Physics [Module 2]

Notes

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Module-2

Physics

ABHISHEK BOHARE

- High Energy Physics
- Condensed Matter Ph
- Nuclear Physics
- Advanced Quantum Physics
- LASER Physics-A
- Electronics-A
- Particle Physics
- Computational Physics
- Presentations
- Special Theory of Relativity
- Environmental Physics

Nuclear Physics

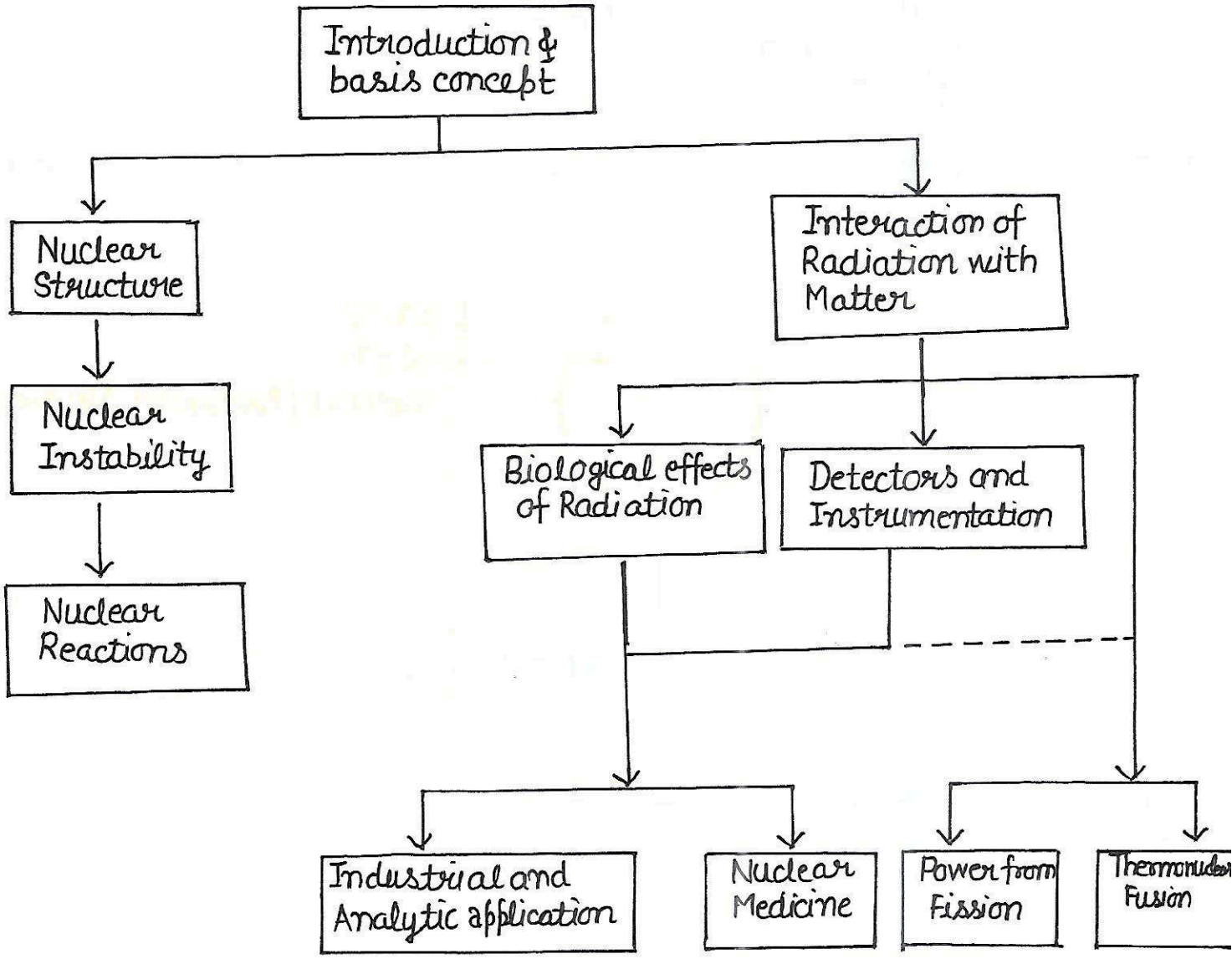
- General Introduction
- UNIT 1 General Properties
- UNIT 2 Nuclear Force/Detector
- UNIT 3 Radioactive Decay
- UNIT 4 Nuclear Reactions

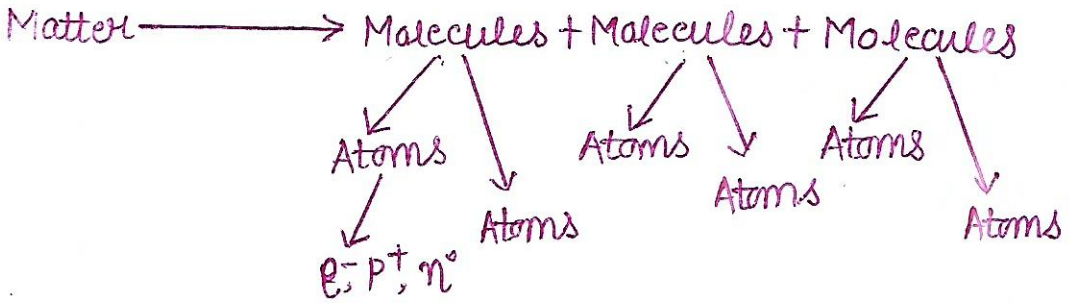
- General properties and Models
- Nuclear Forces and detectors
- Radioactive Decay
- Nuclear Reactions

GENERAL – INTRODUCTION

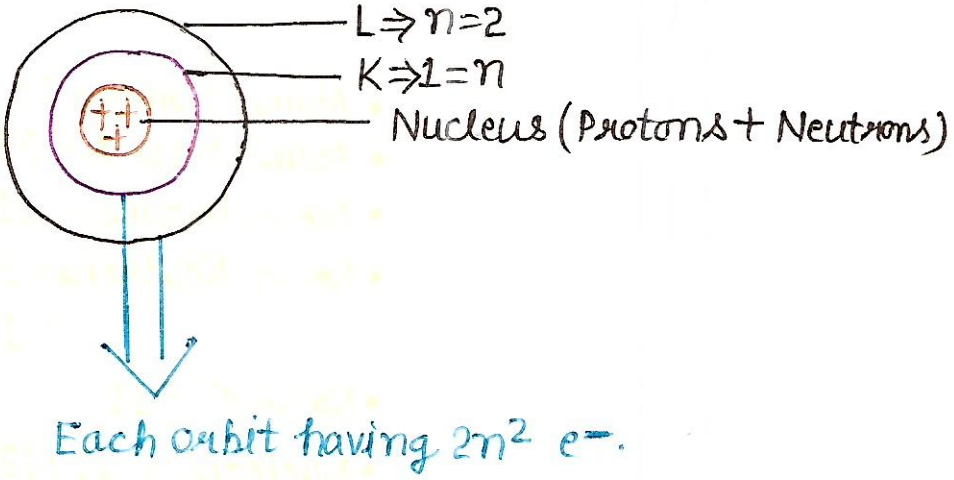
1. General Properties & Models

Flow diagram-

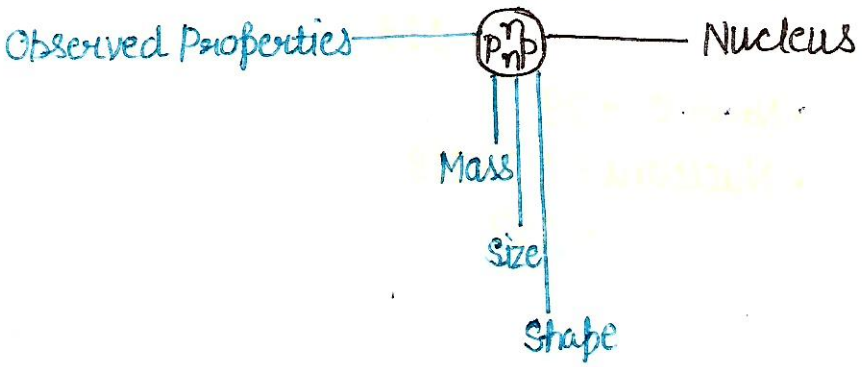




An Atom consists of a small, massive core called the nucleus surramded by orbitting e⁻.



Nuclear Physics, generally explain all aspects of the nucleus, its structure, its behaviour under varias conditions and its effect on nature and mankind.



#Mass, Charge and Constituents of the Nucleus-

- The nucleus is made up of neutrons and protons, two particles which are about 1840 times more massive than electrons. They are spoken of collectively as nucleons.

- No. of Protons = Atomic number Z
- The total number of nucleons A is a integer closest to its atomic weight.

