through a closed surface = $-\frac{d}{dt}$(Flux of B(loop))

3. $\nabla \cdot B = 0$

Flux of B through a closed surface = 0
No magnetic monopole.

4. $c^2 \nabla \times B = \frac{j}{\epsilon_0} + \frac{\partial E}{\partial t}$

c^2(Integral B around a loop) = $\frac{\text{current through loop}}{\epsilon_0} + \frac{\partial(\text{Flux of E})}{\partial t}$

• Classical Physics -

conservation of charge $\nabla \cdot j = -\frac{\partial \rho}{\partial t}$

Force Law $F = q(E + v \times B)$

$$\frac{\partial p}{\partial t} = F \qquad p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Gravitation

$$F = -G \frac{m_1 m_2}{r^2} e_r$$



- A travelling field-

  - Fynmen
  - Travelling Field
  - K-2

- Equation which satisfy wave function-

- When we put the all maxwell equations together, a remarkable new phenomenon occurs: Field generated by moving charge leave the source and travel alone through space.

- B Field as 3D wave

$$\Downarrow$$

$$\nabla^2 B = \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2}$$

$$\nabla^2 B - \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2} = 0$$

- E Field as 3D wave

$$\nabla^2 E = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$$

$$\downarrow$$

$$\boxed{\nabla^2 \psi - \frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2} = 0}$$

- Dynamically varying E_y field in one dimension

$$\rightarrow \frac{\partial^2 E_y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2} = 0$$

→ wave eqn for 1D wave

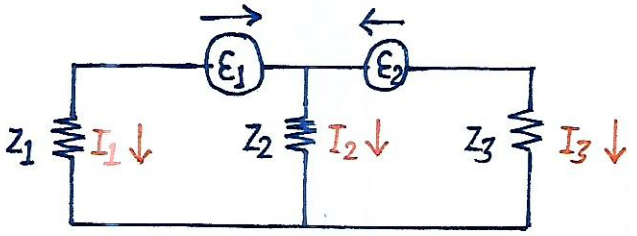
- When charge are accelerated the waves travel outward from the charge.

• Exception-

- There are no rules without exceptions.

$$I_1 + I_2 + I_3 = 0$$

$$I_3 = -(I_1 + I_2)$$



- Voltage eqⁿ :

$$-\mathcal{E}_1 + Z_2 I_2 - Z_1 I_1 = 0$$

$$\mathcal{E}_2 - Z_3(I_1 + I_2) - Z_2 I_2 = 0$$

• Bridge Circuit-

$$\frac{R_2}{R_1} = \frac{Rx}{R_3}$$

• Equivalent Circuit-

$$I = \frac{\mathcal{E}}{Z_{eff}}$$ ← complex number

- Energy of an Ideal Inductor

$$U = \frac{1}{2} L I^2$$

- Energy of a capacitor

$$U = \frac{1}{2} C V^2$$

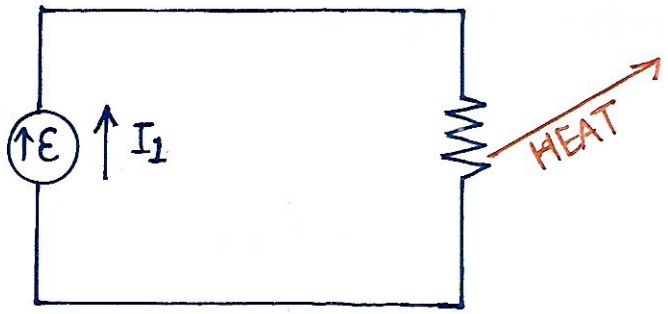
An Ideal capacitor is a non dissipative element.
When a capacitor is in an AC circuit, energy flows in and out of it, but the net energy flow in each cycle is zero.

- Energy of Ideal Generator

$$\frac{dU}{dt} = \mathcal{E} I$$

• Energy of an ideal generator + an ideal resistor -

$$I = \frac{\mathcal{E}}{R}$$



$$\frac{dU}{dt} = I^2R = VI$$

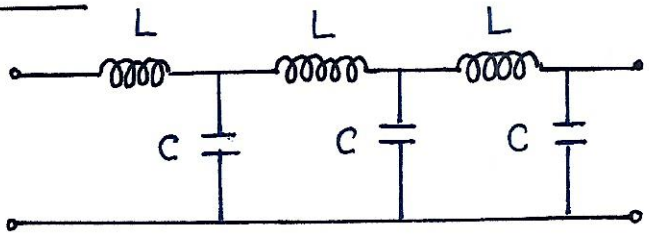
⟹ Energy Loss with resistor only case

$$\text{Energy Loss in AC circuit} = \frac{I^2R}{2}$$

$$\langle P \rangle_{av} = \frac{I^2R}{2} \quad \left[\langle P \rangle_{av} = \frac{1}{T}\int_0^T \mathcal{E}I\,dt\right]$$

- All of the power loss is induced by resistor, not by Inductor or capacitor.
- Any circuit containing only L's and C's has an impedance with an pure imaginary number and will have no energy loss.

• Ladder Network -



$$Z_1 = i\omega L$$
$$Z_2 = \frac{1}{i\omega C}$$

- Analyse in terms of series and parallel.

$$Z_0 = \frac{Z_1}{2} + \sqrt{\left(\frac{Z_1^2}{4}\right) + Z_1Z_2}$$

Characteristic equation for a Ladder network

$$Z_0 = \sqrt{\left(\frac{L}{C}\right) - \left(\frac{\omega^2L^2}{4}\right)}$$

$\omega^2 > \frac{4}{LC}$, $\omega^2 < \frac{4}{LC}$

$$Z_0 = \sqrt{\left(\frac{L}{C}\right) - \left(\frac{\omega^2 L^4}{4}\right)}$$

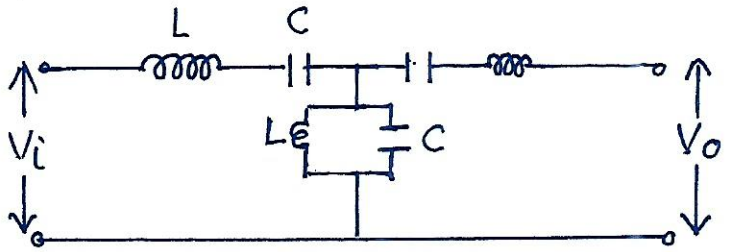
$$Z_0 = \sqrt{\left(\frac{L}{C}\right) - \left(\frac{\omega^2 L^4}{4}\right)}$$

$$Z_0 = \sqrt{\left(\frac{\omega^2 L^4}{4}\right) - \left(\frac{L}{C}\right)}$$

- **Band Pass Filter-**

  - It is possible to make band pass filter that rejects frequencies below some frequency ω_1 and above frequency ω_2.

  - It can be used to tune the broadcasting radio frequency of the program of interest.



- **Transmission Line-**

  - The LC Ladder shown below is an approximate representation of a transmission line.

TV cable

$$Z_0 = \sqrt{\left(\frac{L}{C}\right) - \left(\frac{\omega^2 L^4}{4}\right)} \Rightarrow Z_0 = \sqrt{\left(\frac{\Delta L}{\Delta C}\right) - \left(\frac{\omega^2 \Delta L^4}{4}\right)} \to 0$$

$$Z_0 = \sqrt{\frac{\Delta L}{\Delta C}}$$

$$Z_0 = \sqrt{\frac{L_0}{C_0}}$$ L_0 → L per unit length, C_0 → C per unit length

$$\omega_0 = \sqrt{4/L_0 C_0} \to \infty$$

  - There is no cut off frequency.

Nuclear Physics

Intermediate
- α, β, γ Decay
- Law of Radioactive Decay
- Half Life
- Mass Defect
- Binding Energy
- Nuclear Stability
- Mass Energy Equivalence
- Nuclear Fission
- Nuclear Reactor
- Nuclear Fusion

Advanced

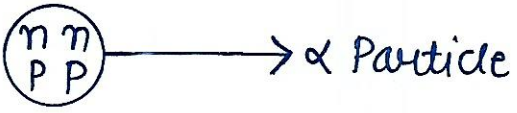
α, β, γ decay-

• α decay-

- α Particle is nucleus of He atom.

$$^{4}_{2}He$$

2P
2n
2e-



- mass :

Atomic mass = 4 amu

- Charge: $C = +2e$

- Emission :

$$^{A}_{Z}X \longrightarrow {}^{4}_{2}He + {}^{A-4}_{Z-2}Y$$

Parent Nucleus — Daughter Nucleus

e.g. $^{238}_{92}U \longrightarrow {}^{4}_{2}He + {}^{234}_{90}Th$ — Stability Increase + Q

$\frac{n}{P} = \frac{146}{92} = 1.58$ $\quad$ $\frac{n}{P} = \frac{144}{90} = 1.6$

- Energy in decay -

↓

mass defect

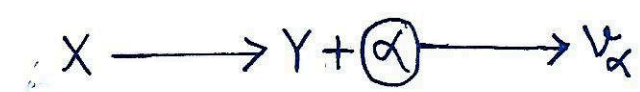
$$^{A}_{Z}X \longrightarrow {}^{4}_{2}He + {}^{A-4}_{Z-2}Y + Q$$

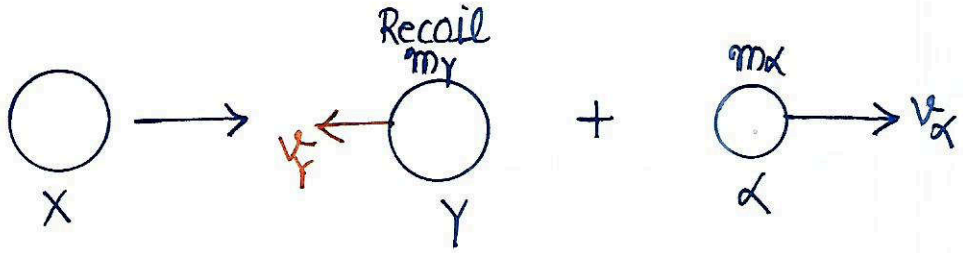
mass defect $\Delta m = [m_X - (m_\alpha + m_Y)]$

$$E = \Delta m c^2$$

$$Q = E = [m_X - (m_\alpha + m_Y)]c^2 \qquad Q = [m_X - (m_\alpha + m_Y)]931.5 MeV$$

- Momentum conservation:





$P_i = 0$ $\qquad$ $P_f = 0$

$$m_\alpha v_\alpha - m_Y v_Y = 0$$

$$\boxed{m_\alpha v_\alpha = m_Y v_Y}$$

$$|P_\alpha| = |P_Y| = P$$

- Kinetic Energy:

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

- Kinetic Energy of emitted α Particle:

$$X \longrightarrow Y + \alpha + Q$$

$$Q = K.E_\alpha + K.E_Y$$

$$Q = \frac{p_\alpha^2}{2m_\alpha} + \frac{p_Y^2}{2m_Y}$$

$$Q = \frac{p^2}{2m_\alpha} + \frac{p^2}{2m_Y}$$

$$2Q = \frac{p^2 m_Y + p^2 m_\alpha}{m_\alpha + m_Y}$$

$$2Qm_\alpha m_Y = p^2(m_\alpha + m_Y)$$

$$p^2 = \frac{2Qm_\alpha m_Y}{m_\alpha + m_Y}$$

$$K.E_\alpha = \frac{Qm_Y}{m_\alpha + m_Y}$$

$$K.E_Y = \frac{Qm_\alpha}{m_\alpha + m_Y}$$

β Decay –

- β^- decay e^-
- β^+ decay e^+

• β^- decay –

$${}^{0}_{-1}e \text{ or } {}^{0}_{-1}\beta$$

$${}^{A}_{Z}X \longrightarrow {}^{0}_{-1}e + {}^{A}_{Z+1}Y + Q + \bar{\nu}$$

antineutrino

$${}^{14}_{6}C \longrightarrow {}^{0}_{-1}e + {}^{14}_{7}N + \bar{\nu}$$

** Actually β^- decay is conversion of a ~~β~~ neutron to a proton inside nucleus.

$${}^{1}_{0}n \longrightarrow {}^{1}_{1}P + {}^{0}_{-1}e^- + \bar{\nu}$$

$\bar{\nu}$ → Antineutrino

e^- → β particle

Antineutrino was suggested by W. Pauli.

↓

Antineutrino is emitted for energy conservation and spin momentum conservation.

• β^- decay Energy –

∵ e^- is very small, it takes nearly all the energy.

$$Q \approx K.E_\beta$$

** Experimentally it was observed that all the β particle do not have same Kinetic energy.

So, antineutrino came into picture.

$$Q = K.E._{\beta} + K.E_{\bar{\nu}}$$

$$^{1}_{0}n \longrightarrow {}^{1}_{1}P + {}^{0}_{-1}e + \bar{\nu}$$

$\frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad -\frac{1}{2}$

$-\frac{1}{2} \quad -\frac{1}{2} \quad -\frac{1}{2} \quad +\frac{1}{2}$

$\frac{1}{2} \quad \frac{1}{2} \quad -\frac{1}{2}$

$-\frac{1}{2} \quad \frac{1}{2} \quad -\frac{1}{2} \quad -\frac{1}{2}$

$\bar{\nu}$ → Antineutrino → mass = 0
speed = c
charge = 0
High penetration power
spin = $+\frac{1}{2}$

ν → neutrino →

• **β^+ decay -** $^{0}_{+1}e$ $^{0}_{+1}\beta$, charge = +c, mass = 0

$$^{A}_{Z}X \longrightarrow {}^{0}_{+1}e + {}^{A}_{Z-1}Y + \nu + Q$$

neutrino

$$^{12}_{7}N \longrightarrow {}^{0}_{+1}e + {}^{12}_{6}C + \nu + Q$$

Actually β^+ decay is conversion of a proton into neutron inside nucleus.

$$^{1}_{1}P \longrightarrow {}^{1}_{0}n + {}^{0}_{+1}e$$

β^- → mass decrease

$${}^{1}_{0}n \longrightarrow {}^{1}_{1}P + {}^{0}_{-1}e + \bar{\nu}$$

β^+ → mass increase

$${}^{1}_{1}P \longrightarrow {}^{1}_{0}n + {}^{0}_{1}e + \nu$$

In β^+ decay, energy is obtained from Binding energy of nucleus.

$$\frac{\Delta E}{c^2} = \Delta m$$

e^- capture | Proton capture e^-, convert to neutron.

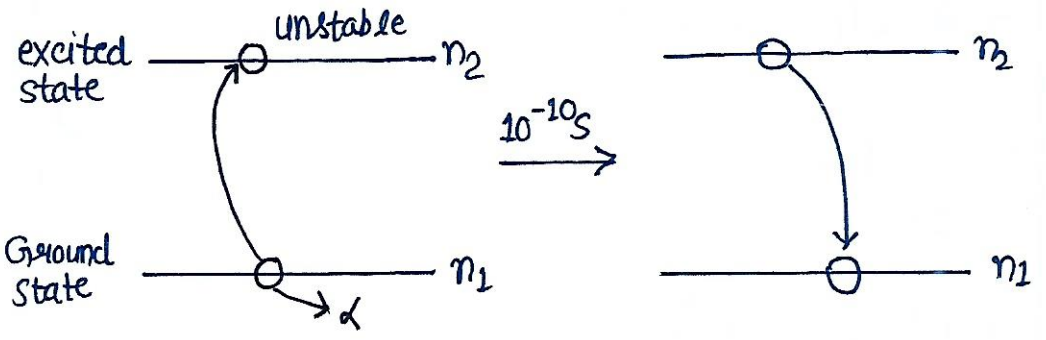
$${}^{1}_{1}P + {}^{0}_{-1}e \longrightarrow {}^{1}_{0}n + \nu$$

$${}^{1}_{1}P \longrightarrow {}^{1}_{0}n + {}^{0}_{+1}e + \nu$$

 # γ decay-

γ → Electromagnetic wave

When a nucleus emit α or β particle, it gets excited to a higher energy level. This excited nucleus tries to return to its ground state while doing so, it emits a highly energatic radiation whose frequency $f = \frac{E_{n_2} - E_{n_1}}{h}$



$E = E_{n_2} - E_{n_1}$

$E = hf$

↑

γ radiation

↑

γ spectrum

$$ {}^{A}_{Z}X \longrightarrow \alpha + Y^{*} $$

$$ Y^{*} \longrightarrow \gamma + Y $$

excited state — Ground State

Law of Radiactive Decay-

→ Radioactivity is a random process.