Hence No. of Neutrons = A - Z

$_{11}Na^{23}$

- Atomic Number Z = 11
- Atomic Mass A = 23
- No. of Protons = 11
- No. of Neutrons = 23 - 11
  = 12
- No. of e^- = 11
- Nucleons = 11 + 12 = 23

$_{79}Au^{197}$

- Atomic Number Z = 79
- Atomic Mass A = 197
- No. of Protons = 79
- No. of Neutrons = 197 - 79
  = 118
- No. of e^- = 79
- Nucleons = 79 + 118
  = 197

- Mass of the nucleus is very nearly equal to the mass of the atom; in kg it is the atomic weight devided by Avogadro's number $6.03 \times 10^{23+3}$
  $\rightarrow 6.03 \times 10^{26}$

- The nucleus was first discovered in 1911 in experiments conducted by Lord Rutherford and his associates on scattering of α particles of atoms. He found that the scattering pattern could be explained if atoms consist of a small, massive, positively charged core surrounding by orbiting electrons. While most of his results on the basis of an infinitely small nucleous, derivations indicates that the nuclear size is of order 10^{-14}m. Since this is 10000 times smaller than the diameter of atoms, it is small enough to be negligible in practically all atomic problems.

Unit 1 to 4 Topicwise Questions

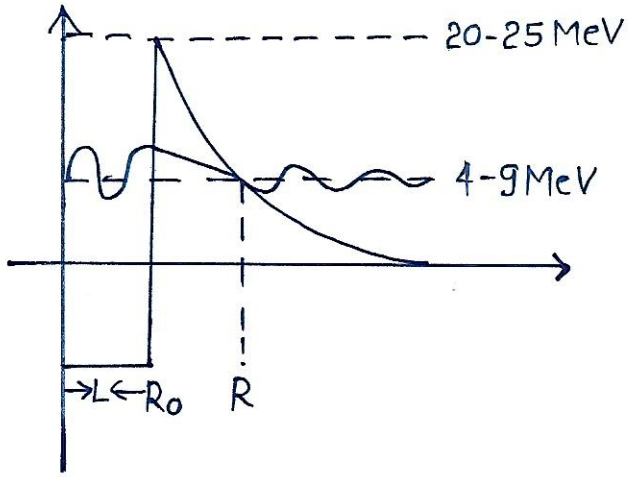
#Gamow Theory of Alpha decay and Geiger Nuttal Law-

α decay is a spontaneous decay process in which a large sized nucleus ($A > 210$) undergo decay process which leads to emission of α-particle.

$$_{Z}X^{A} \longrightarrow {}_{Z-2}Y^{A-4} + {}_{2}He^{4} + \text{Energy}$$

$A > 210$; α-particle 4-9 MeV

α-particle is basically a Helium nucleus having charge $+2e$ where $e = 1.6 \times 10^{-19} C$. Before emission, an α-particle can be assumed to exist inside the nucleus.



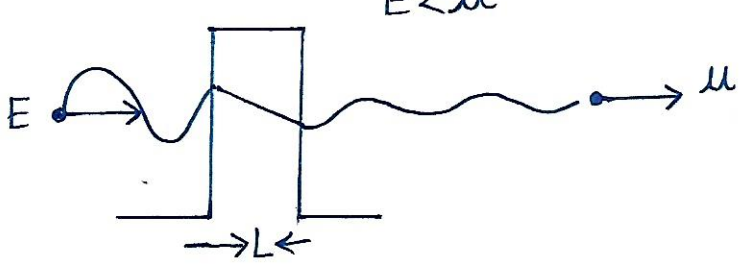
$$\mu = \frac{(2e)(ze)}{4\pi\epsilon_0 r} = \frac{2ze^2}{4\pi\epsilon_0 r}$$

At $r = R$

$$\mu = E$$

$$E = \frac{2ze^2}{4\pi\epsilon_0 R} \Rightarrow R = \frac{2ze^2}{4\pi\epsilon_0 E} \quad \ldots\ldots\ldots (1)$$

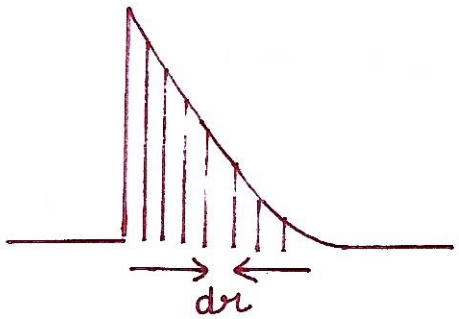
$E < \mu$



Quantum Tunneling

$$T = e^{-2K_2 L} \quad \ldots\ldots\ldots (2)$$

$$K_2 = \sqrt{\frac{2m(U-E)}{\hbar^2}} \quad \ldots\ldots\ldots (3)$$



$$T = e^{-2K_2 L}$$

$$\ln T = -2K_2 L$$

$$\ln T = -2K_2\, dr$$

$$\ln T = \int_{R_0}^{R} -2K_2\, dr$$

$$\ln T = -2\int_{R_0}^{R} K_2\, dr$$

$$\ln T = -2\int_{R_0}^{R} \left[\frac{2m[U-E]}{\hbar^2}\right]^{1/2} dr \quad [\text{From eqn (3)}]$$

$$\ln T = -2\int_{R_0}^{R} \left(\frac{2m}{\hbar^2}\right)^{1/2} (U-E)^{1/2}\, dr.$$

$$\ln T = -2\left(\frac{2m}{\hbar^2}\right)^{1/2} \int_{R_0}^{R} (U-E)^{1/2}\, dr$$

$$\ln T = -2\left(\frac{2m}{\hbar^2}\right)^{1/2} \int \left[\frac{2Ze^2}{4\pi\epsilon_0 r} - \frac{2Ze^2}{4\pi\epsilon_0 R}\right]^{1/2} dr$$

$$\ln T = -2\left(\frac{2m}{\hbar^2}\right)^{1/2} \int \left(\frac{2Ze^2}{4\pi\epsilon_0}\right)^{1/2} \left(\frac{1}{r} - \frac{1}{R}\right)^{1/2} dr \cdot \frac{R}{R}$$

$$\ln T = -2\left(\frac{2m}{\hbar^2}\right)^{1/2} E^{1/2} \int \left(\frac{R}{r} - 1\right)^{1/2} dr$$

$$\ln T = -2\left(\frac{2mE}{\hbar^2}\right)^{1/2} \int_{R_0}^{R} \left(\frac{R}{r} - 1\right)^{1/2} dr \quad \ldots\ldots\ldots (4)$$

So

$$I = \int_{R_0}^{R} \left(\frac{R}{r} - 1\right)^{1/2} dr \quad \ldots\ldots\ldots (5)$$

Let suppose

$$\frac{R}{r} = \frac{1}{x}$$

$$\frac{1}{R} dr = dx$$

$$dr = R\,dx \quad \ldots\ldots\ldots (6)$$

$$I = R \int \left(\frac{1}{x} - 1\right)^{1/2} dx$$

$$I = R \int \frac{\sqrt{1-x}}{\sqrt{x}}\, dx$$

Let suppose

$$\sqrt{x} = u$$

$x = u^2$

$\frac{1}{2\sqrt{x}} dx = du$

$\frac{1}{2u} dx = du$

$dx = 2u\,du$

$I = R \int \frac{\sqrt{1-u^2}}{u} 2u\,du$

$I = 2R \int \sqrt{1-u^2}\, du$

$u = \sin(t)$

$t = \sin^{-1} u$

$du = \cos t\, dt$

$I = 2R \int \sqrt{1-\sin^2 t}\, dt \cos t$

$I = 2R \int \sqrt{\cos^2 t}\, \cos t\, dt$

$I = 2R \int \cos^2 t\, dt$

$I = 2R \int \left(\frac{1+\cos 2t}{2} \right) dt$

$I = R \left[\int dt + \int \cos 2t\, dt \right]$

$I = R \left[t + \frac{1}{2} \sin 2t \right]$

$$I = R\left[t + \frac{1}{2} \times 2 \sin t \cos t\right]$$

$$I = R\left[t + \sin t \cos t\right]$$

$$I = R\left[\sin^{-1} u + \sin(\sin^{-1} u)\cos(\sin^{-1} u)\right]$$

$$I = R\left[\sin^{-1} u + u\sqrt{1-x^2}\right]$$

$$I = R\left[\sin^{-1}\sqrt{x} + \sqrt{x}\sqrt{1-x^2}\right]$$

$$I = R\left[\sin^{-1}\sqrt{\frac{r}{R}} + \sqrt{\frac{r}{R}}\sqrt{1-\frac{r}{R}}\right] \quad \ldots\ldots\ldots (7)$$

Transition Probability

$$\ln T = -2\left(\frac{2mE}{\hbar^2}\right)^{1/2} R\left[\sin^{-1}\sqrt{\frac{r}{R}} + \sqrt{\frac{r}{R}}\sqrt{1-\frac{r}{R}}\right]_{R_0}^{R}$$

$$\ln T = -2\left(\frac{2mE}{\hbar^2}\right)^{1/2} R\left[\frac{\pi}{2} - \sin^{-1}\sqrt{\frac{R_0}{R}} - \sqrt{\frac{R_0}{R}}\sqrt{1-\frac{R_0}{R}}\right]$$

Assume

$$R >>> R_0$$

$$\frac{R_0}{R} <<< 1$$

$$\ln T = -2\left(\frac{2mE}{\hbar^2}\right)^{1/2} R\left[\frac{\pi}{2} - \sqrt{\frac{R_0}{R}} - \sqrt{\frac{R_0}{R}}\right]$$

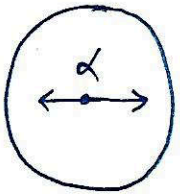
$$\boxed{\ln T = -2\left(\frac{2mE}{\hbar^2}\right)^{1/2}\left[\frac{\pi}{2} - 2\sqrt{\frac{R_0}{R}}\right]}$$

$$R = \frac{2ze^2}{4\pi\epsilon_0 E}$$

$$\ln T = -2\left(\frac{2mE}{\hbar^2}\right)^{1/2} \times \frac{\pi}{2} \times R + 4\left(\frac{2mE}{\hbar^2}\right)^{1/2}\left(\frac{R_0}{R}\right)^{1/2} \times R$$

$$\ln T = 4\left(\frac{m}{\pi\epsilon_0}\right)^{1/2}\left(\frac{e}{\hbar}\right) z^{1/2} R_0^{1/2} - \left(\frac{m}{2}\right)^{1/2}\left(\frac{e^2}{\hbar\epsilon_0}\right) zE^{1/2}$$

$$\boxed{\ln T = 1.29 z^{1/2} R_0^{1/2} - 1.72 zE^{1/2}}$$



$$v = \frac{v_0}{2R_0}$$

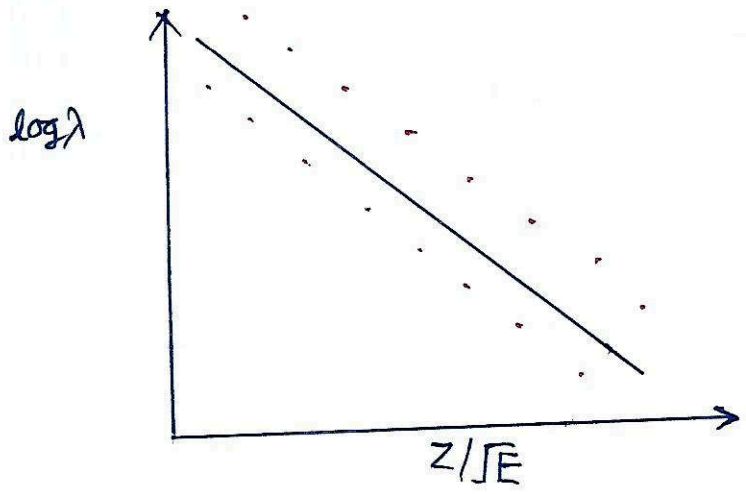
$$\lambda = v \times T$$

$$\log \lambda = \log\left(\frac{v_0}{2R_0}\right) + \log T$$

$$\boxed{\log \lambda = \log\left(\frac{v_0}{2R_0}\right) + 1.29 z^{1/2} R_0^{1/2} - 1.72 zE^{1/2}}$$

Geiger Nutter Law

$$\log \lambda = -a_1 \frac{z}{\sqrt{E}} + a_2$$

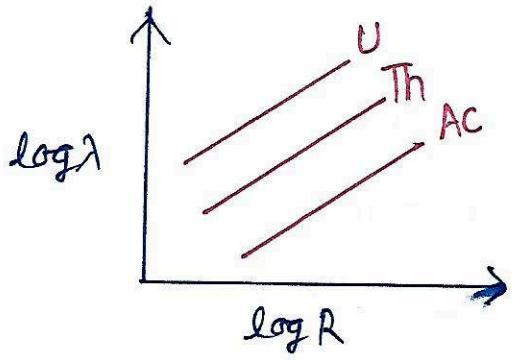


$E_1 > E_2$

$T_{1/2}^{(1)} < T_{1/2}^{(2)}$

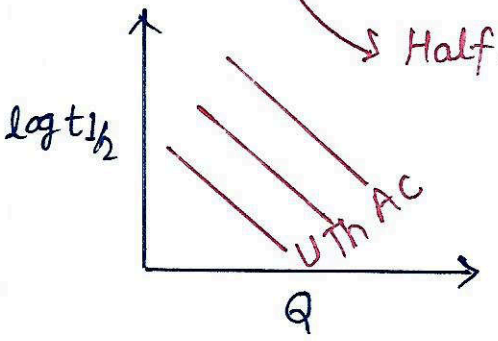
Short lived α particles have greater Kinetic Energy and long lived α particles have less Kinetic Energy. This is experimentally proven.

$$\log\lambda = a + b\log R$$

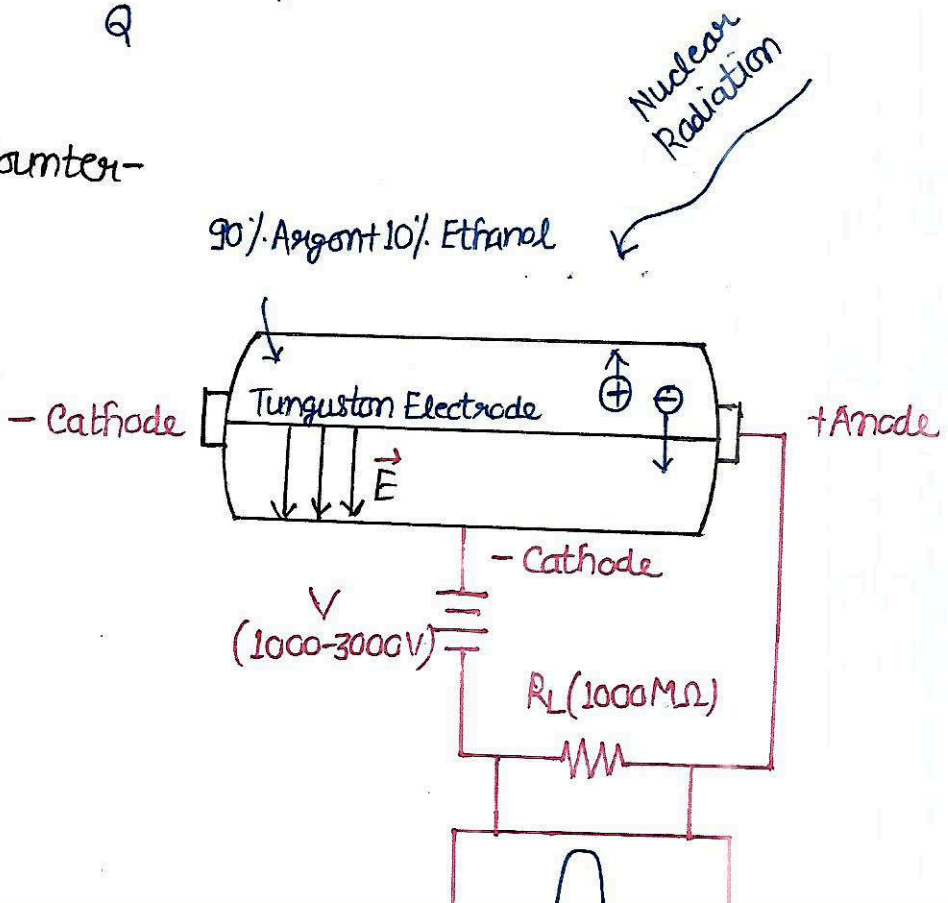


$$\log t_{1/2} = a + b\frac{Z}{\sqrt{Q_1}}$$

Half life

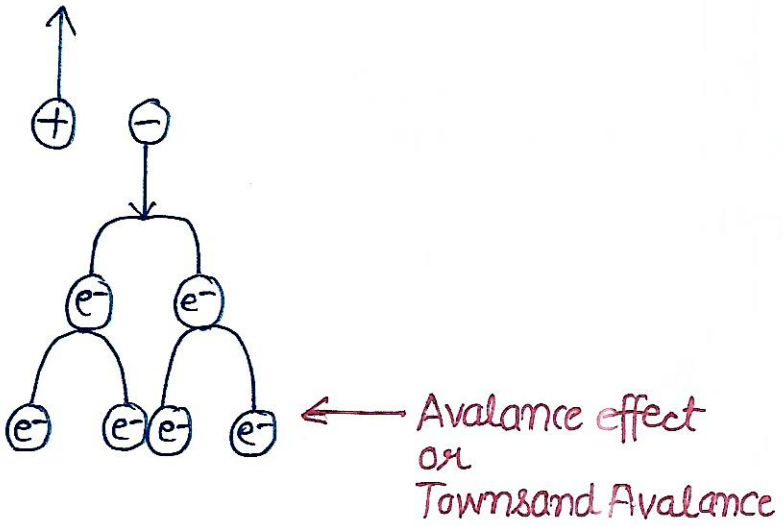


GM Counter-



→ Nuclear detector

→ α, β, γ, nucleon detection



→ dead time

- Inactive
- Inefficient

100μs to 200μs

→ Quentching

Removal of Sheath of +ve ions are called quentching

- Chemical Quentching
- Self Quentching

→ Recovery time-

Ready to produce new pulse

→ Rosalving time = Recovery time + Dead time

Liquid drop Model–

→ Comparison of Liquid drop and nucleus.