→ A particular nuclei can decay to anytime between $t=0$ and ∞.

→ It is impossible to predict when a particular nucleus will decay/ disintegrate.

→ Rutherford & Soddey conducted experimental study on different kind of nuclei and gave a statistical law.

"When there is a large number of nuclei, rate of decay/disintegration is directed proportional to number of nuclei in sample".

Let in time dt, dN nuclei decays

$$ \text{Rate of decay} = \frac{\text{No. of nuclei decay}}{\text{time}} = -\frac{dN}{dt} $$

$$ -\frac{dN}{dt} \propto N \longrightarrow \text{Undecayed} $$

$$ \boxed{-\frac{dN}{dt} = \lambda N} \longrightarrow \text{Undecayed (no. of nuclei)} $$

λ → disintigration constant/ Radioactive constant (depends on choice of element and isotope)

First order reaction $[R] = K[A]^{1}$

$$ \int_{N_0}^{N} \frac{dN}{N} = \int_{0}^{t} -\lambda \, dt $$

$$ [\log_e N]_{N_0}^{N} = -\lambda (t)_0^t $$

$$\log N - \log N_0 = -\lambda(t-0)$$

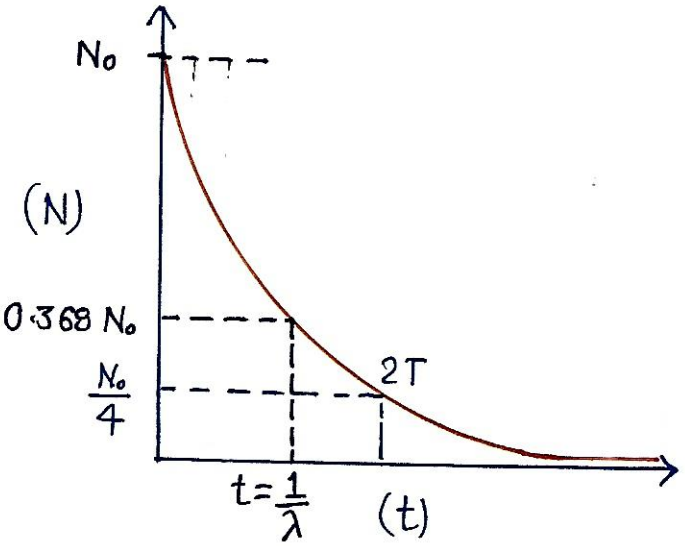
$$\boxed{\log\left(\frac{N}{N_0}\right) = -\lambda t}$$

$$\log\left(\frac{A}{A_0}\right) = kt$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\boxed{N = N_0 e^{-\lambda t}}$$

- t ▷ time
- λ ▷ decay constant
- N ▷ No. of nuclei undecayed at t
- N_0 ▷ Initial number of nuclei ($t=0$)



$$N = N_0 e^{-\lambda\left(\frac{1}{\lambda}\right)} \qquad \text{Put } t = \frac{1}{\lambda}$$

$$N = N_0 e^{-1} = 0.368\, N_0$$

λ is reciprocal of time.

λ Unit ⇨ sec^{-1}

#Half life of Decay -

The time in which half of nuclei decays & half remain unchanged.

$$t = t_{1/2}$$

$$N = \frac{N_0}{2}$$

$$N = N_0 e^{-\lambda t}$$

$$\log\left(\frac{N}{N_0}\right) = -\lambda t$$

$$\log\left(\frac{\frac{N_0}{2}}{N_0}\right) = -\lambda t_{1/2}$$

$$\log\left(\frac{1}{2}\right) = -\lambda t_{1/2}$$

$$\log(2)^{-1} = -\lambda t_{1/2}$$

$$\log_e 2 = \lambda t_{1/2}$$

$$\boxed{t_{1/2} = \frac{\log_e 2}{\lambda}}$$

$$\log_e 2 = 0.693$$

$$\boxed{t_{1/2} = \frac{0.693}{\lambda}}$$

#Number of nuclei left after n half life -

After 1 half life $\Rightarrow N = \frac{N_0}{2} \Rightarrow \frac{N_0}{(2)^1}$

After 2 ——— $\Rightarrow N = \frac{N_0}{4} \Rightarrow \frac{N_0}{(2)^2}$

After 3 ——— $\Rightarrow N = \frac{N_0}{8} \Rightarrow \frac{N_0}{(2)^3}$

After 4 ——— $\Rightarrow N = \frac{N_0}{16} \Rightarrow \frac{N_0}{(2)^4}$

Number of nuclei left after n half lives = $\frac{N_0}{(2)^n}$

$$\boxed{N = \frac{N_0}{2^n}}$$

Activity of radio active substance-

$R = R_0 e^{-\lambda t}$ $\qquad R = \lambda N$

R → Activity at time t

R_0 → Initial Activity

Unit-

1. Becquerel = 1 Bq = 1 disintigration/second
2. Rutherford = 10^6 disintigration/second = 10^6 Bq
3. 1 curie = 1 Ci = 3.7×10^{10} disintigration/second = 3.7×10^{10} Bq

Mean or Avarage of Life of Decay-

It is the avg of lives of all nuclei of the sample.

$$T = \frac{1}{\lambda}$$

Avg life > Half Life

$T = 1.443\, t_{1/2}$

Mass-Energy Equivalence-

- 1905, Einstein in his special theory of relativity, he suggested that mass is a form of energy. He predicted mass and energy are equivalent.

$$E = mc^2$$

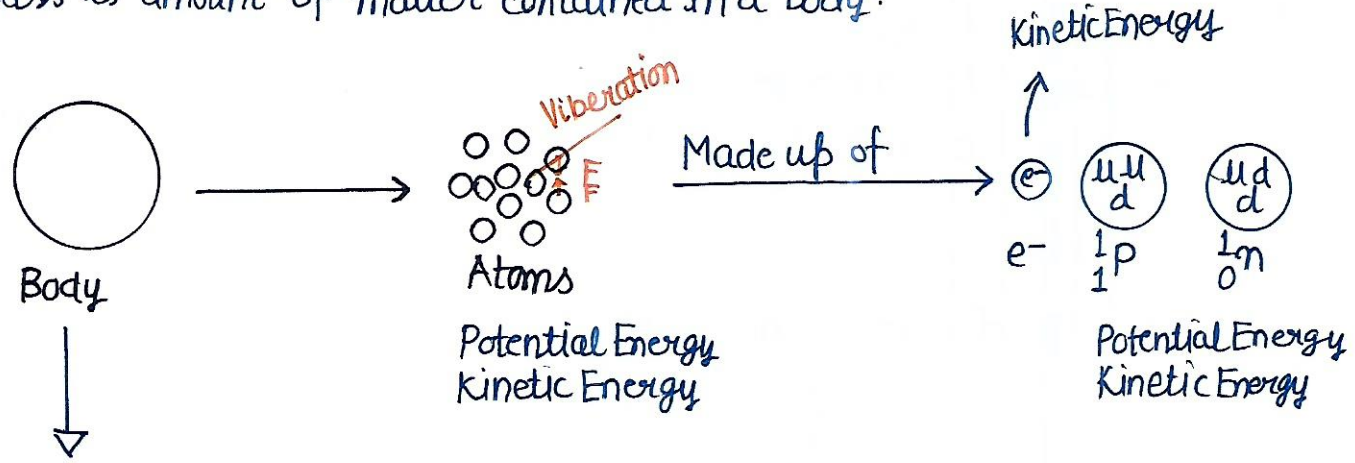
$$m = \frac{E}{c^2}$$

- $m_{moving} = m_{stationary} + m_{extra}$

m_{extra} → $\frac{\text{Kinetic Energy}}{c^2} + \frac{\text{Heat Energy}}{c^2}$

→ Very small

- Mass is amount of matter contained in a body.



$$\text{mass of object} = \frac{\text{Potential Energy} + \text{Kinetic Energy}}{c^2}$$

99% contribution in mass

1% Intrinsic mass → Due to fields (Maybe)

Mass defect and Binding Energy-

→ The mass of nucleus is less than the sum of all nucleus in it.



mass > mass of nucleons

→ Some mass disappears during formation of nucleus.

→ The mass that disappered in termed as Mass Defect.

$$\Delta m = m_{nucleons} - m_{nucleus}$$

$$\Delta m = m_{Proton} + m_{Neutron} - m_{nucleus}$$

$$\Delta m = (Zm_p + (A-Z)m_N - m_X)$$

m_H

$_Z^A X$

$P = Z$

$N = A-Z$

→ $\Delta E = \Delta m c^2$

$$\Delta E = [Zm_p + (A-Z)m_N - m_X]c^2$$

↓

Binding Energy

(Amount of Energy required to seperate nucleons from the nucleus)

→ More Binding Energy ⟶ More energy release ⟶ more stable nucleus

$$\Delta E = \Delta m c^2$$

$$\Delta E = 1 \times 1.67 \times 10^{-27}\ \cancel{kg} \times (3\times10^8)^2\ \cancel{m^2/s^2} \times \frac{1.6\times10^{19}\ \cancel{eV}}{10^6}\ MeV$$

$$\Delta E = 931.6\ MeV$$

$$\boxed{\Delta E = \Delta m \times 931.6\ MeV}$$

↓

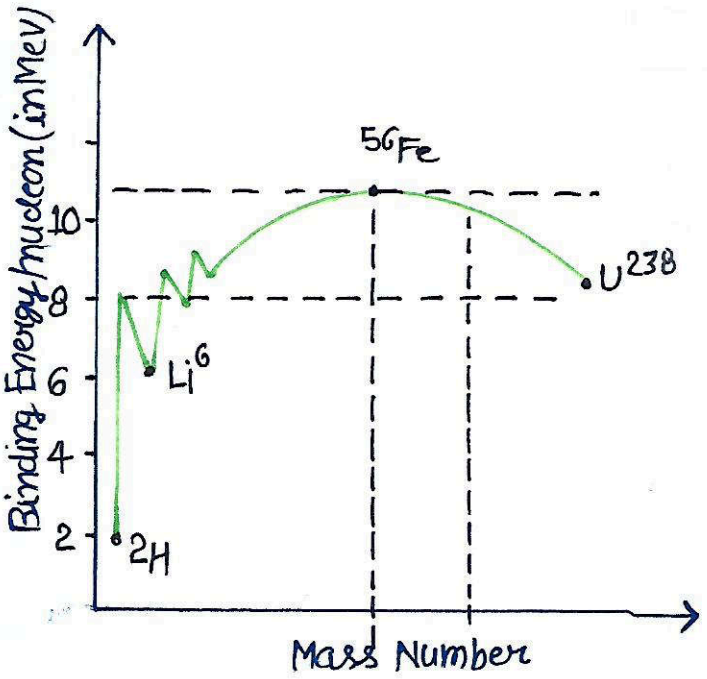
in amu

#Nuclear Stability or Binding Energy Per nucleon–

- The Binding energy of heavy hydrogen ^{2}H is 1.1 MeV & that of ^{238}U is 1800 MeV.
- Binding energy increase with mass number (A).
- ^{56}Fe with binding energy 492 MeV

  ^{238}U with binding energy 1800 MeV → Less stable

- Binding energy per nucleon is criteria for Nuclear stability.



(1) From A = 50 to A = 80

Binding Energy/Nucleus ≈ 8.5 MeV & these nucleus are stable.

(2) Fe (A = 56)

B.E./Nucleon ≈ Highest ≈ Most stable nucleus.

(3) From A > 80,

B.E/Nucleons ≈ decrease slowly ≈ Unstable → Nuclear Fission (Stable)

(4) For A < 50,

B.E/ nucleons ≈ decrease sharply ≈ Unstable

↓

Try to gain stability through Nuclear Fusion

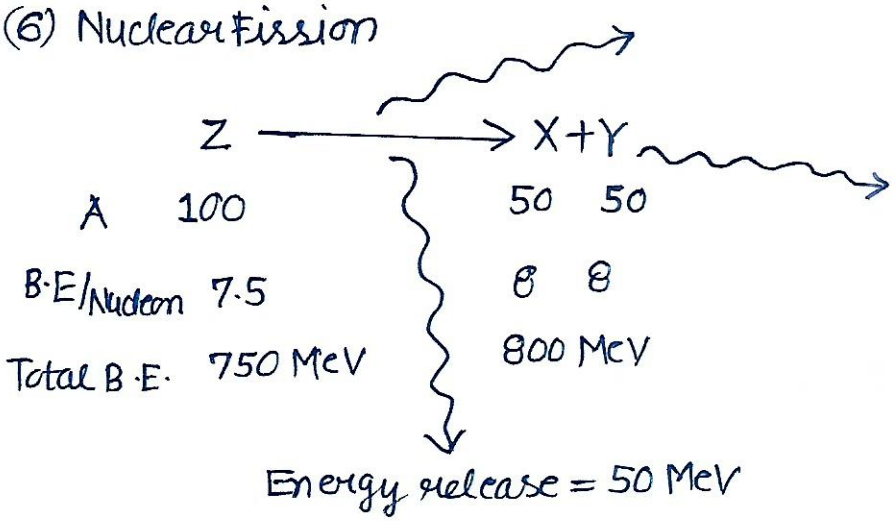
(5) B.E ↑ = Stability ↑

$$X \xrightarrow[\text{Released}]{20\,MeV} Y$$

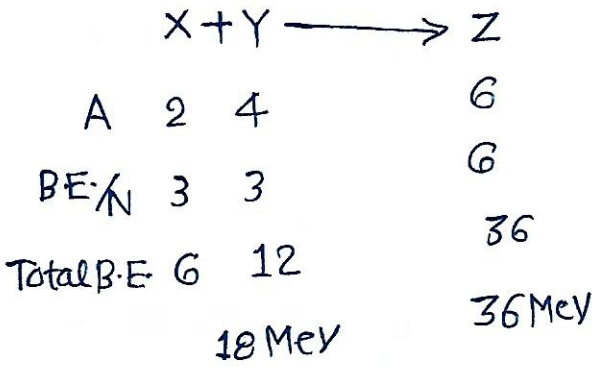
100 MeV 120 MeV

- B.E. increase in reaction, energy will release.