→ Nucleus is a quantum particle of n^0, p^+, e^-.

→ Liquid drop is collection of molecules.

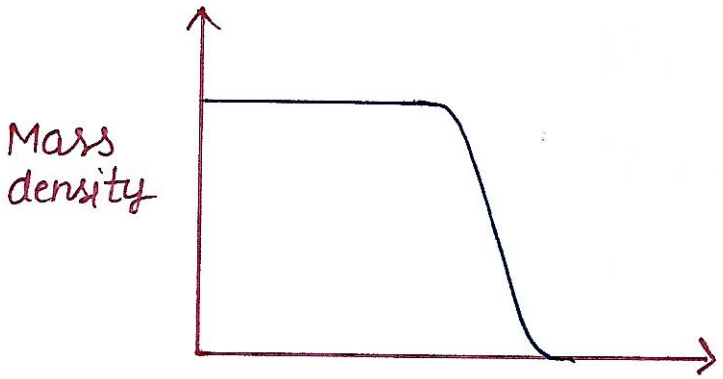
→ Both are similar.

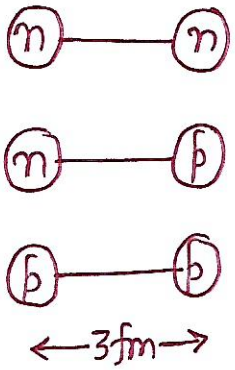
→ Both contains large no. of particles.

→ Both are Homogenous

→ Both are incompressible.

→ Nuclear mass densities are same.





Strong nuclear force.

→ Surface tension.

→ Heat or Vaporization of Liquid = Binding Energy of Nucleus.

→ Fission and Fusion.

1. Volume Energy -

$$\frac{BE}{\text{Nucleons}} = \frac{1}{2}u$$

$$\boxed{E_V = a_1 A}$$

2. Surface Energy -

Surface Area $= 4\pi R^2$

$$= 4\pi \left[R_0 A^{1/3} \right]^2$$

$$= 4\pi R_0^2 A^{2/3}$$

Surface Energy $\propto A^{2/3}$

$$\boxed{E_S = -a_2 A^{2/3}}$$

3. Columb Energy -

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{e}{r}$$

$$V = \frac{Z(Z-1)e^2}{8\pi\epsilon_0} \cdot \frac{3}{2R}$$

$$\boxed{E_C = -a_3 \frac{Z(Z-1)}{A^{1/3}}}$$

4. Assymmetric Energy -

Assymmetric Term ——Reflects——→ Stability of Nucleus

$$E_A \propto \frac{(N-Z)^2}{A}$$

$$\boxed{E_A = -a_4 \frac{(A-2z)^2}{A}}$$

5. Pairing Energy -

$$\boxed{E_p = a_5 A^{-3/4}}$$

Then

$$\boxed{M = ZM_p + NM_n - a_1 A + a_2 A^{2/3} + a_3 \frac{Z(Z-1)}{A^{-1/3}} + a_4 \frac{(A-2z)^2}{A} - a_5 A^{2/3}}$$

#Achievements-

→ Predict Atomic mass.
→ Predict Binding Energy.
→ Emission of α, β particles.
→ Explain Fission correctly.

#Failure-

→ Fail to explain magic no.

Radioactive Equilibrium–

The disintigration of radioactive element leads to the formation of a daughter element. If daughter element is radioactive, it will also undergo to the disintigration to produce another daughter element. The process goes on till the stable non-radioactive element is formed.

$$A \xrightarrow[\text{of B}]{\text{Rate of formation}} B \xrightarrow[\text{of B}]{\text{Rate of formation}} C$$

$$\underset{N_A}{A} \longrightarrow \underset{N_B}{B} \longrightarrow \underset{N_C}{C}$$

$$-\frac{dN_A}{dt} = -\frac{dN_B}{dt} = -\frac{dN_B}{dt}$$

But $-\frac{dN}{dt} = \lambda N$

$$\lambda_A N_A = \lambda_B N_B$$

$$\frac{\lambda_A}{\lambda_B} = \frac{N_B}{N_A}$$

$$\frac{N_A}{N_B} = \frac{\lambda_B}{\lambda_A}$$

$$\lambda = \frac{0.693}{t_{1/2}}$$

$$\boxed{\frac{N_A}{N_B} = \frac{\lambda_B}{\lambda_A} = \frac{t_{1/2}\,A}{t_{1/2}\,B}}$$

This equation is known as radioactive equilibrium eqn.

• Application-

1. If we know the amount of two radioactive elements and if the half life of one is known, we can find out the other.
2. If we know the amount of two radioactive elements and if we know decay constant of one, we can find out the other.

Equilibrium

Secular Equilibrium

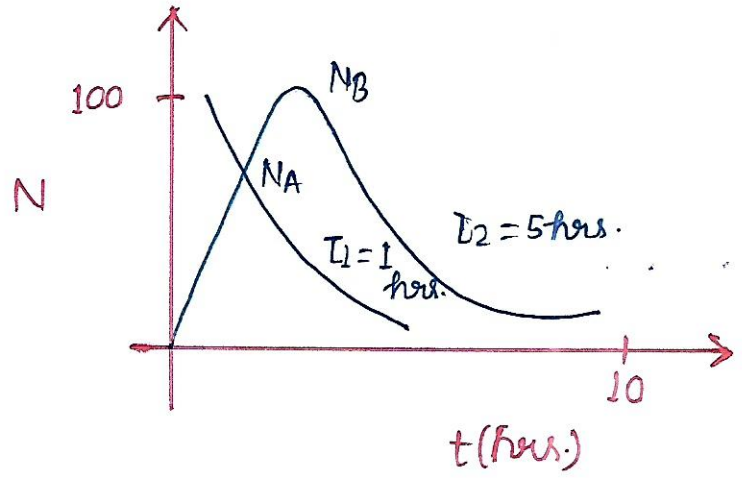
Transient Equilibrium

1. Secular Equilibrium-

In this type, the half of parents element is many time greater than that of the daughter element by the factor of 10^4 or more.

$$^{238}U \xrightarrow[4.5\times10^4 \text{ years}]{\alpha} {}^{234}Th \xrightarrow[24.1 \text{ days}]{\beta^-} Pa$$

$$N_B = \frac{\lambda_A}{\lambda_B} N_{10}\{1-e^{-\lambda_B t}\}$$



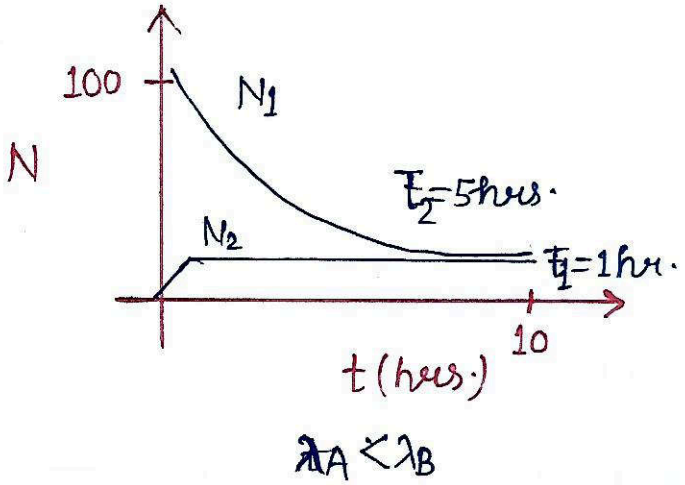
$$\lambda_A >>> \lambda_B$$

2. Transient Equilibrium-

$$N_B = \frac{\lambda_A}{\lambda_B - \lambda_A} N_0 e^{-\lambda_A t}$$

In this type, the half life of the parent and daughter element differ by a small factor.

$$^{234}Th \xrightarrow[24.1\ days]{\beta^-} Pa \xrightarrow[1.18\ min]{\beta^-} Series$$



$\lambda_A < \lambda_B$

QUANTUM MECHANICS

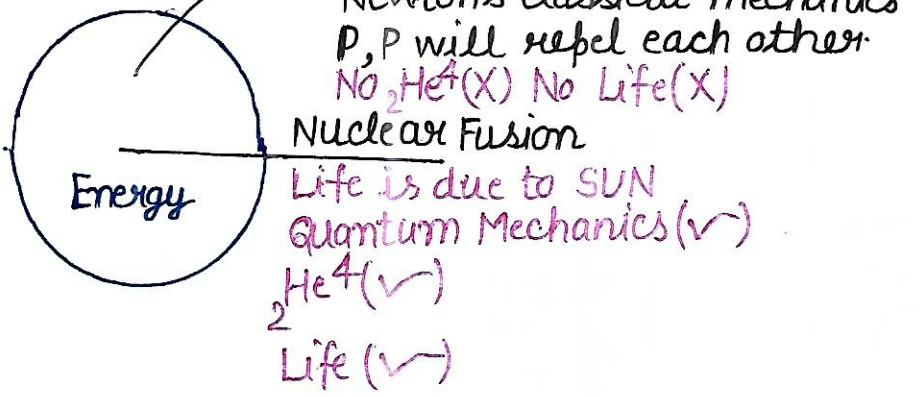
- Basic Principle of Quantum Mechanics
- UNIT-1 SCATTERING THEORY
- UNIT-2 IDENTICLE PARTICLE
- UNIT-3 RELATIVISTIC QUATUM MECHANICS
- UNIT-4 QUAINTIZATION OF WAVE FIELD

Basic Principle of Quantum Mechanics

- By Prof. H.C. Verma

#0 / Introduction

- → Add Imagination
- → Will Answer every question
- → Microscopic details of Universe
- → Spectrum of Hydrogen Atom
- → Schrodienger Contribution



#1 / Arrival of Quantum Ideas-

- Identical Experiments need not give identical results-

Motion of Planets
Motion of Cricket Ball
→ Newton's Law

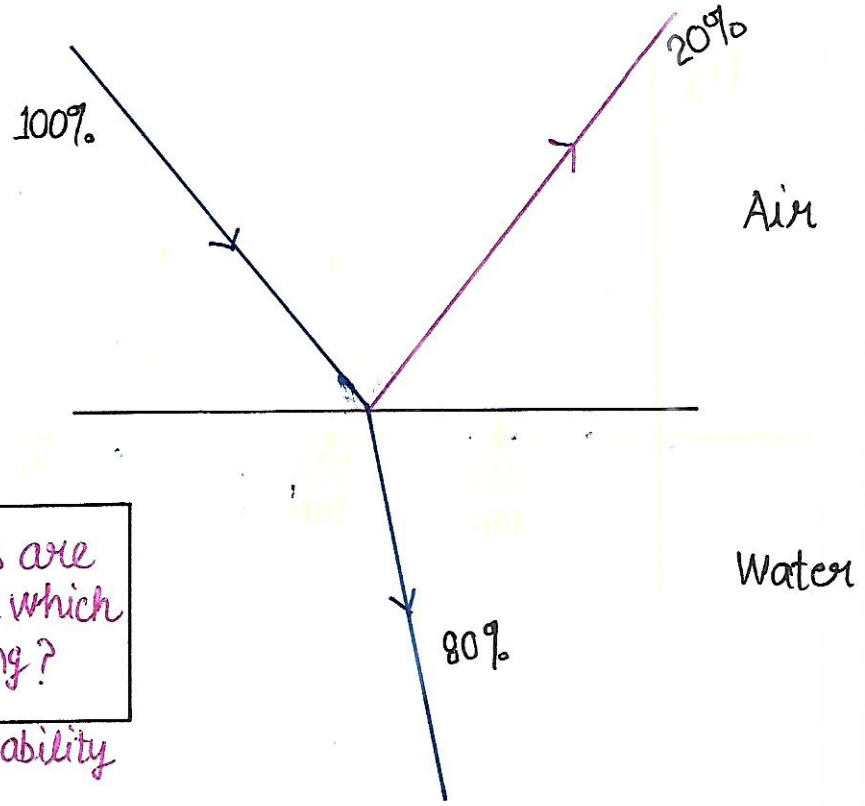
Classical Physics

Small Particles
(P, n, e⁻, Photons)
→ Behaviour of small particles
Quantum Mechanics

High speed [Speed of light]
→ Relativity
→ Space
→ Time

Failure of Newton's Law [Speed of light (Particles)]

→ Truely Identical experiments don't give identical results.



Which Photons are reflecting and which are refracting?

depends on Probability

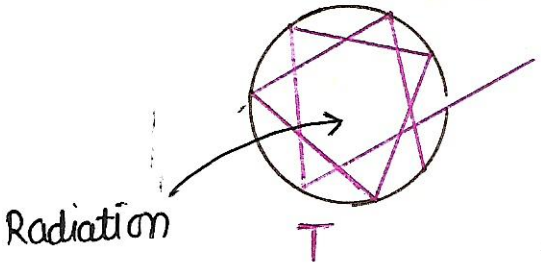
• Basics of Blackbody Radiation-

→ Quantum Mechanics tells us the probability of any problem.
→ Light is also Radiation

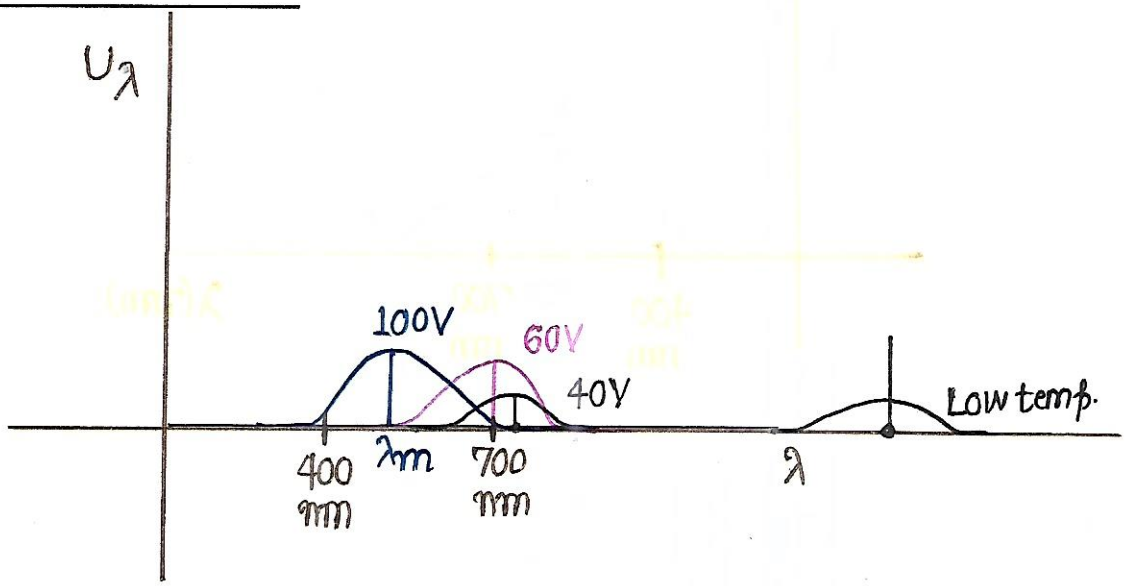
Light — Waves ~~~→
Light — Particle (Photons like hν)

Blackbody (कृष्णिका) absorbs all the radiations.

If the temperature of Blackbody is not rising while absorbing the radiation, it means the blackbody is also emitting the radiation.



Wein's displacement Law-



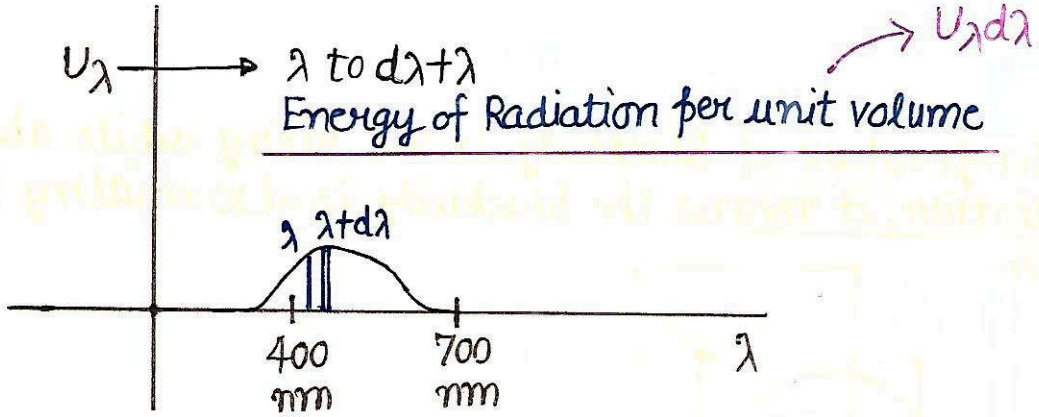
$\lambda_m T$ = Constant

$\lambda_m T = 2.898 \times 10^{-3}$ mK

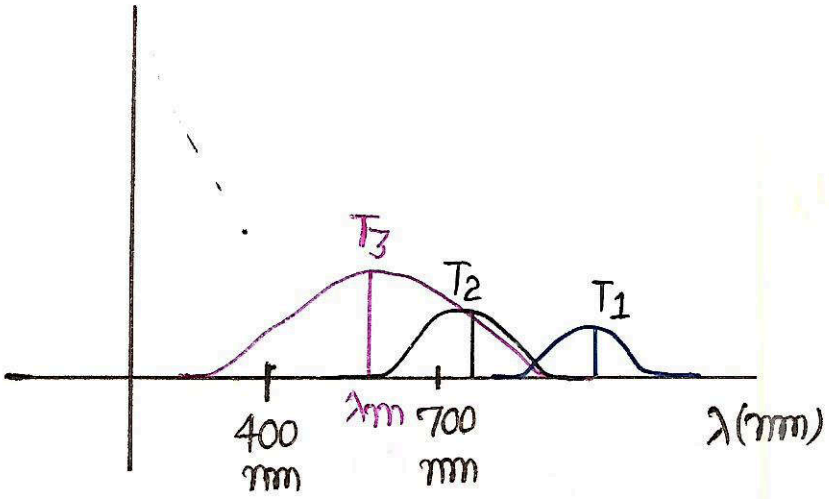
#Stefan's Law-

For blackbody $U = \sigma T^4$

$U = \sigma T^4$
$\lambda_m T$ = Constant } Classical Physics explain this.