(6) Nuclear Fission

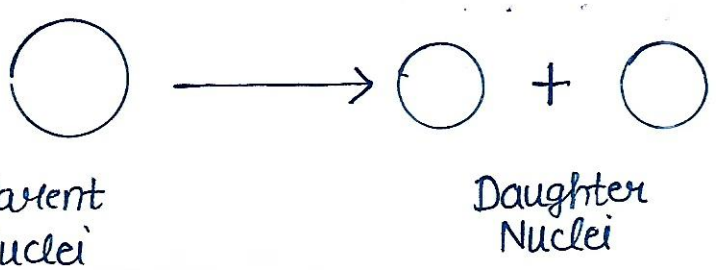


(7) Nuclear Fusion



Nuclear Fission-

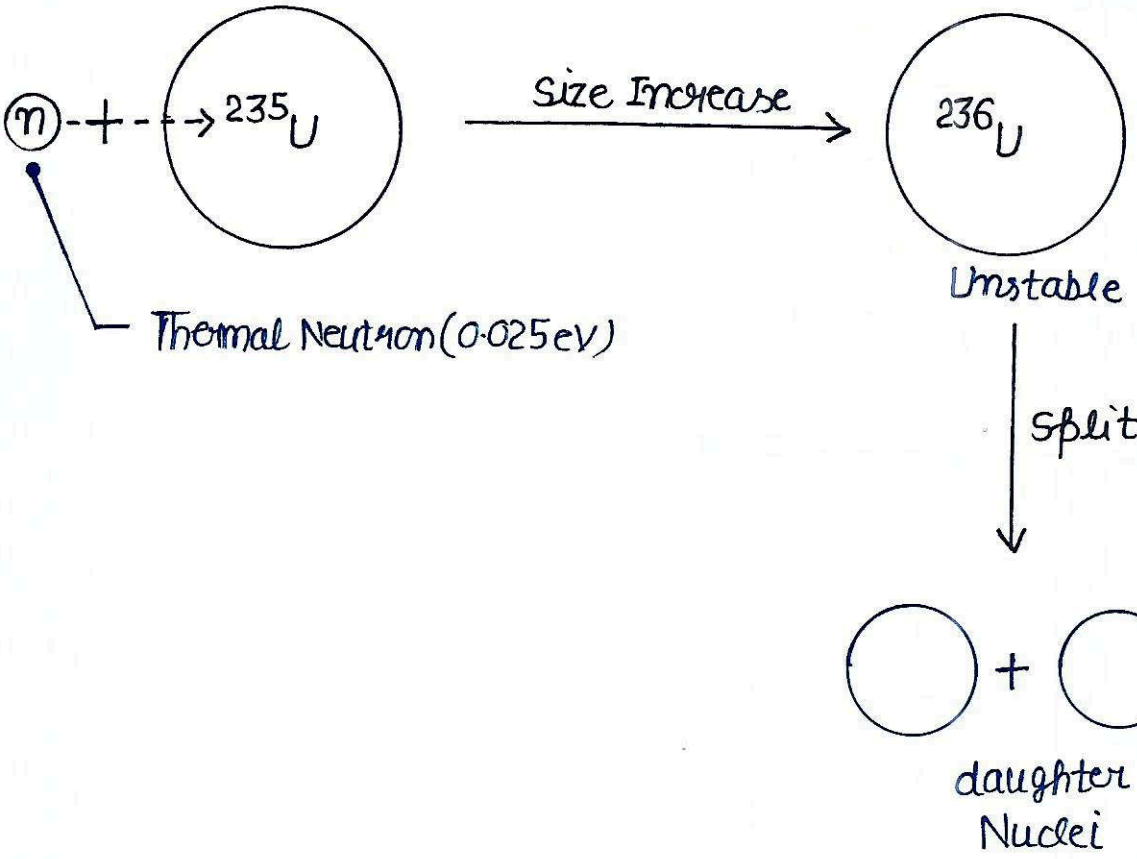
Nuclear Fission is a nuclear reaction in which a heavy nucleus splits into two or more smaller nuclei.



* Most of the nuclei fission are binary.
* The daughter nuclei are of comparable size.
* Spontaneous fission ⟶ Radioactivity

* Induced Fission ⟶ Induced by β, n, α.

• Fission induced by neutron –



They are radioactive. Further β, γ emission.

$$^{1}_{0}n + {}^{235}_{92}U \longrightarrow {}^{236}_{92}U \xrightarrow{\text{Split}} {}^{144}_{56}Ba + {}^{89}_{36}Kr + 3\,{}^{1}_{0}n + Q(\text{energy})$$

Unstable

$$^{133}_{51}Sb + {}^{99}_{41}Nb + 4\,{}^{1}_{0}n + Q$$

$$^{140}_{54}Xe + {}^{99}_{38}Sr + 2\,{}^{1}_{0}n + Q$$

Most Probable

⇒ On avarage, 2.5 neutrons are released per fission of ^{235}U.

• Energy in Fission-

e.g. Stability of nuclei

$$X \longrightarrow Y + Z$$

Mass No.	240	120	120
$\frac{\text{B.E.}}{n+p}$	7.6	8.5	8.5
Total B.E.	1824 MeV	1020 MeV	1020 MeV

2040 MeV

Energy liberate = 2040 − 1824 = 216 MeV ≈ 200 MeV

1 nucleus of ^{235}U on fission release 216 MeV of energy.

⇓

$240 \times 1.6 \times 10^{-24}$ g of Uranium

∵ Energy is liberated thus mass has disappered.

931.5 MeV ⟶ 1 amu

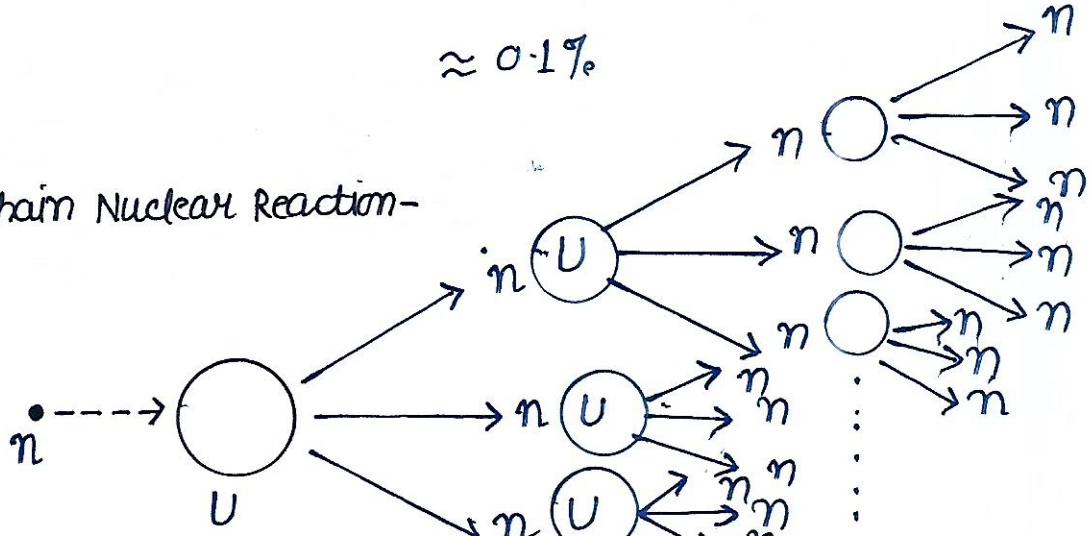
216 MeV ⟶ ??

$$\frac{216}{931.5} = 0.23 \text{ amu}$$

% of mass disappeared ≈ $\frac{0.23}{240} \times 100$

≈ 0.1%

• Chain Nuclear Reaction-


n
U
n
U
n
U
n
n
n

Chain Nuclear Reaction

Uncontrolled
Energy will multiply in ms.

→ Atom Bomb

→ 50kg U

→ 10^{15} J Energy

→ Hirosfima & Nagashaki

Controlled
When one fission cause one fission

→ Constant Energy Liberation

→ Nuclear Reactor

→ Electricity

→ 3% of India

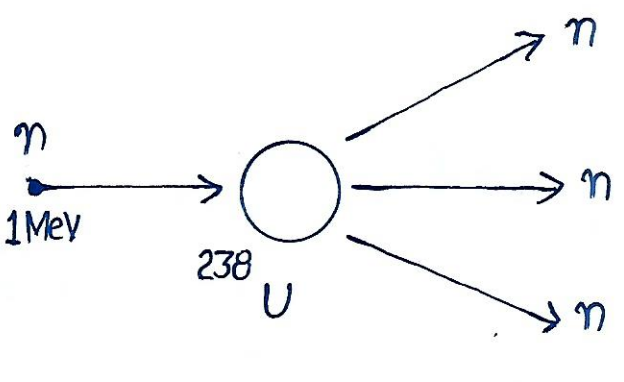
→ 10% of World

- Difficulties in chain reaction-

1. Poor concentration of Fuel (U)

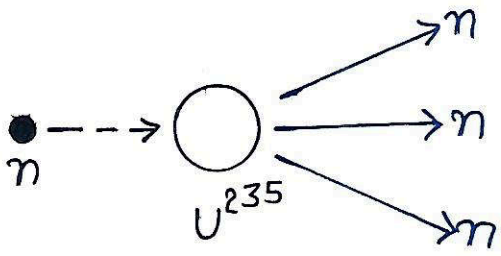
Natural Uranium: 99.3% ^{238}U, 0.7% ^{235}U

2. ^{238}U is Non-Fissile. It can be only fissionable with high energy neutron (1 MeV or more)



Slow neutrons less energatic due to inelastic scattering → These neutrons do not have 1 MeV energy and hence they can't sustain chain reaction.

3. The Fuel U^{235} is fissionable by only slow neutron.



Each of these neutron have 2 Mev energy each ← We have to Slow down the neutrons to low energy so that they can cause further fission

4. Critical mass –

So even slow neutrons move at 8 times speed of sound. If the layer of fuel is thin, the neutrons may pass through it without causing fission.

There must be limiting mass of fuel which can sustain chain reaction. The ~~maximum~~ minimum mass of fuel required for chain reaction is called critical mass.

- depends on geometry of fuel & speed of neutrons.
- Use moderator.

5. Controlling the rate of fission –

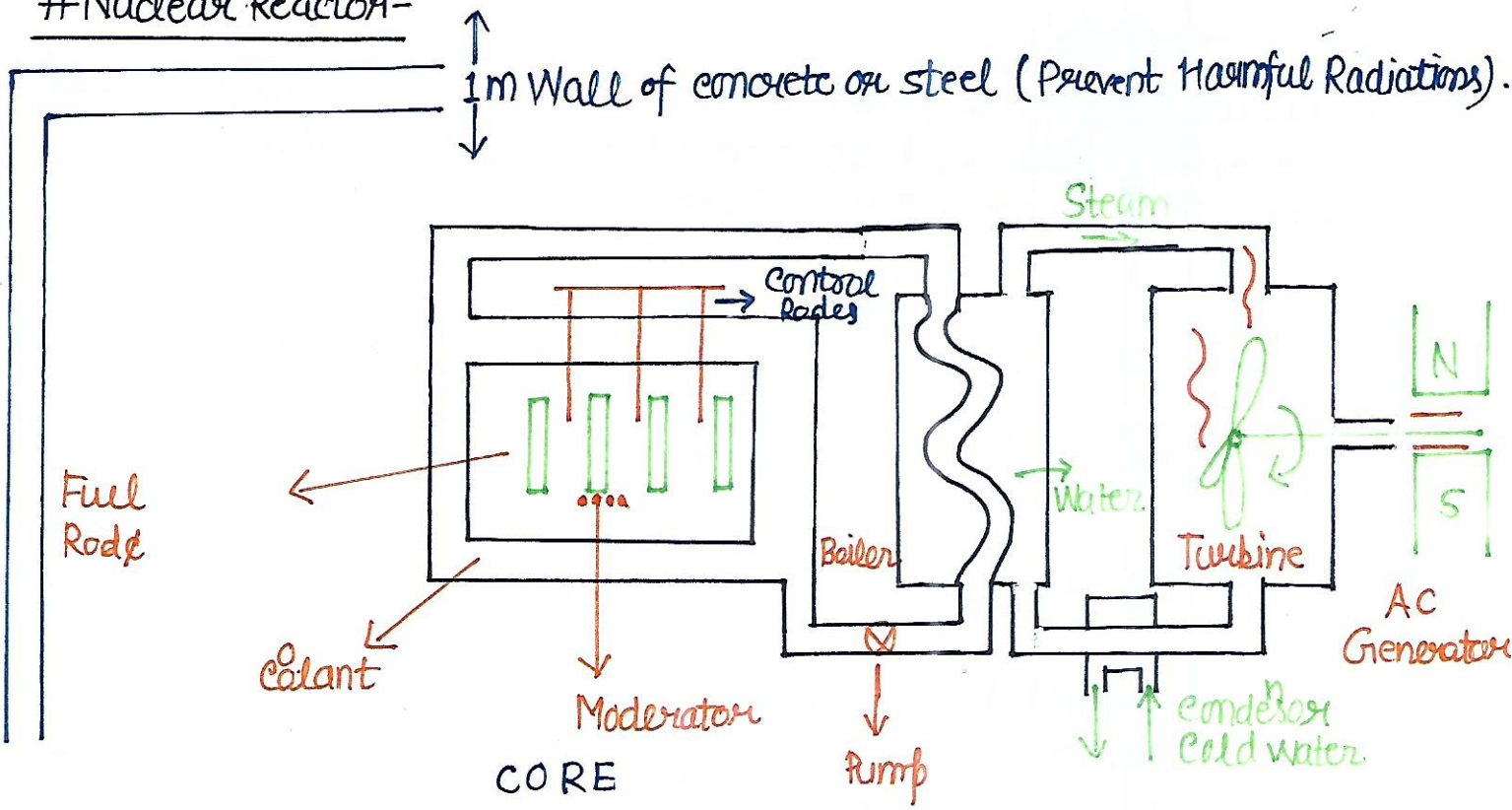
$$K = \frac{\text{No. of fission in present generation}}{\text{No. of fission in previous generation}}$$

$K = 1$ → One fission causes one fission (✓)
→ Rate of energy liberated is constant

$K > 1$ → One fission causes more than one fission. → Explosion.
→ Rate of energy liberated is increased.

$K < 1$ → No. of fission decreased. → Reaction stop.
→ Rate is decreased.

#Nuclear Reactor-



1. Fuel Rod ^{235}U ⟶ Fissile
2. Moderator → Slows down fast moving neutrons to thermal energy. so that they can cause fission.
   - Water, Heavy water, Graphite

   Apsara Reactor (BARC) ⟶ Water
   Chernobyl ⟶ Graphite
   Rest 21 reactor in India ⟶ Heavy water.
3. Control Rods ⟶ control the rate of fission (Maintain K Value)
   - Neutron absorbing material.
   - Boron, cadmium, Hafnium.
4. Coolant ⟶ circulates around core in 2-3 loops and absorbs heat from fission, and carries heat to boiler.
   - Water, CO_2, Air, Na(l).