#Plank Radiation Law-



$\lambda_m T = 2.898 \times 10^{-3}$ mK
$U \propto T^4$
$U = \sigma T^4$

$U_\lambda d\lambda$ = Energy per unit Volume in wavelength range λ to $\lambda + d\lambda$

$$\lambda = \frac{c}{\nu}$$

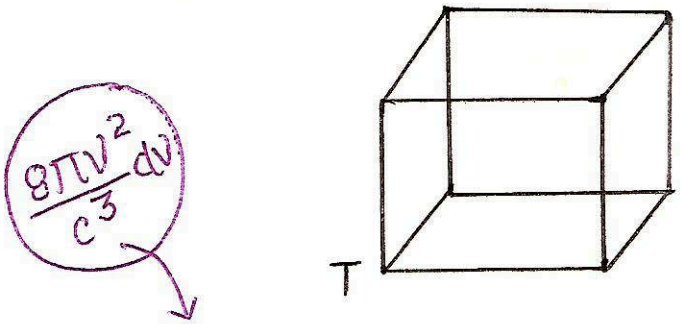
$$d\lambda = \frac{c}{\nu^2} d\nu$$

Now in frequency range ν to $\nu + d\nu$

$U_\nu d\nu = ?$

$U_\nu d\nu = a\nu^3 e^{-b\nu/T}$ (Wein Radiation Law)

- **Rayleigh-Jeans Law-**



→ Radiations are waves.
→ Energy b/w ν to $\nu + d\nu$

Oscillations
Standing waves

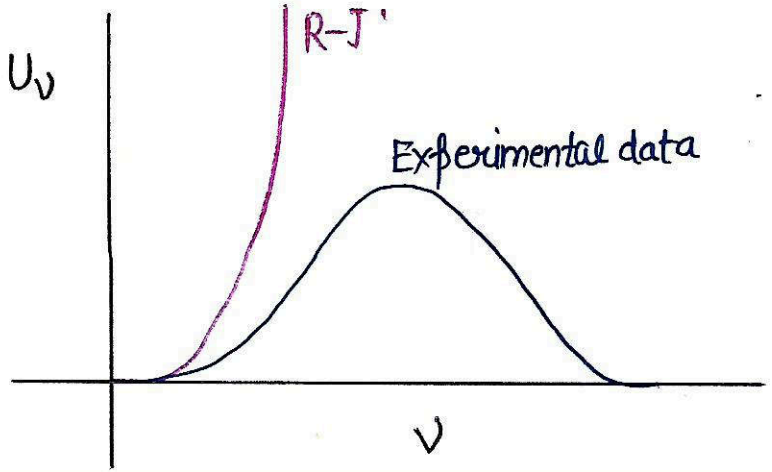
$$\langle E \rangle \Rightarrow \frac{\int_0^\infty E e^{-E/KT} dE}{\int_0^\infty e^{-E/KT} dE}$$

$$\langle E \rangle = KT$$

$$U_\nu d\nu = \frac{8\pi\nu^2}{c^3} KT \, d\nu$$

$$\boxed{U_\nu = \frac{8\pi\nu^2}{c^2} KT}$$

→ Ultraviolet catastrophe



⇒ Energy emitted or absorbed in radiation at frequency ν is $n\hbar\nu$.

where $n = 1, 2, 3, 4, 5, \ldots$

$\hbar$ = Universal constant.

$$\langle E \rangle = n\hbar\nu\, e^{-n\hbar\nu/KT} / e^{-n\hbar\nu/KT}$$

$$\langle E \rangle = \frac{\sum_{n=0}^{\infty} n\hbar\nu\, e^{-n\hbar\nu/KT}}{\sum_{n=0}^{\infty} e^{-n\hbar\nu/KT}}$$

$$\boxed{\langle E \rangle = \frac{\hbar\nu}{e^{\hbar\nu/KT} - 1}}$$

$$U_\nu d\nu = \frac{8\pi\nu^2}{c^3} \frac{\hbar\nu}{e^{\hbar\nu/KT} - 1} d\nu$$

$$\boxed{U_\nu = \frac{8\pi\nu^2}{c^3} \frac{\hbar\nu}{e^{\hbar\nu/KT} - 1}}$$ Plank Radiation Law

#2/Wave or Particle?

- Light is wave and also Particle-

$$U_\nu = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{h\nu/KT}-1}$$

U_ν = Energy contained in frequency range ν to $\nu+d\nu$

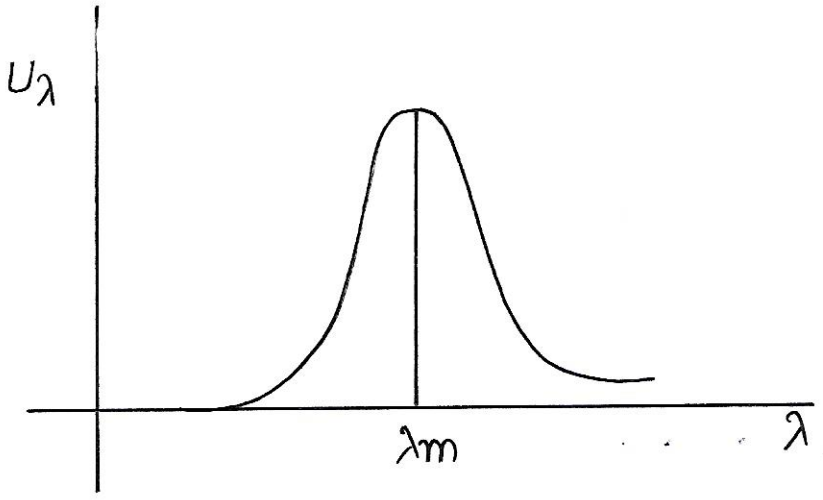
$$\nu = \frac{c}{\lambda}$$

$$d\nu = \frac{c}{\lambda^2}d\lambda$$

$$U_\lambda d\lambda = \frac{8\pi}{\lambda^3}\cdot\frac{1}{e^{hc/\lambda KT}-1}\cdot\frac{c}{\lambda^2}d\lambda$$

$$U_\lambda \cancel{d\lambda} = \frac{8\pi hc}{\lambda^5}\cdot\frac{1}{e^{hc/\lambda KT}-1}\cancel{d\lambda}$$

$$\boxed{U_\lambda = \frac{8\pi hc}{\lambda^5}\cdot\frac{1}{e^{hc/\lambda KT}-1}}$$



$$\lambda_m T = b$$

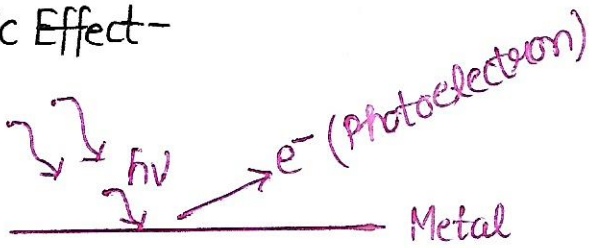
$$\frac{dU_\lambda}{d\lambda} = 0$$

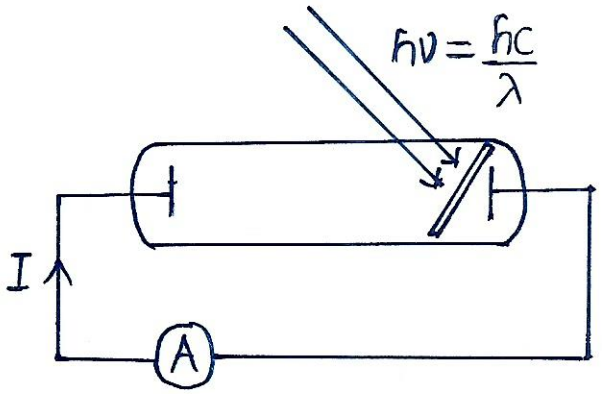
$$\lambda_m T = 2.898\times10^{-3}\ mK$$