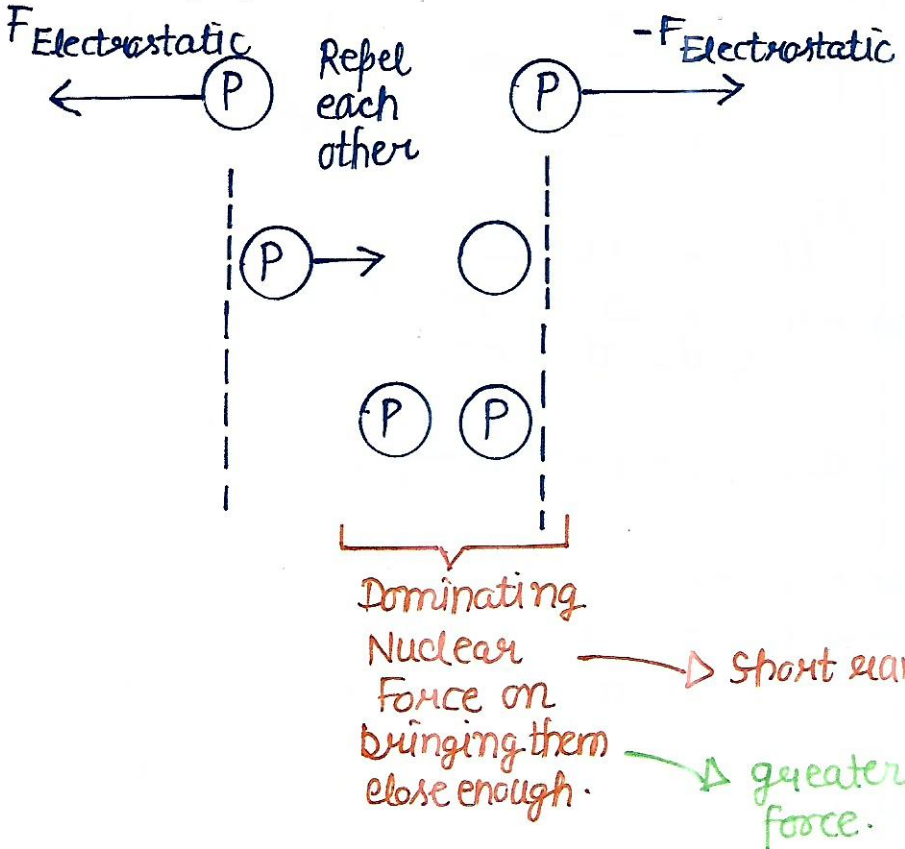
#Nuclear Fusion-

- sun, stars etc.
- Nuclear Fusion is a process in which two or more lighter nuclei combines to form a heavy nucleus, alongwith simultanious release of large amount of energy.
- e.g.

$$_1^1H + {}_1^1H \longrightarrow {}_1^2H + e^+ + \nu + 0.42\ MeV$$

(P) (P) (P)(n)

- blending by melting



$$K.E = \frac{1}{2}KT$$

↓ Baltzmann's Constant

$K.E = 400\ KeV$

$T \approx 3 \times 10^9\ K$

$H_2 \longrightarrow H + H$

Atoms are completely ionised, bare nuclei with e⁻ cloud around them are obtained. (PLASMA STATE)

Solid ⟶ Liquid ⟶ Gas ⟶ Plasma

→ Nuclear fusion is exothermic process.

→ Hydrogen Bomb

↓

Atom bomb provide initial energy of temp. 10^9 K

↓

Basic principle

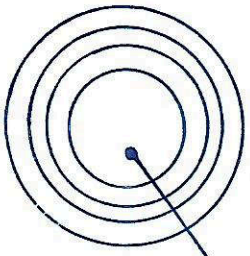
↓

Destructive use.

→ Such High temperature is maintained in stars. e.g → Sun.

↙ ↘

90% (H) + He) 10% (other elemen + organic comple



Core ($E = 1.5 \times 10^7$ K) → K.E. 2keV

↓

> 400KeV

↓

Maxwell Boltzmann Law

#Nuclear Fusion in sun-

$$_1^1H + _1^1H \longrightarrow _1^2H + e^+ + \upsilon + 0.42\ MeV$$

$$e^+ + e^- \xrightarrow[\text{Annihilation}]{\text{matter}} \gamma + \gamma + 1.02\ MeV$$

to balance momentum, 2 γ emit.

$$_1^2H + _1^1H \longrightarrow _2^3He + \gamma + 5.49\ MeV$$

$$_2^3He + _2^3He \longrightarrow _2^4He + _1^1H + _1^1H + 12.86\ MeV$$

(P)(P) (n) + (P)(P) (n) → (P)(P) (n)(n) + (P) + (P)

Net reaction:

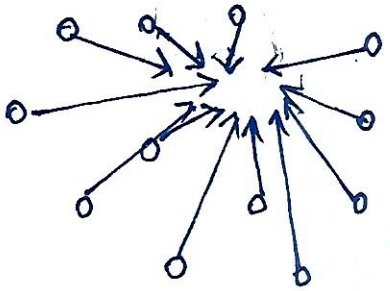
$$4\,_1^1H + 2e^- \longrightarrow _2^4He + 2\upsilon + 6\gamma + 26.7\ MeV$$

* Fusion gives much more energy than fission.

#Life cycle of stars-

Birth →

When massive clouds of gases (H and He) & other elements start contracting due to gravitational pull.



K.E is very high
T is very high

↓

Molecules breakes into atoms.

↓

Bare nuclei with e^- clouds

↓

Fusion begins

Sequence phase

↓

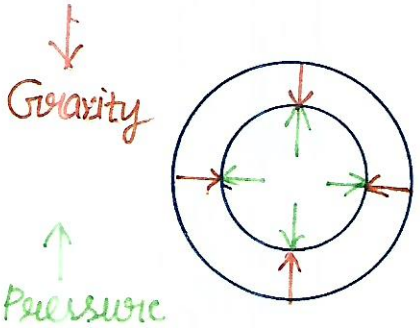
Heat releases

↓

Expansion

↓

Stable due to Hydrostatic Equilibrium



↓

10 Billions years

↓

Spent 5 Billion years

↓

Rest life 5 Billion years → Yellow

↓

Calling down → Still some hydrogen rest in shell

↓

RED

↓

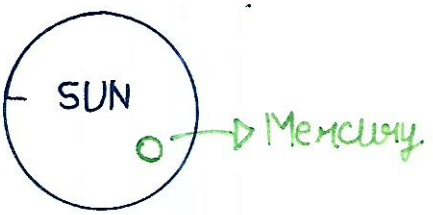
Core →←

↓

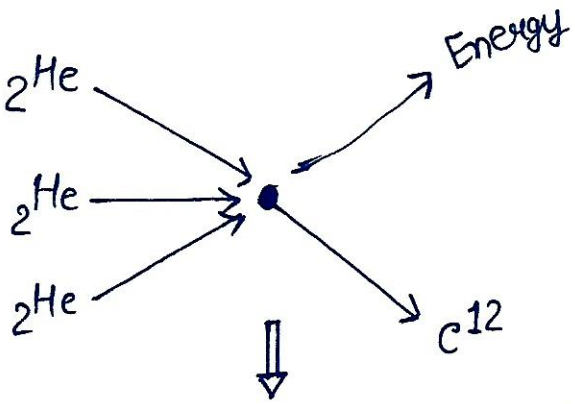
Red Giant Phase

↓

Still expends (Shell)



$M \leq 8 M_{\odot}$



⇓

K.E. is not that much high in such stars that it can fuse carbon nuclei

⇓

Death of SUN

↓

^{12}C

↓

Core contraction → Some He is rest in outer core

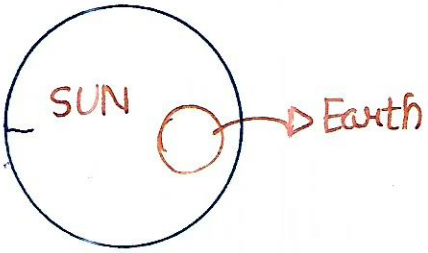
↓

Heat

↓

Outer core expends

↓



↓

Sun looses mass from outer layer.

↓

e^- degeneracy

↓

Stable

White Dwarf (Nebula)

↓

10 billion years

↓

Black dwarf — Black lumps of C^{12}.

$M \geq 10 M_\odot$

C^{12} Fusion — CNO cycle

↓

N-N→O

↓

Fe (Most stable Nuclei)

↓

Death star

↓

Supernova

↓

Implosion — An inward explosion

↓

Fe core

↓

Neutron star: $14 M_\odot \leq x \leq 5 M_\odot$

↓

Emit X rays

Black Hole: $x \geq 5 M_\odot$ → Strong gravitational pull

↓

core falls onto itself

Escape Velocity $V_e = \sqrt{\frac{2GM}{r}}$

M ↑↑↑

r ↓↓↓

Nuclear Physics -

→ Study of Atomic nucleus.

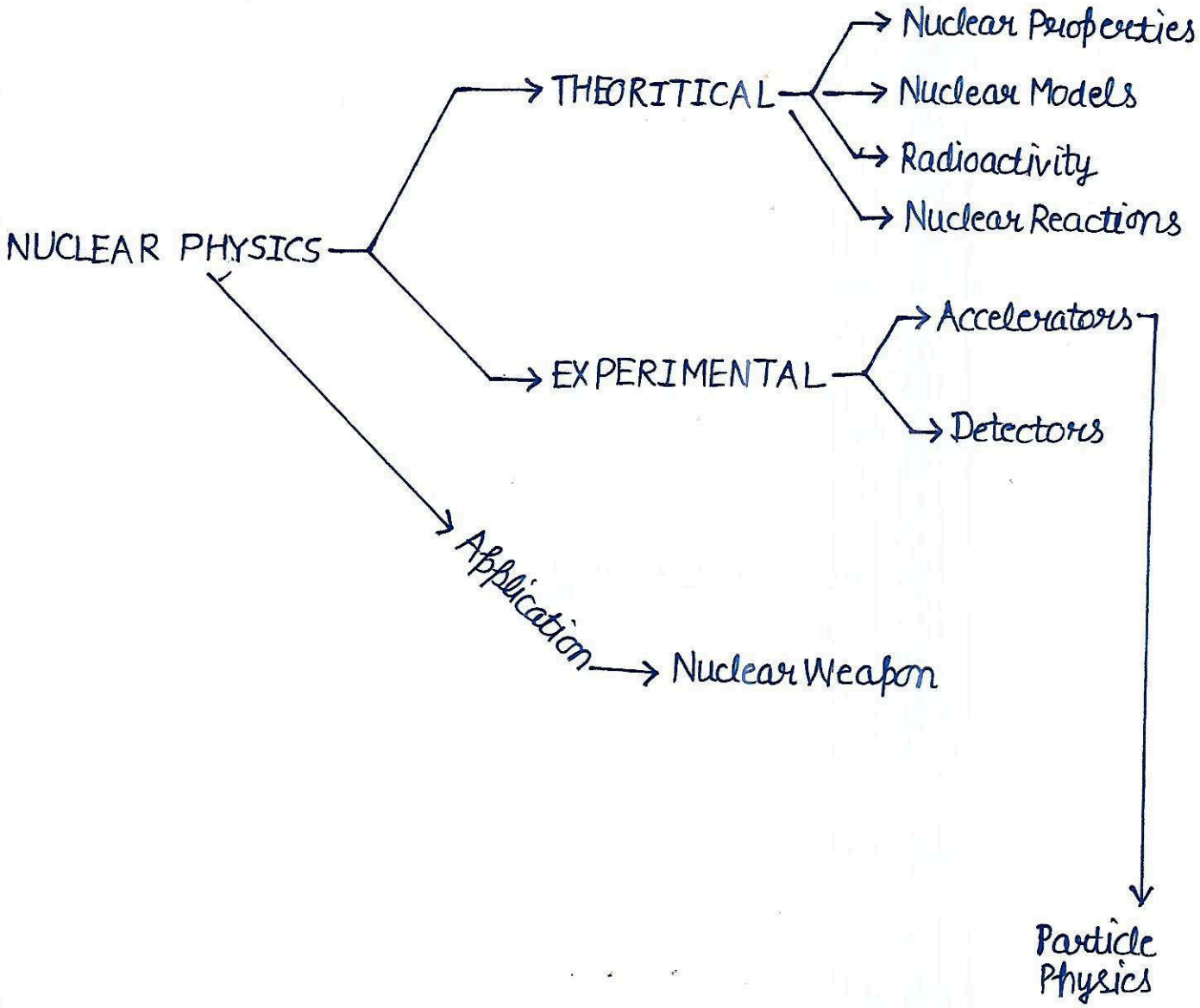
→ Discovery of Radioactivity.

→ Discovery of e^-.

→ $E = mc^2$

→ Discovery of Neutron

→ Yukava Theory of Nuclear Force.



Nuclear Size -

H ; Nucleus → 0.8 fm

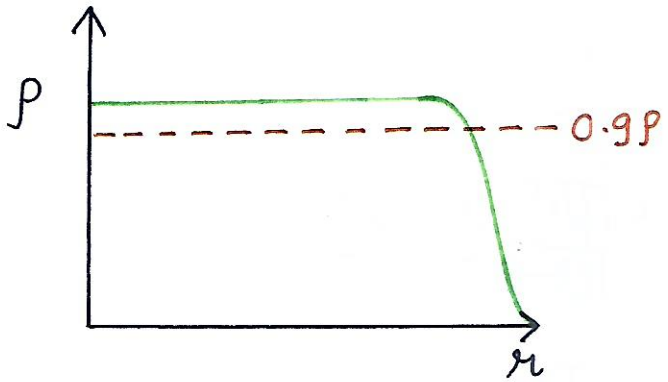
U ; Nucleus → 7 fm

H atom → 0·5 Å ≈ 50000 fm

$$\text{Ratio} = \frac{50000\ fm}{0.8\ fm} \approx 62{,}500$$

• Methods of measuring atomic radius-

→ Electron scattering
→ X-ray isotop shift
→ Muonic X-rays
→ Mirror Nuclei
→ Alpha decay
→ π-mesonic X rays



ρ = Constant

$$\frac{A}{\frac{4}{3}\pi R^3} = \text{constant}$$

$$R^3 \propto A$$

$$R \propto A^{1/3}$$

$$R = R_0 A^{1/3}$$

$$R_0 \approx 1{\cdot}2\ fm$$

#Quantization of Angular Momentum-

- Orbital Angular momentum
- Spin Angular momentum

$\vec{L} = \vec{r} \times \vec{p}$ (Continuous Values/Not quantized)

→ No restrictions

→ Not quantized

$$I = \sum_i m_i r_i^2$$

$$S = I\omega$$

ATOM/ELECTRON-

- orbital Angular momentum-

(a) Magnitude $L = \sqrt{l(l+1)}\,\hbar$

$l = 0, 1, 2, \ldots (n-1)$

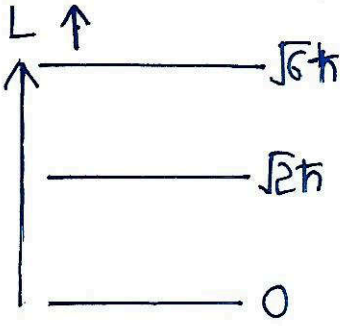
↓ ↓ ↓

s p d

$s: l=0 \quad : \quad L = \sqrt{l(l+1)}\,\hbar = 0$

$p: l=1 \quad : \quad L = \sqrt{1(1+1)}\,\hbar = \sqrt{2}\,\hbar$

$d: l=2 \quad : \quad L = \sqrt{2(2+1)}\,\hbar = \sqrt{6}\,\hbar$



(b) Space or direction



$L_z = m_l \hbar$

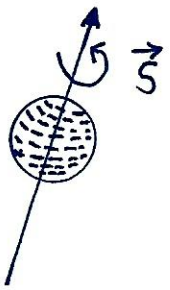
$m_l = -l, -(l-1), 0, l$

$l = 1$

$m_l = -1, 0, 1$

$L_z = -\hbar, 0, \hbar$

- Spin Angular momenta-



$$S = \sqrt{s(s+1)}\,\hbar$$

$$S = \sqrt{\frac{1}{2}\left(\frac{1}{2}+\frac{1}{2}\right)}\,\hbar$$

$$S = \frac{\sqrt{3}}{2}\hbar$$

$$S_z = m_s \hbar$$

$$S_z = \hbar m_s = \pm\frac{\hbar}{2} \quad \left[m_s = \pm\frac{1}{2}\right]$$

- Angular momentum is always conserved. Not necessarily S, L is seperately conserved.

Total Angular momentum $\vec{J} = \vec{L} + \vec{S}$

$$J = \sqrt{j(j+1)}\,\hbar$$

$$j = l+s \text{ or } l-s$$

$$j = l+\frac{1}{2} \text{ or } l-\frac{1}{2}$$

For p orbital : $l = 1$

$$j = l+\frac{1}{2} \text{ or } \left|l-\frac{1}{2}\right|$$

$$j = \frac{3}{2} \text{ or } \frac{1}{2}$$

$$J = \frac{\sqrt{15}}{2}, \frac{\sqrt{3}}{2}$$

$$J_z = m_j \hbar$$

$$m_j = -j, 0, j$$

$$j = \frac{1}{2} : m_j = -\frac{1}{2}, \frac{1}{2}$$

$$j = \frac{3}{2} : m_j = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$$

#Nuclear Spin and Angular momentum-

Spin Angular momentum $= L + S$

$$= \sum_i l_i + \sum_i s_i$$

Magnitude $= \sqrt{I(I+1)}\hbar$

$I = 0, \frac{1}{2}, 1, \frac{3}{2}$

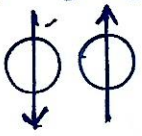
→ Nuclear Spin

$$I_z = M_z \hbar$$

$$M_z = -I, 0, I$$

Pairing Effect -

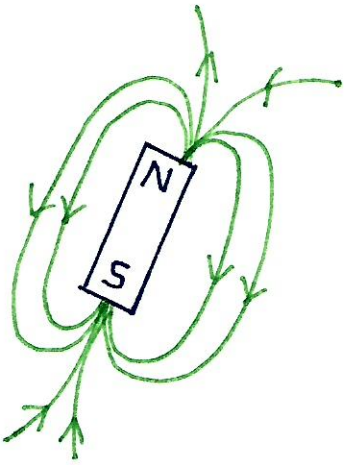
Even-Even nuclei $I = 0$ — $^4He, ^{12}C, ^{64}Cd$

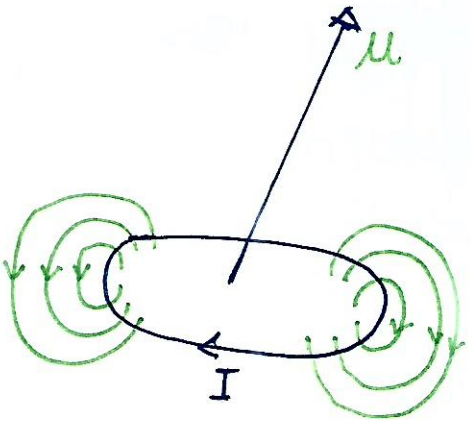


Even-odd nuclei $I = \frac{1}{2}$ — $^1H, N^{15}$

Odd-odd nuclei $I = 1$ — $^2H, N^{14}$

#Nuclear Magnetic Moment-





$$I = \frac{-e}{T} \quad \text{Where } T = \frac{2\pi r}{v}$$

$$I = \frac{-ev}{2\pi r}$$

$\mu = IA$ ..... Magnetic diapole moment

$$\mu = \frac{-ev}{2\pi r} \cdot A$$

$$\mu = \frac{-ev\,\pi r^2}{2\pi r}$$

$$\mu = \frac{-e(mvr)}{2m}$$

$$\boxed{\mu = \frac{-eL}{2m}}$$

$$\mu = \frac{-eL}{2m}$$

$\mu \propto L$

Intrintic charge
+
Angular Momentum

- Orbital Magnetic momentum

$$\mu = -g_l \frac{eL}{2m} \quad (g_l = 1)$$

- Spin Magnetic momentum

$$\mu = -g_s \frac{eL}{2m} \quad (g_s = 2)$$

* Electron behave as a tiny bar magnet:

$$\mu = -g_l \left(\frac{e\hbar}{2m}\right) \frac{L}{\hbar}$$

↓ Bohr magneton

$$\mu = -g_l \mu_B \frac{L}{\hbar}$$

$$\mu = -g_s \left(\frac{e\hbar}{2m}\right) \frac{S}{\hbar}$$

$$\mu = -g_s \mu_B \frac{S}{\hbar}$$

Bohr magneton (for e^-) $\mu_B = \frac{e\hbar}{2m_e} = 5.788 \times 10^{-5}\ eV/T$

Bohr magneton (for nuclear particle) $\mu_N = \frac{e\hbar}{2m_p} = 3.15 \times 10^{-8}\ eV/T$

Neutron $\mu_N = g\mu_N \frac{S}{\hbar}$

Proton $\mu_p = g\mu_p \frac{S}{\hbar}$

Nucleus $\mu = g\mu_N \frac{I}{\hbar}$

Binding Energy curve-

$$^{2}_{1}H \longrightarrow {}^{1}_{1}H + {}^{1}_{0}n$$

$m(^{2}_{1}H) \longrightarrow 2.01402\ u$

$m(^{1}_{1}H) \longrightarrow 1.007824\ u$

$m(^{1}_{0}n) \longrightarrow 1.008665\ u$

$m \longrightarrow 2.016490\ u$

⇒ Expected mass - Actual mass

⇒ 2.016490 - 2.01402

⇒ 0.002388 u

↑ mass defect

mass-energy equivalence-

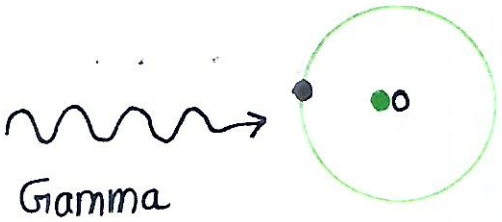
$E = \Delta m c^2$

$E = 931.49\ MeV/c^2$

So

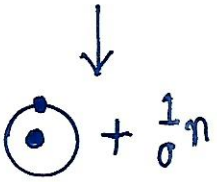
$E = 0.002388 \times 931.49$

$E = 2.224\ MeV$



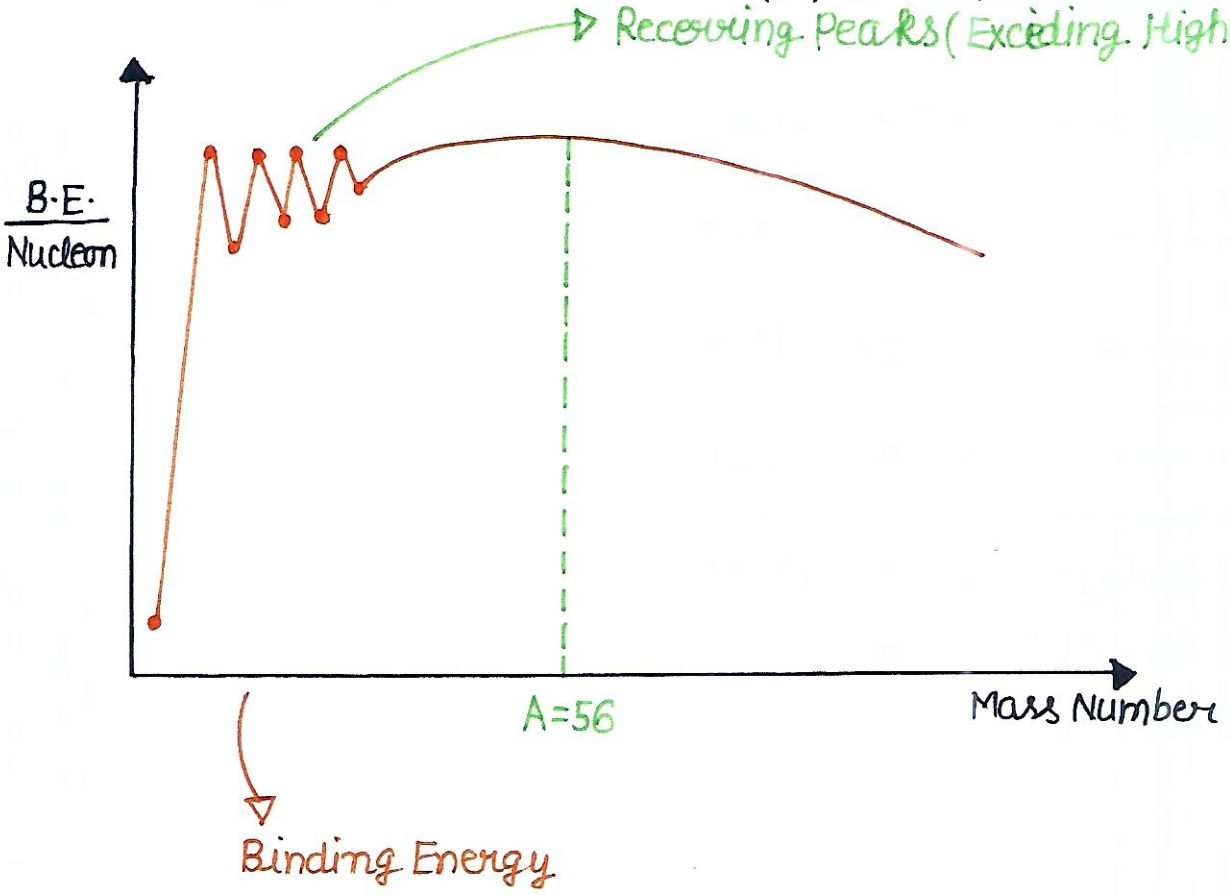
Gamma Photon

$E_\gamma = 2.224\ MeV$



Nuclear Binding energy-

$$B.E. = [Z m(^1_1H) + N m(^1_0n) - m(^A_ZX)] \times 931.4\,g$$



Higher BE/N ⟶ Higher Stability

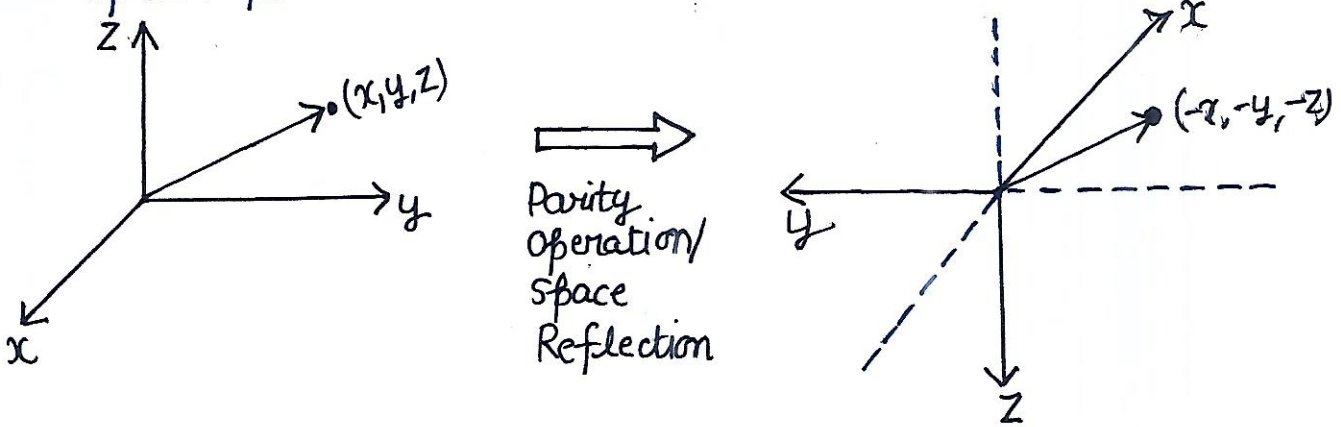
Lesser BE/N ⟶ Higher + Energy BE/N

Binding Energy is the energy required to break apart a nucleus into its constituents particles.

#Parity of Wave Function-

→ Reflection in mirror (Inverted)

→ Space Reflection



Parity : $(x, y, z) \longrightarrow (-x, -y, -z)$

Schrodinger Eq^n

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

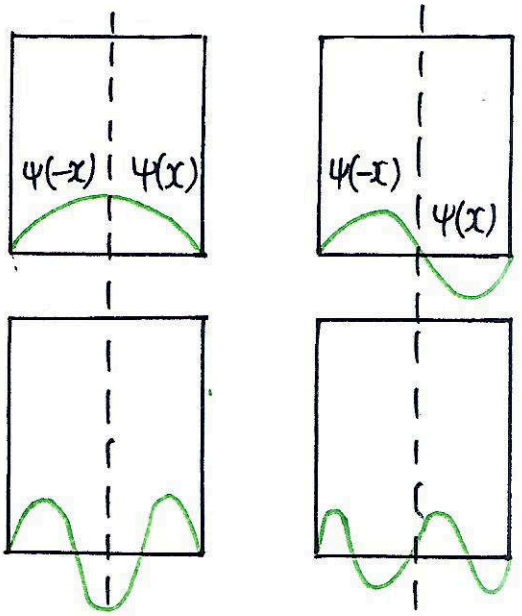
under parity

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(-x)}{dx^2} + V(+x)\psi(-x) = E\psi(-x)$$

$V(-x) = V(x)$: $|\psi(-x)|^2 = |\psi(x)|^2$

$$\Downarrow$$

$$\psi(-x) = \pm\psi(x)$$



Square Well Potential

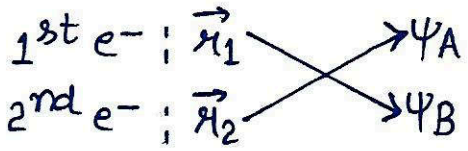
Symmetric & Anti-symmetric wave function-

1st e$^-$: $\vec{r}_1 \longrightarrow \psi_A$

2nd e$^-$: $\vec{r}_1 \longrightarrow \psi_B$

Combined wave f^n solution

$$\psi(\vec{r}_1, \vec{r}_2) = \psi_A(\vec{r}_1)\psi_B(\vec{r}_2)$$



$$\psi(\vec{r}_2, \vec{r}_1) = \psi_A(\vec{r}_2)\,\psi_B(\vec{r}_1)$$ [After exchange of particles]

$$|\psi(\vec{r}_1, \vec{r}_2)|^2 = |\psi(\vec{r}_2, \vec{r}_1)|^2$$

$$\psi(\vec{r}_1, \vec{r}_2) = \pm\psi(\vec{r}_2, \vec{r}_1)$$

- **Symmetric ;**

$$\psi(\vec{r}_1, \vec{r}_2) = \psi(\vec{r}_2, \vec{r}_1)$$ → Bosons $\psi_{sym} = \frac{1}{\sqrt{2}}[\psi(\vec{r}_1, \vec{r}_2) + \psi(\vec{r}_2, \vec{r}_1)]$

- **Assymmetric ;**

$$\psi(\vec{r}_1, \vec{r}_2) = -\psi(\vec{r}_2, \vec{r}_1)$$ → Fermions $\psi_{asy} = \frac{1}{\sqrt{2}}[\psi(\vec{r}_1, \vec{r}_2) - \psi(\vec{r}_2, \vec{r}_1)]$

- **Special case ;**

1st e⁻ ; $\vec{r}_1 \longrightarrow \psi_A$

2nd e⁻ ; $\vec{r}_2 \longrightarrow \psi_A$

$$\psi(\vec{r}_1, \vec{r}_2) = \psi_A(\vec{r}_1)\,\psi_A(\vec{r}_2)$$

$$\psi(\vec{r}_2, \vec{r}_1) = \psi_A(\vec{r}_2)\,\psi_A(\vec{r}_1)$$

ψ_{sym} = some value

$\psi_{asy} = 0$ → Not possible.

⇓

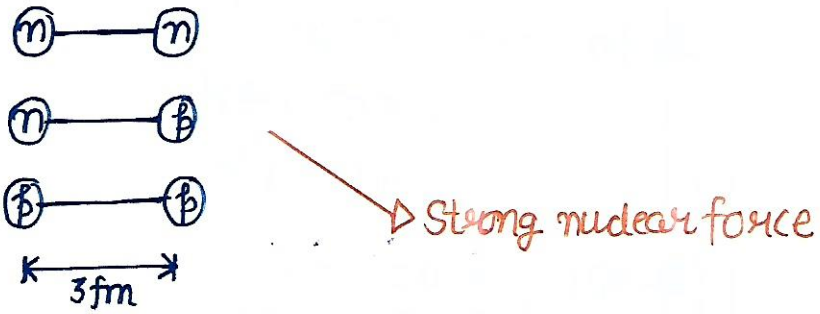
Pauli Exclusion Principle.