$U = \sigma T^4$

कभी-कभी मजबूरी आपके सिद्धान्त बन जाते हैं।

#=> Photoelectric Effect-





1. Emission of Photoelectron is almost Instantaneous.
2. If $\lambda >>> \lambda_0$
3. $E_{K_{max}} = \frac{hc}{\lambda} - W = h\nu - W$

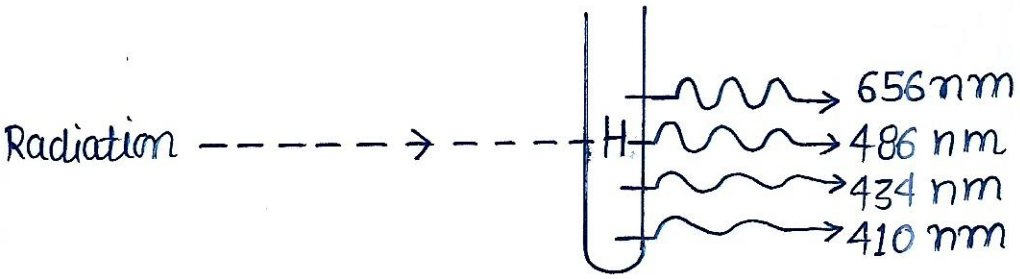
Linear Momenta $P = \frac{E}{c}$

$$P = \frac{E}{c} = \frac{\frac{hc}{\lambda}}{c}$$

$$\boxed{P = \frac{h}{\lambda}}$$

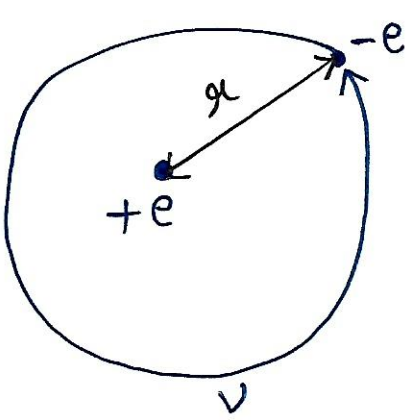
Light can behave like wave or particle.

• Bohr's Model of Hydrogen Atom-



Barmar $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$

$n = 3, 4, 5, 6$



$E = \frac{1}{2}mv^2 - \frac{e^2}{4\pi\epsilon_0 r}$

$\frac{mv^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2}$

$mv^2 = \frac{e^2}{4\pi\epsilon_0 r}$

$E = \frac{1}{2}\cdot\frac{e^2}{4\pi\epsilon_0 r} - \frac{e^2}{4\pi\epsilon_0 r}$

$E = \frac{1}{2}mv^2 - mv^2$

$E = -\frac{1}{2}mv^2$

$v = 2\pi\nu r$

$E = \frac{1}{2}mv^2$

$mv^2 = \frac{e^2}{4\pi\epsilon_0 r}$

$v = 2\pi\nu r$

Bohr model of Quantum Physics

$$-E = n\hbar\frac{\nu}{2}$$

Eliminate ν, r, V

$$V = \frac{v}{2\pi r}$$

$$-E = \frac{n\hbar v}{2 \times 2\pi r} \qquad \left[\because mv^2 = \frac{e^2}{4\pi\epsilon_0 r}\right]$$

$$-E = \frac{n\hbar v}{4\pi} \frac{(mv^2)\, 4\pi\epsilon_0}{e^2}$$

$$-E = \frac{(n\hbar v)\, mv^2 \epsilon_0}{e^2}$$

$$-E = \frac{n\hbar m v^3 \epsilon_0}{e^2}$$

$$v = \left[\frac{-Ee^2}{n\hbar\epsilon_0 \cdot m}\right]^{1/3}$$

$$E = \frac{1}{2}mv^2$$

$$E = \frac{-1}{2}m\left[\frac{Ee^2}{n\hbar\epsilon_0 \cdot m}\right]^{2/3}$$

$$E^{1/3} = -\frac{1}{2}m\left[\frac{e^2}{n\hbar\epsilon_0 \cdot m}\right]^{2/3}$$

$$E = -\frac{1}{2}m\left[\frac{e^4}{n^2\hbar^2\epsilon_0^2 m^2}\right]$$

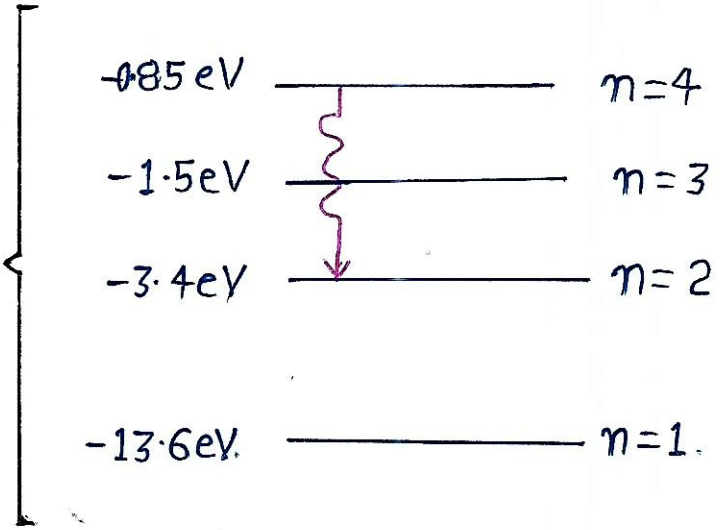
$$\boxed{E = \frac{-me^4}{8\epsilon_0^2 m^2 \hbar^2 n^2}}$$

$$E = -\frac{me^4}{8\epsilon_0^2 h^2 n^2}$$

$n = 1$

$E = -13.6\ eV$

For Transition of e^- in Hydrogen Atom



$\Delta E = E_{n_1} - E_{n_2}$

$-3.4\ eV$

$$\Delta E = \frac{-8\, me^4}{8\epsilon_0^2 h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\lambda = \frac{hc}{\Delta E}$$

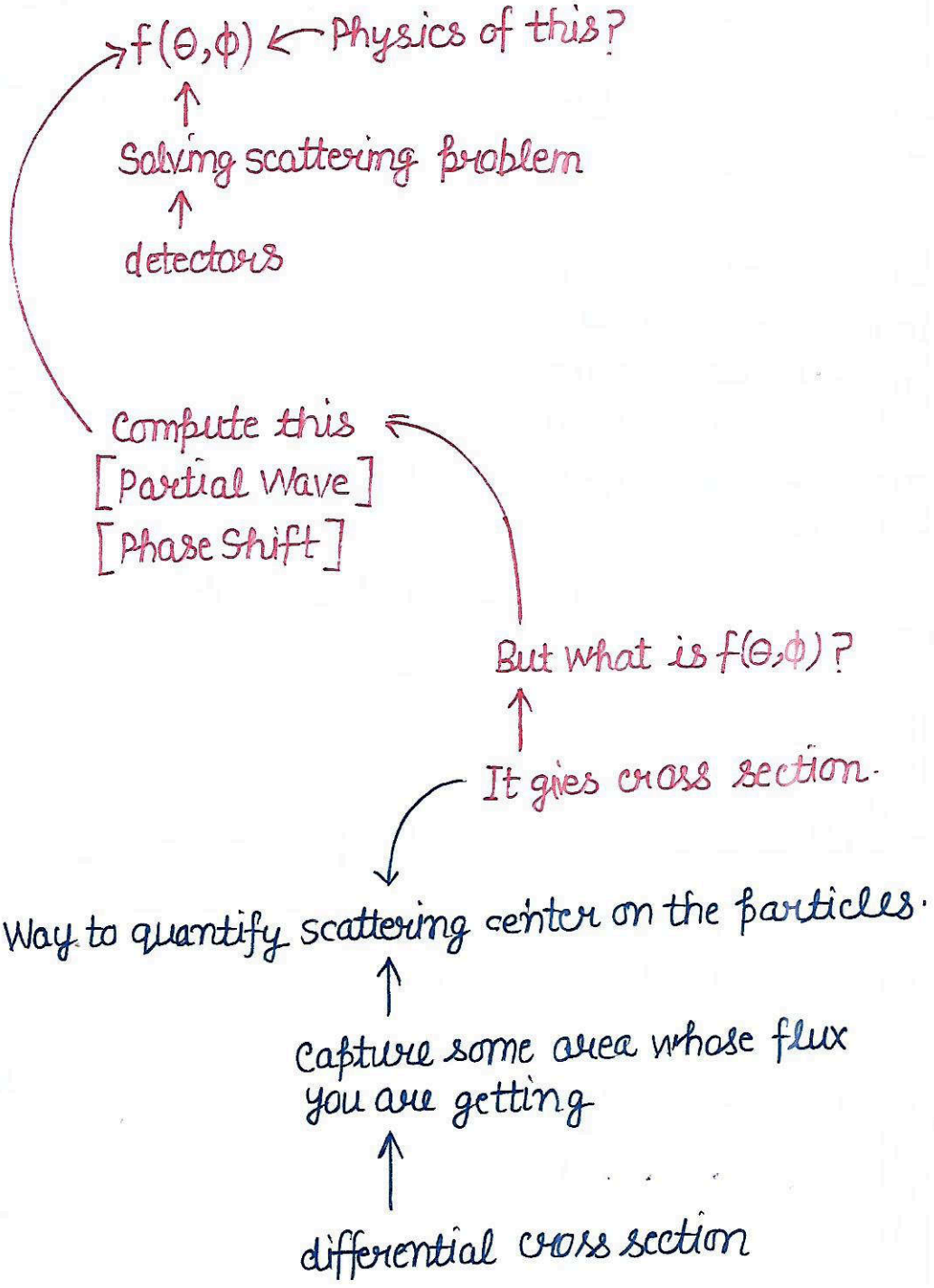
In Bose Einstein distribution →

$$l = mvr = n\frac{h}{2\pi}$$

UNIT - 1.