Quantum Particles

Bosons	Fermions
Symmetric wave fn	Assymmetric wave fn
Don't follow exclusion principle	Follow exclusion principle
Spin 0, 1, 2	Spin $\frac{1}{2}, \frac{3}{2}$
photons	e^-; p, n
BE statistics	Fermi Dirac statistics

#Liquid drop Model of Nucleus-

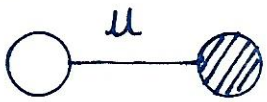
1. Both the nucleus and drop of a liquid contain a large number of particles.
2. Both are homogenous and incompressible.
3. The nuclear mass density, is constant throughout its entire volume and drops to zero near its boundary, just like drop of liquids.



4. Each nucleon inside the nucleus interacts strongly with its nearest neighbouring nucleons, just as molecules do in a liquid.
5. Both show surface tension effect.

6. Heat of vaporization of liquid = Binding energy of nucleus.
7. Evaporation of molecules from a liquid drop is similar to loss of nucleons from the nucleus when provided external energy.
8. The fusion of small drops into larger one and the breaking up/fission of large drops into smaller one is analogous to fusion and fission of nuclei respectively.

(A) Volume Energy :



$E = u$

$\frac{B.E.}{\text{nucleon}} = \frac{1}{2}u \Rightarrow \frac{1}{2} u \times 12 = 6u$ [For 13 nucleon, 12 neighbour]

$B.E = A \times \frac{B.E.}{\text{Nucleon}}$

$B.E = 6uA$

$$\boxed{E_V = a_1 A}$$

(B) Surface Energy -

Surface area $= 4\pi R^2$

$= 4\pi (R_0 A^{1/3})^2$

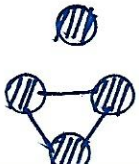
$= 4\pi R_0^2 A^{2/3}$

Surface Energy $\propto A^{2/3}$

$$\boxed{E_S = -a_2 A^{2/3}}$$

(C) columb Energy -





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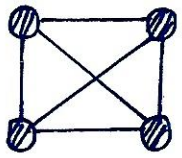
3



$$V = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r}$$

For Z protons

$$V = \frac{1}{4\pi\epsilon_0} \times \frac{Z(Z-1)e^2}{2} \times \left(\frac{1}{r}\right)_{av}$$

$$V = \frac{Z(Z-1)e^2}{8\pi\epsilon_0} \times \left(\frac{1}{r}\right)_{av}$$

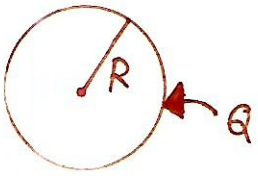
$$V = \frac{Z(Z-1)e^2}{8\pi\epsilon_0} \times \frac{3}{2R}$$

$$E_C = -a_3\frac{Z(Z-1)}{R}$$

$R \to R_0 A^{1/3}$

$$\boxed{E_C = -a_3\frac{Z(Z-1)}{A^{1/3}}}$$

$$\left(\frac{1}{r}\right)_{av} = \frac{1}{Q}\int_0^R \frac{dq}{r}$$



$$\rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$\rho = \frac{q}{\frac{4}{3}\pi r^3}$$

$$\rho = \frac{dq}{3\times\frac{4}{3}\pi r^3 dr}$$

$$\Rightarrow dq = \rho\frac{4}{3}\pi r^3 dr \times 3$$

$$\left(\frac{1}{r}\right)_{av} = \frac{1}{Q}\times\frac{Q}{\frac{4}{3}\pi R^3}\int_0^R \frac{\frac{4}{3}3r^2dr}{r}$$

$$\left(\frac{1}{r}\right)_{av} = \frac{3}{2R}$$

$$E_B = E_V + E_S + E_C$$

$$E_B = a_1 A - a_2 A^{2/3} - a_3 \frac{Z(Z-1)}{A^{1/3}}$$

$$\boxed{\frac{E_B}{A} = a_1 - a_2\left(\frac{1}{A}\right)^{1/3} - a_3 \frac{Z(Z-1)}{A^{4/3}}}$$

Corrections in Liquid drop model/Semi-Empirical mass formula-

(D) Assymetry Energy-

$A = 12$	$A = 12$
$N = 6$	$N = 10$
$Z = 6$	$Z = 2$

To create an access of nuetrons $(N-Z) = 8$ neutrons

$$\left(\frac{N-Z}{2}\right) = 4 \text{ protons to neutrons}$$

Therefore

$$\frac{\text{Energy}}{\text{Nucleon}} = 2E$$

$$= \frac{4E}{2}$$

$$= \frac{1}{2} \times 4 \times E$$

$$= \frac{1}{2}\left(\frac{N-Z}{2}\right) E$$

Energy difference b/w both configuration

$$E = \frac{1}{2}(N-Z) \times \frac{1}{2}\left(\frac{N-Z}{2}\right) E$$

$$E = \frac{1}{8}(N-Z)^2 E$$

$$E = \frac{1}{8}(A-Z-Z)^2 E \qquad [\because N = A-Z]$$

$$E = \frac{1}{8}(A-2Z)^2 E$$

$$\boxed{E_a = \frac{1}{8}(A-2Z)^2 E = -a_4 \frac{(A-2Z)^2}{A}} \qquad \left[\because E \propto \frac{1}{A}\right]$$

(E) Pairing Energy -

Even-Even Nuclei > Even-odd or odd-even Nuclei > odd-odd Nuclei

$$\propto A^{-3/4}$$

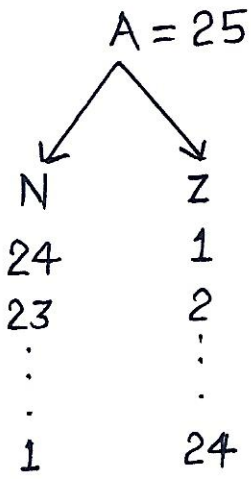
$$\boxed{E_P = (\pm, 0)\, a_5 A^{-3/4}}$$

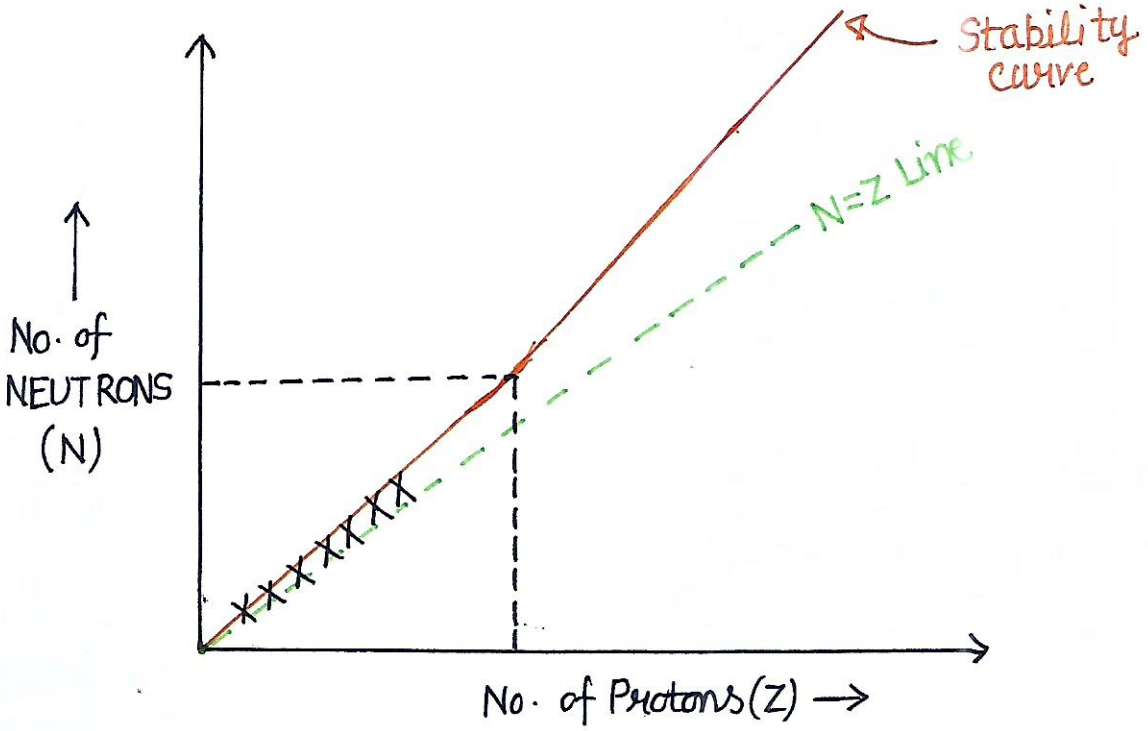
$$E_B = E_V + E_S + E_C + E_A + E_P$$

$$\boxed{E_B = a_1 - a_2\left(\frac{1}{A}\right)^{1/3} - a_3 \frac{Z(Z-1)}{A^{4/3}} - a_4 \frac{(A-2Z)^2}{A}(\pm, 0)\, a_5 A^{-3/4}}$$

Semi Empirical Mass Formula

#NZ Graph (Maximizing Binding Energy)–





Observations →

1. For smaller nuclei (A<20), the NZ graph coincides with the N=Z line, i.e. the number of neutrons is equal to the number of protons.
2. As mass number keeps on increasing, the NZ graph deviates from the N=Z line, i.e. numbers of neutrons start progressively exceeding the number of protons.

e.g. $^{12}_{5}B$ $^{12}_{6}C$

Z=5	Z=6
N=7	N=6
Unstable	Stable

$$E_B = a_1 A - a_2 A^{2/3} - a_3 \frac{Z(Z-1)}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} (\pm, 0) \frac{a_5}{A^{3/5}}$$

derivative of Binding Energy with respect to Z

$$\frac{dE_B}{dZ} = 0$$

$$0 - 0 - \frac{a_3}{A^{1/3}} \frac{d}{dZ} Z(Z-1) - \frac{a_4}{A} \frac{d}{dZ} (A-2Z)^2 + 0 = 0$$

$$-\frac{a_3(2Z-1)}{A^{1/3}} + \frac{4a_4}{A} (A-2Z) = 0$$

$$\frac{4a_4}{A} (A-2Z) - \frac{a_3(2Z-1)}{A^{1/3}} = 0$$

$$\frac{a_3}{a_4} \frac{A}{A^{1/3}} = \frac{4(A-2Z)}{2Z-1}$$

$$\frac{1}{2} \cdot \frac{a_3}{a_4} \cdot A^{2/3} = \frac{A-2Z}{\left(Z-\frac{1}{2}\right)} \approx \left(\frac{A-2Z}{Z}\right)$$

$$\frac{N+Z-2Z}{Z} = \frac{1}{2} \frac{a_3}{a_4} A^{2/3}$$

$$\boxed{\frac{N}{Z} = \frac{1}{2} \frac{a_3}{a_4} A^{2/3} + 1}$$

$a_3 = 0.595$ MeV

$a_4 = 19$ MeV

$$\boxed{\frac{N}{Z} = 1 + 0.0156 A^{2/3}}$$

What is the most stable isobar for A = 25 ?

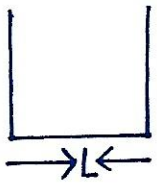
$$\frac{N}{Z} = 1 + \frac{a_3 A^{2/3}}{2a_4}$$

$Z = 12$
$N = 13$

$$\boxed{{}^{25}_{12}Mg}$$

Fermi Energy of Nucleus -

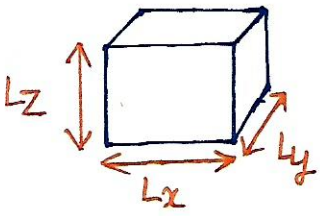
1D



$\psi = A\sin kx + B\cos kx$

$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} = \frac{\hbar^2k^2}{2m}$ (where $K = \frac{n\pi}{L}$)

3D



$$E_n = \frac{\pi^2\hbar^2}{2m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right)$$

$$E_n = \frac{\hbar^2}{2m}\left(K_x^2 + K_y^2 + K_z^2\right)$$

Where $K_x = \frac{n_x\pi}{L_x}$

$K_y = \frac{n_y\pi}{L_y}$

$K_z = \frac{n_z\pi}{L_z}$

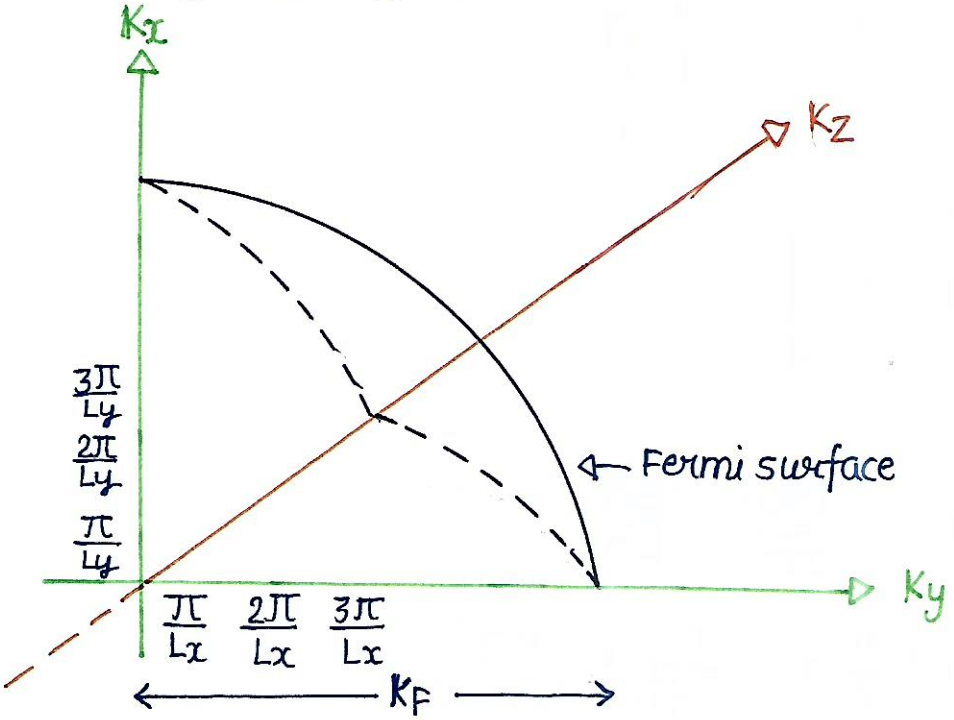
$K^2 = K_x^2 + K_y^2 + K_z^2$

$n_x = 1, 2, 3, \ldots$
$n_y = 1, 2, 3, \ldots$
$n_z = 1, 2, 3, \ldots$

$$K_x = \frac{\pi}{L_x}, \frac{2\pi}{L_x}, \frac{3\pi}{L_x}$$

$$K_y = \frac{\pi}{L_y}, \frac{2\pi}{L_y}, \frac{3\pi}{L_y}$$

$$K_z = \frac{\pi}{L_z}, \frac{2\pi}{L_z}, \frac{3\pi}{L_z}$$



Volume of the unit cell $= \frac{\pi}{L_x} \times \frac{\pi}{L_y} \times \frac{\pi}{L_z}$

$$= \frac{\pi^3}{L_x L_y L_z}$$

$$= \frac{\pi^3}{V}$$ ← Volume of the potential

Each unit cells → 8

→ $8 \times \frac{1}{8}$ Energy eigen state

→ 1

→ N no. of neutrons/protons

Volume of occupied energy state $= \frac{N}{2} \times \frac{\pi^3}{V}$

$$\frac{N}{2} \times \frac{\pi^3}{V} = \frac{1}{8} \times \frac{4}{3}\pi K_F^3$$

$$\frac{K_F^3}{3} = \left(\frac{N}{V}\right)\pi^2$$

$$K_F^3 = \left(\frac{N}{V}\right)3\pi^2 \qquad \left[\because \frac{N}{V} = \rho\right]$$

$$K_F^3 = \rho\, 3\pi^2$$

$$\boxed{K_F = (3\pi^2\rho)^{1/3}}$$

Fermi Energy,

$$E_F = \frac{\hbar^2 K_F^2}{2m} = \frac{\hbar^2}{2m}(3\pi^2\rho)^{2/3}$$

$$\rho = \frac{N}{V} = \frac{0.6A}{\frac{4}{3}\pi R^3} = \frac{0.6A}{\frac{4}{3}\pi R_0^3 A}$$

$$E_F = \frac{\hbar^2}{2m}\left[3\pi\left(\frac{0.6}{\frac{4}{3}\pi R_0^3}\right)^2\right]^{1/3}$$

$$\boxed{E_F \simeq 43\ \text{MeV}}$$

$$\rho = \frac{0.4A}{\frac{4}{3}\pi R^3}$$

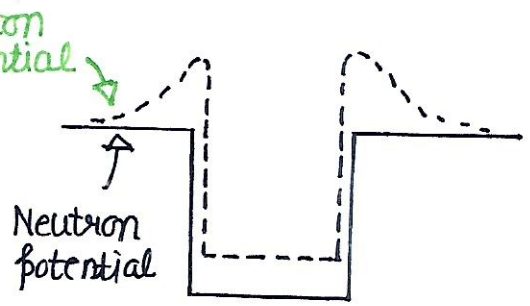
$$\boxed{E_F = 33\ \text{MeV}}$$

Fermi Gas Model of Nucleus—

- Fermi gas is a collection of non interacting fermion inside a potential well.

FERMIONS
→ e^-, p^+, n
→ Half Integer Spin
→ Pauli Exclusion principle

Assumption :

1. The nuclear potential is approximated by a 3D finite square well potential.
2. 

   Since n and p are distinct from each other, potential well will be different
3. Since n and p are fermion, each energy level will be occupied by two of each type of particles of opposite spin.
4. Collision of two identical particles will no result in net charge.

Fermi Energy for $\begin{pmatrix} p \\ 33\ MeV \end{pmatrix}$ $\begin{pmatrix} n \\ 44\ MeV \end{pmatrix}$

Binding Energy/A = 7-8 MeV

Success :

1. Nucleons exist in quantised energy levels.
2. Even-Even > Even-odd > odd-odd
3. It explains β decay.

#SHELL MODEL OF NUCLEUS-

Atomic Structure → similarity

Magic Numbers → 2, 8, 20, 28, 50, 82, 126

↓

No. of p, n, A

↓

Stability

↓

Large no. of stable Isotops, Isotons

$^{4}_{2}He$ $Z=2, N=2$

$^{16}_{8}O$ $Z=8, N=8$

$^{40}_{20}Ca$ $Z=20, N=20$

→ 6 stable Isotopes

$^{100}_{50}Sn$ $Z=50, N=50$

→ 10 stable Isotopes, Isotones

Radioactive decay series → Uranium series
→ Thorium series
→ Actinium series

↓

$Z=82$
$N=126$

↓

Stable

Electric Quadrupole Moment

↓

$Q=0$ → Spherical symmetric

$Q<0$ → Deformed shape

$Q>0$ ↗

NATURE OF STRONG NUCLER FORCE-

- One of the four fundamental force.
- Work at very small distances.
- Within proton, neutron.

Inside Neutrons and protons → Strong Nuclear Force

Outside Neutrons and protons → Residual Nuclear Force

(P) (P)

Columb force $F = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$ $\quad r = 1\,fm$

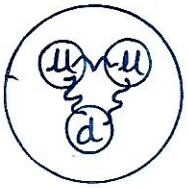
$\boxed{F = 230\,N}$

↓

Yukawa or Meson Theory (Exchange of particles) → π^- Meson

↓

Catch-Catch Example

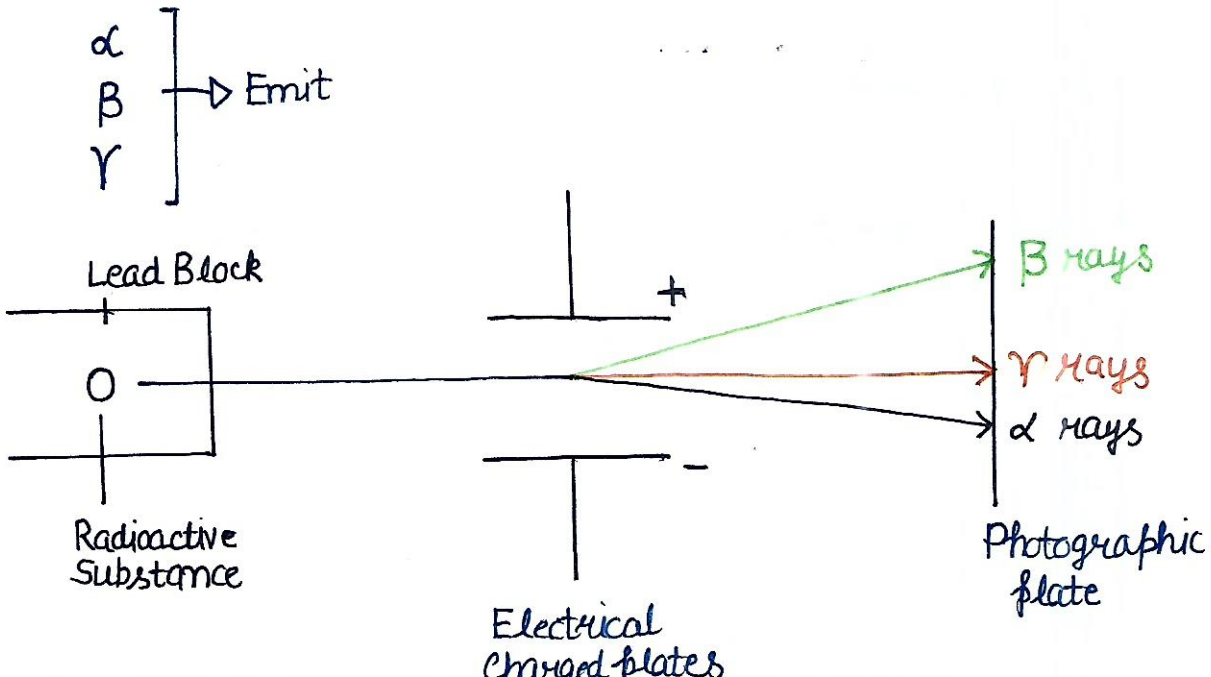


quarkes

↓

Exchange of particles → Gluons

#ALPHA, BETA AND GAMMA DECAY-



α decay :

→ A > 210

→ $^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He + Q$

$$Q = \Delta m c^2$$

$$Q = \left[m\left({}^{A}_{Z}X\right) - m\left({}^{A-4}_{Z-2}Y\right) - m\left({}^{4}_{2}He\right) \right] c^2$$

→ happen to large size nuclei

→ Nuclear Force > Columb Repulsion (Short distances) → A < 210

→ Nuclear Force < Columb Repulsion → A > 210

↓

Quantum Tunneling

↓

Gamow's Theory of α decay

β decay :

β^- ; $n \longrightarrow p$

β^+ ; $p \longrightarrow n$

e^- capture ; $p \longrightarrow n$

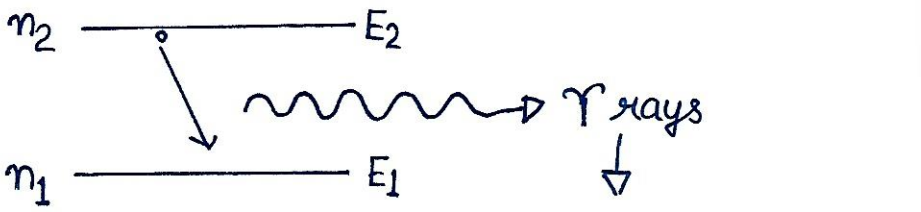
Lesser Energy

↓

Greater Stability

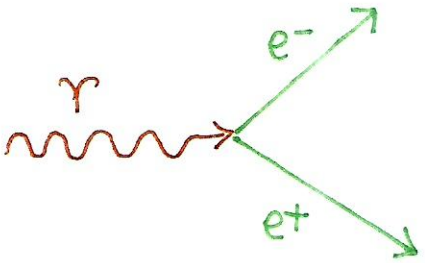
Inverse β decay

γ Decay :



$$^{27}_{13}Al^{*} \longrightarrow {}^{27}_{13}Al + \gamma$$

- Internal conversion
- Internal pair creation (pair production)



#GAMOY THEORY OF ALPHA DECAY-

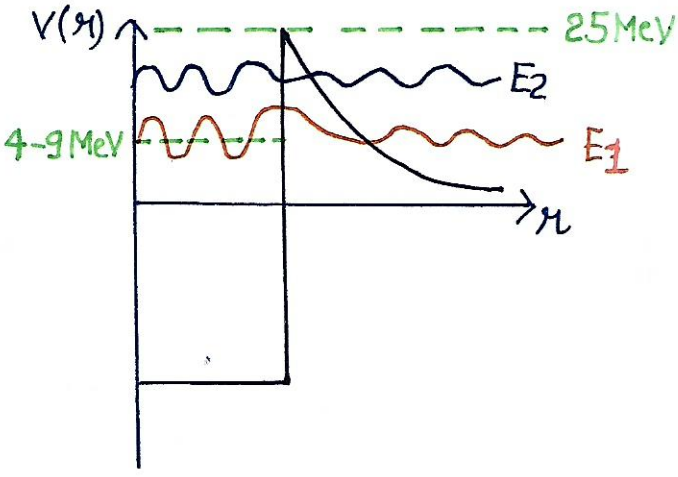
$$^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}X + {}^{4}_{2}He + Q$$

$Q \rightarrow$ 4-9 MeV

$\rightarrow$ Stable

$^{4}_{2}He \rightarrow \alpha$ Particle

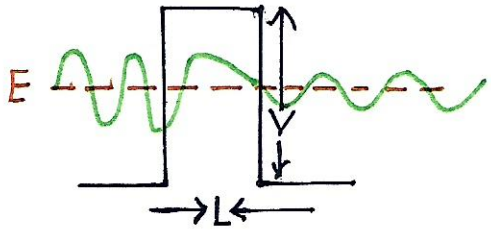
A>210 $\rightarrow$ Unstable



- classically we can't explain.
- Quantum Physics

↓

Quantum Tunneling



$$TP \simeq e^{-2k_2L}$$

$$k_2 = \sqrt{\frac{2m(V-E)}{\hbar}}$$

↓

Borrowed by Gamow

↓

Gamow theory of α decay

↓

Multiple decays

↓

Geiger Nuttal Law ⟶ Two alpha particle

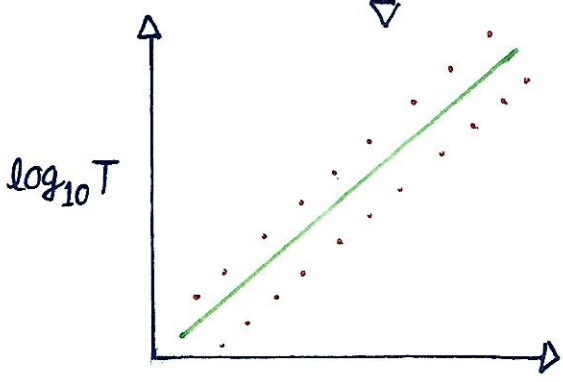
$E_2 > E_1$

$TP(E_2) > TP(E_1)$

$T_{1/2}(E_2) < T_{1/2}(E_1)$

- Short lived α particle have greater E_K.
- Long lived α particle have lesser E_K.

↓



$$\log_{10} T_{1/2} = \frac{Z}{\sqrt{E}} a_1 + a_2$$

• Derivation –

$${}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

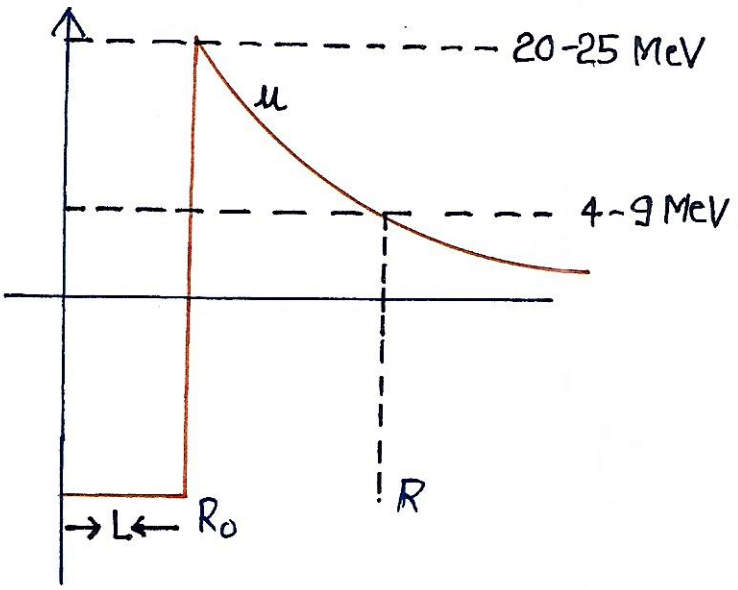
(4–9 MeV)

α particle

• Borrowed Idea

$$T = e^{-2K_2 L}$$

$$K_2 = \sqrt{\frac{2m(U-E)}{\hbar^2}}$$

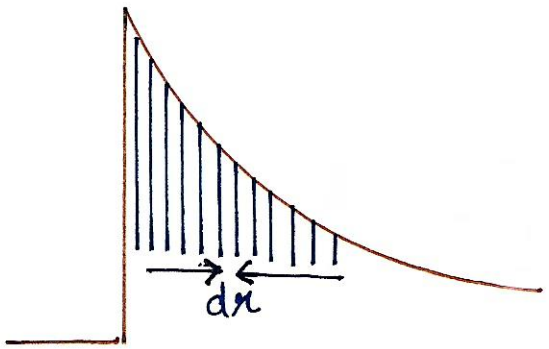


$$U = \frac{(2e)(z)e}{4\pi\epsilon_0 r} = \frac{2ze^2}{4\pi\epsilon_0 r}$$

at $r = R$

$U = E$

$$E = \frac{2ze^2}{4\pi\epsilon_0 R} \Rightarrow R = \frac{2ze^2}{4\pi\epsilon_0 E} \quad \ldots\ldots\ldots (1)$$



$$T = e^{-2K_2 L}$$

$$\ln T = -2K_2 L$$

$$\ln T = -2K_2 dr$$

$$\ln T = \int_{r=R_0}^{R} -2K_2\,dr$$

$$\ln T = -2\int_{R_0}^{R}\left[\frac{2m}{\hbar^2}(U-E)\right]^{1/2} dr$$

$$\ln T = -2\left(\frac{2m}{\hbar^2}\right)^{1/2}\int_{R_0}^{R}(U-E)^{1/2}\,dr$$

$$\ln T = -2\left(\frac{2m}{\hbar^2}\right)^{1/2}\int_{R_0}^{R}\left[\frac{2Ze^2}{4\pi\epsilon_0 r} - \frac{2Ze^2}{4\pi\epsilon_0 R}\right]^{1/2} dr$$

$$\ln T = -2\left(\frac{2m}{\hbar^2}\right)^{1/2}\int\left(\frac{2e^2Z}{4\pi\epsilon_0}\right)^{1/2}\left[\frac{1}{r} - \frac{1}{R}\right]^{1/2} dr$$

$$\ln T = -2\left(\frac{2m}{\hbar^2}\right)^{1/2}\int\left(\frac{2Ze^2}{4\pi\epsilon_0 R}\right)^{1/2}\left(\frac{R}{r}-1\right)^{1/2} dr$$

$$\ln T = -2\left(\frac{2mE}{\hbar^2}\right)^{1/2}\int\left(\frac{R}{r}-1\right)^{1/2} dr \quad \ldots\ldots\ldots (2)$$

$$\Rightarrow I$$

$$I = \int_{R_0}^{R}\left(\frac{R}{r}-1\right)^{1/2} dr \quad \ldots\ldots (3)$$

$$I \Rightarrow \qquad \left[\because \text{Let } \frac{R}{r} = \frac{1}{x}\right]$$

$$\frac{r}{R} = x$$

$$\frac{1}{R}\,dr = dx$$

$$dr = R\,dx \quad \ldots\ldots (4)$$

$$I = R\int\left(\frac{1}{x}-1\right)^{1/2} dx$$

$$I = R\int\left(\frac{1-x}{x}\right)^{1/2} dx$$

$$I = R\int\frac{\sqrt{1-x}}{\sqrt{x}}\, dx$$

Let suppose

$$\sqrt{x} = u$$

$$x = u^2$$

$$\frac{1}{2\sqrt{x}}\, dx = du$$

$$dx = 2u\, du \quad \ldots\ldots (5)$$

$$I = 2R\int\frac{\sqrt{1-u^2}}{x}\, 2x\, du$$

$$I = 2R\int\sqrt{1-u^2}\, du$$

Let suppose

$$u = \sin t \Rightarrow t = \sin^{-1} u \quad \ldots (6)$$

$$du = \cos t\, dt$$

Then

$$I = 2R\int\sqrt{1-u^2}\, du$$

$$I = 2R\int\sqrt{1-\sin^2 t}\,\cos t\, dt$$

$$I = 2R\int\sqrt{\cos^2 t}\,\cos t\, dt$$

$$I = 2R\int\cos^2 t\, dt$$

$$\cos^2 t = 2\cos^2 t - 1 = \frac{1+\cos 2t}{2}$$

$$I = 2R\int\left(\frac{1+\cos 2t}{2}\right) dt$$

$$I = R\left[t + \frac{1}{2}\sin^2 t\right] = R\left[t + \frac{1}{2}\times 2\sin t\cos t\right]$$

$$I = R\left[t + \sin t\cos t\right] = R\left[\sin^{-1}u + \sin(\sin^{-1}u)\cos(\sin^{-1}u)\right]$$

$$I = R\left[\sin^{-1}u + u\cos(\sin^{-1}x)\right]$$

$$I = R\left[\sin^{-1}u + u\sqrt{1-u^2}\right]$$

$$I = R\left[\sin^{-1}u + u\sqrt{1-u^2}\right]$$

$$I = R\left[\sin^{-1}\sqrt{x} + \sqrt{x}\sqrt{1-x}\right]$$

$$x = \frac{r}{R}$$

$$I = R\left[\sin^{-1}\sqrt{\frac{r}{R}} + \sqrt{\frac{r}{R}}\sqrt{1-\frac{r}{R}}\right]$$

Assume:

$\theta = \sin^{-1}u$

$\sin\theta = u$

$\sin^2\theta = u^2$

$1-\cos^2\theta = u^2$

$1-u^2 = \cos^2\theta$

$\sqrt{1-u^2} = \cos\theta$

$\cos(\sin^{-1}x) = \sqrt{1-x^2}$

So, The Transmission probability

$$\ln T = -2\left(\frac{2mE}{\hbar^2}\right)^{1/2} R\left[\sin^{-1}\sqrt{\frac{r}{R}} + \sqrt{\frac{r}{R}}\sqrt{1-\frac{r}{R}}\right]_{R_0}^{R}$$

$$\ln T = -2\left(\frac{2mE}{\hbar^2}\right)^{1/2} R\left[\sin^{-1}(1) + \sqrt{\frac{R}{R}}\sqrt{1-\frac{R}{R}} - \sin^{-1}\sqrt{\frac{R_0}{R}} - \sqrt{\frac{R_0}{R}}\sqrt{1-\frac{R_0}{R}}\right]$$

$$\Downarrow$$

$$0$$

$$\ln T = -2\left(\frac{2mE}{\hbar^2}\right)^{1/2} R\left[\frac{\pi}{2} - \sin^{-1}\sqrt{\frac{R_0}{R}} - \sqrt{\frac{R_0}{R}}\sqrt{1-\frac{R_0}{R}}\right]$$

Assume

$R >>> R_0$

$\frac{R_0}{R} \cdots\cdots 1 \quad \Rightarrow \frac{\pi}{2} - \sin^{-1}\sqrt{\frac{R_0}{R}} \approx \frac{\pi}{2} - \sqrt{\frac{R_0}{R}}$

$\Rightarrow \sqrt{\frac{R_0}{R}}\sqrt{1-\frac{R_0}{R}} \approx \sqrt{\frac{R_0}{R}}\left(1-\frac{R_0}{R}\right)^{1/2} \approx \sqrt{\frac{R_0}{R}}(1+\ldots)$

Then

$$\ln T = -2\left(\frac{2mE}{\hbar^2}\right)^{1/2} R\left[\frac{\pi}{R} - \sqrt{\frac{R_0}{R}} - \sqrt{\frac{R_0}{R}}\right]$$

$$\boxed{\ln T = -2\left(\frac{2mE}{\hbar^2}\right)^{1/2} R\left[\frac{\pi}{2} - 2\sqrt{\frac{R_0}{R}}\right]}$$

$$R = \frac{2Ze^2}{4\pi\epsilon_0 E}$$

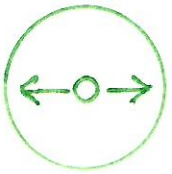
$$\ln T = -2\left(\frac{2mE}{\hbar^2}\right)^{1/2} R \times \frac{\pi}{2} + 4\left(\frac{2mE}{\hbar^2}\right)^{1/2}\left(\frac{R_0}{R}\right)^{1/2}$$

$$\ln T = -2\left(\frac{2mE}{\hbar^2}\right)^{1/2} R\,\frac{\pi}{2} + 4\left(\frac{2mE}{\hbar^2}\right)^{1/2} R_0^{1/2}\, 4\pi\epsilon_0 E\left(\frac{2Ze^2}{4\pi\epsilon_0 E}\right)^{1/2}$$

$$\log T = 4\left(\frac{m}{\pi\epsilon_0}\right)^{1/2}\left(\frac{e}{\hbar}\right) Z^{1/2} R_0^{1/2} - \left(\frac{m}{2}\right)^{1/2}\left(\frac{e^2}{\hbar\epsilon_0}\right) Z E^{-1/2}$$

Then

$$\boxed{\ln T = 1.29\, Z^{1/2} R_0^{1/2} - 1.72\, Z E^{-1/2}}$$



Nucleus

$R_0 \cong 10^{-14}\,m$

$$\nu = \frac{v_0}{2R_0}$$

Radioactive decay constant

$$\lambda = \nu \times T$$

$v_0 = 2 \times 10^{-7}\ m/s$

$$\nu = \frac{v_0}{2R_0}$$

$$\boxed{\nu = 10^{21} s^{-1}}$$

$$\lambda = \nu \times T$$

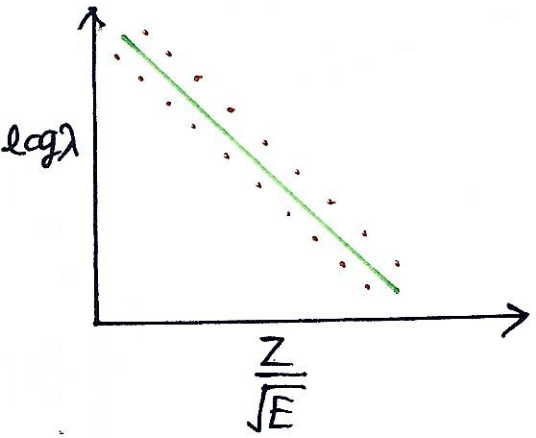
$$\log \lambda = \log \nu + \log T$$

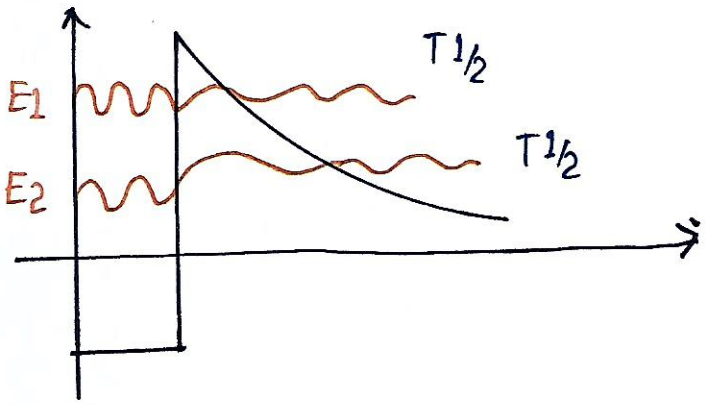
$$\log \lambda = \log\left(\frac{v}{2R_0}\right) + \log T$$

$$\boxed{\log \lambda = \log\left(\frac{v}{2R_0}\right) + 1.27 Z^{1/2} R_0^{1/2} - 1.72 Z E^{-1/2}} \quad \ldots\ldots (7)$$

Geiger Nuttal Law :

$$\log_{10} \lambda = -a_1 \frac{Z}{\sqrt{E}} + a_2 \quad \ldots\ldots (8)$$





$E_1 > E_2$

$T^{(1)}_{1/2} < T^{(2)}_{1/2}$

#Q Value | KE of α Decay (Kinematics)

$$^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He + Q$$

e.g.

$$^{238}_{92}U \longrightarrow {}^{234}_{90}Th + {}^{4}_{2}He + Q$$

(m, v for Th; m_α, v_α for He)

Q value/Disinigation Energy.

$$Q = \left[m\left({}^{A}_{Z}X\right) - m\left({}^{A-2-2}_{Z-2}Y\right) - m\left({}^{4}_{2}He\right) \right] \times c^2$$

Conservation of Linear momentum:

$$mv = m_\alpha v_\alpha \quad \ldots\ldots\ldots (1)$$

Conservation of Energy:

$$Q = \frac{1}{2} m_\alpha v_\alpha^2 + \frac{1}{2} m v^2 \quad \ldots\ldots (2)$$

$$Q = \frac{1}{2} m_\alpha v_\alpha^2 + \frac{1}{2} \frac{m^2 v_\alpha^2}{m}$$

$$Q = \frac{1}{2} m_\alpha v_\alpha^2 \left[1 + \frac{m_\alpha}{m}\right]$$

$$Q = KE_\alpha \left[\frac{m + m_\alpha}{m}\right]$$

$$\boxed{KE_\alpha = \left(\frac{m}{m + m_\alpha}\right) Q}$$

$$KE_\alpha = \left(\frac{A-4}{A-4+4}\right) Q$$

$$\boxed{KE_\alpha = \left(\frac{A-4}{A}\right) Q}$$

#BETA DECAY & NEUTRINO HYPOTHESIS:

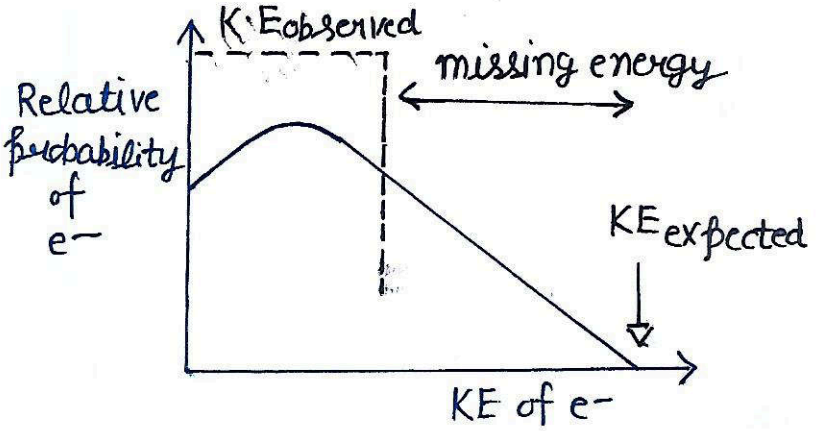
β^- decay:

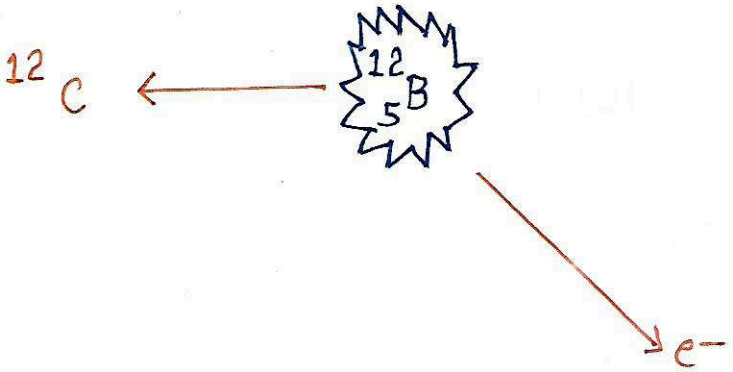
$$^{12}_{5}B \longrightarrow {}^{12}_{6}C + e^-$$

$$^{1}_{0}n \longrightarrow {}^{1}_{1}P + {}_{1}e^- \quad \text{(Charge conserved)}$$

VOILATION OF CONSERVATION LAWS

1. Energy conservation





2. Angular momentum conservation

$$n \longrightarrow p + e^-$$

$$s \quad \frac{1}{2} \qquad \frac{1}{2} \quad \frac{1}{2}$$

⇓

Wolfgang Pauli → Fermi

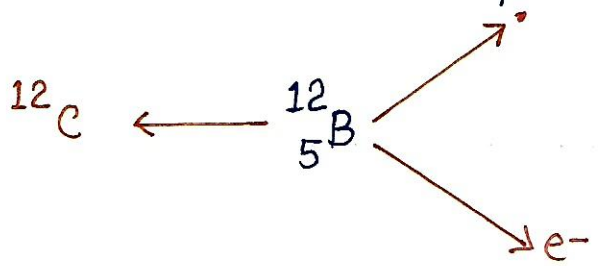
↓

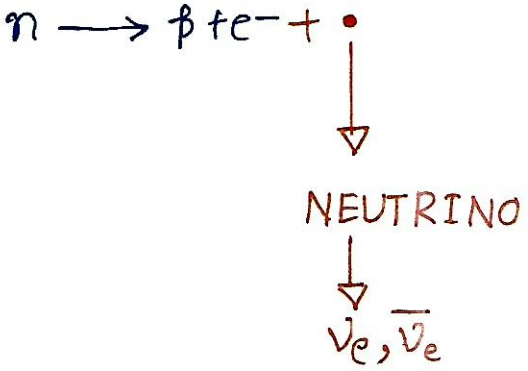
1931

→ Uncharged in nature

→ Tiny, small rest mass

Missing Energy = $KE_{expected} - KE_{observed}$





β^+ decay

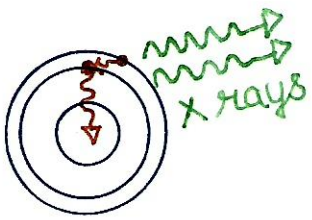
$$^{23}_{12}Mg \longrightarrow {}^{23}_{11}Na + e^+ + \nu_e$$

$$^{1}_{1}p \longrightarrow {}^{1}_{0}n + e^+ + \nu_e$$

e^- capture

$$^{81}_{36}Kr + e^- \longrightarrow {}^{81}_{35}Be + \nu_e$$

$$^{1}_{1}p + e^- \longrightarrow {}^{1}_{0}n + \nu_e$$



Inverse β decays -

$$^{1}_{0}n + \nu_e \longrightarrow {}^{1}_{1}P + e^-$$

$$^{1}_{1}P + \bar{\nu}_e \longrightarrow {}^{1}_{0}n + e^+$$

NOTES!

ABHISHEK BOHARE