SCATTERING THEORY

- **Differential and Total Cross Section -**

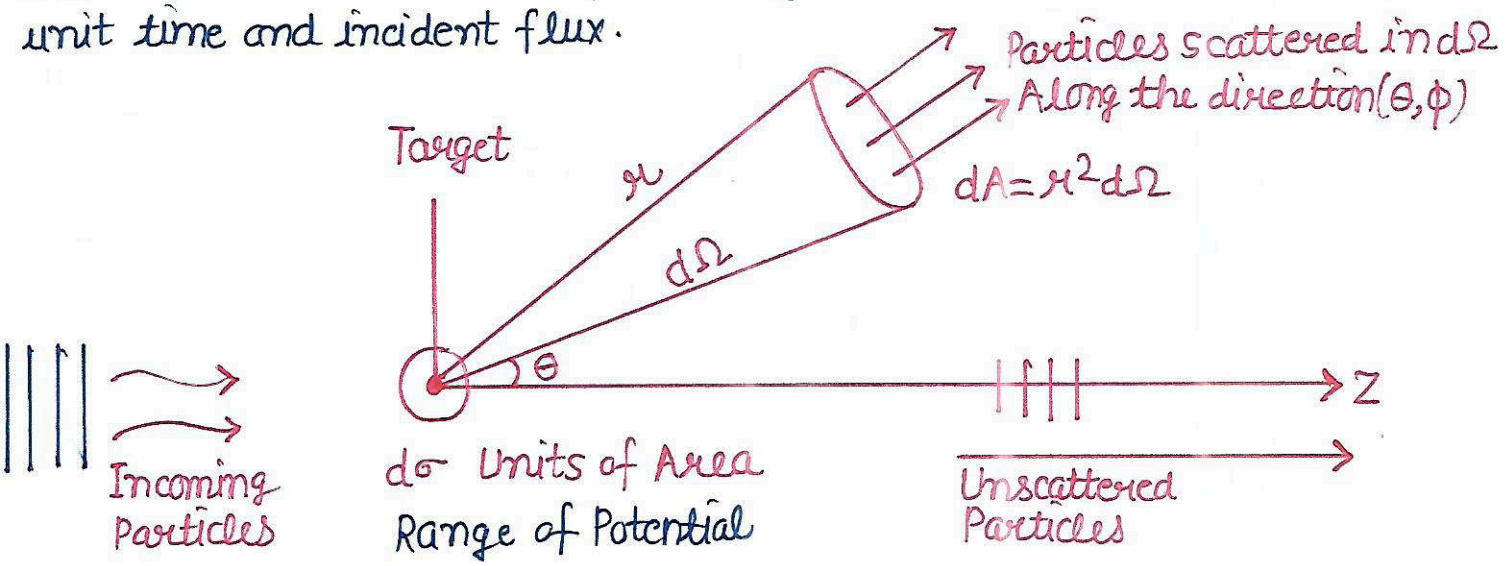


Much of our understanding about the structure of matter is extracted from the scattering of particles. Had it not been for scattering, the structure of (matter) the microscopical world would have remained inaccessible to humans. It is through scattering experiments that important building blocks of matter, such as the atomic nucleus, and the various quarks, have been discovered.

In the scattering experiment, one observe the collision b/w a beam of incident particles and a target material. The total number of collisions over the duration of the experiment is proportional to the no. of incident particles and to the number of target particles per unit area in the path of the beam.

In these experiments, one counts the collision products that comes out of the target. After scattering, those particles that do not interact with the target continue their motion (undisturbed) in the forward direction, but those that interact with the target get scattered at some angle as depicted in figure.

The number of particles scattered into an element of solid Angle is proportional to the quantity that plays a center role in the physics of scattering: The differential cross section. The differential cross section, which is denoted by $d\sigma(\theta,\phi)/d\Omega$ is defined as the number of particles scattered into an element of solid Angle $d\Omega$ in the direction (θ,ϕ) per unit time and incident flux.



$$d\sigma = \frac{\text{Particles scattered per unit time into solid Angle } d\Omega}{\left(\text{Flux of incident particles} = \frac{\text{No. of Particles per unit}}{\text{area time}}\right)}$$

↑ differential cross Section

We are not working with wave packets, we are working with energy eigen states.

$$\text{Incident Flux} = \frac{\hbar}{m} \text{Im}\left[\phi^*(\vec{r}) \nabla \phi(\vec{r})\right] \qquad \phi = e^{ikz}$$

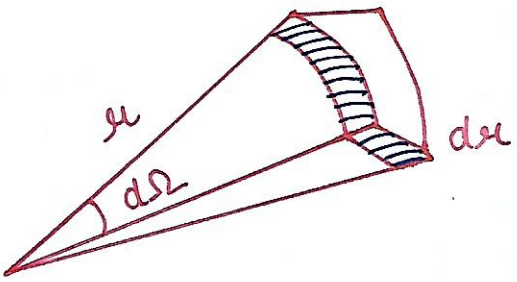
$$\text{Incident Flux} = \frac{\hbar k}{m} \hat{z}$$

↑ J_{inc}

$$\rho \vec{v} = \hat{z} 1 \cdot \frac{p}{m} = \frac{\hbar k}{m} \hat{z}$$

↑ $|\phi|^2 = 1$

Intuition in form of current.



$$\psi(\vec{r}) \simeq \phi(\vec{r}) + \psi_{scatter}(\vec{r}) = e^{ikz} + f(\theta,\phi)\frac{e^{ikr}}{r}$$

↑ consider

dn = No. of particles in the little volume

$$dn = \underbrace{\left| \frac{f(\theta,\phi)\, e^{ikr}}{r} \right|^2}_{\psi_{scatter}} r^2 \, d\Omega \, dr$$

$$dn = |f(\theta,\phi)|^2 \, d\Omega \, dr$$

$\frac{dn}{dt}$

$$dt = \frac{dr}{v} = \left(\frac{dr}{\frac{\hbar k}{m}}\right)$$

↑ Time to go through the box

$$\frac{dn}{dt} = \frac{\hbar k}{m} |f(\theta,\phi)|^2 \, d\Omega$$

$$d\sigma = \frac{\frac{\hbar k}{m} |f(\theta,\phi)|^2 \, d\Omega}{\frac{\hbar k}{m} \hat{z}}$$

$$\boxed{d\sigma = |f(\theta,\phi)|^2 d\Omega}$$ ← Differential cross section

$$\frac{d\sigma}{d\Omega} = |f(\theta,\phi)|^2$$

$$\boxed{\sigma = \int d\sigma = \int |f(\theta,\phi)|^2 d\Omega}$$ ← Total cross section

- **Born Approximation and its Validity–**

If the potential $V(\vec{r})$ is weak enough, it will distort only slightly the incident plane wave. The First Born Approximation consists then of approximating the scattered wave function $\psi(\vec{r})$ by a plane wave. The approximation corresponds to the first iteration of scattering wave function $\psi(\vec{r})$

$$\psi(\vec{r}) \simeq \phi_{inc}(\vec{r}) - \frac{\mu}{2\pi\hbar^2}\int \frac{e^{iK(\vec{r}-\vec{r}')}}{|\vec{r}-\vec{r}'|} V(\vec{r}')\,\phi_{inc}(\vec{r}')\,d^3r' \quad \ldots\ldots (1)$$

Thus, we can write the scattering amplitude and the differential cross section in the first Born Approximation as follows:

$$f(\theta,\phi) = \frac{-\mu}{2\pi\hbar^2}\int e^{-iK\vec{r}}\, V(\vec{r}')\,\phi_{inc}(\vec{r}')\,d^3r' = \frac{-\mu}{2\pi\hbar^2}\int e^{iq\vec{r}'}\, V(\vec{r}')\,d^3r' \ldots (2)$$

$$\frac{d\sigma}{d\Omega} = |f(\theta\phi)|^2 = \frac{\mu^2}{4\pi^2\hbar^4}\left|\int e^{iq\vec{r}'} V(\vec{r}')\,d^3r'\right|^2 \quad \ldots\ldots (3)$$

Scattering Amplitude

differential cross section

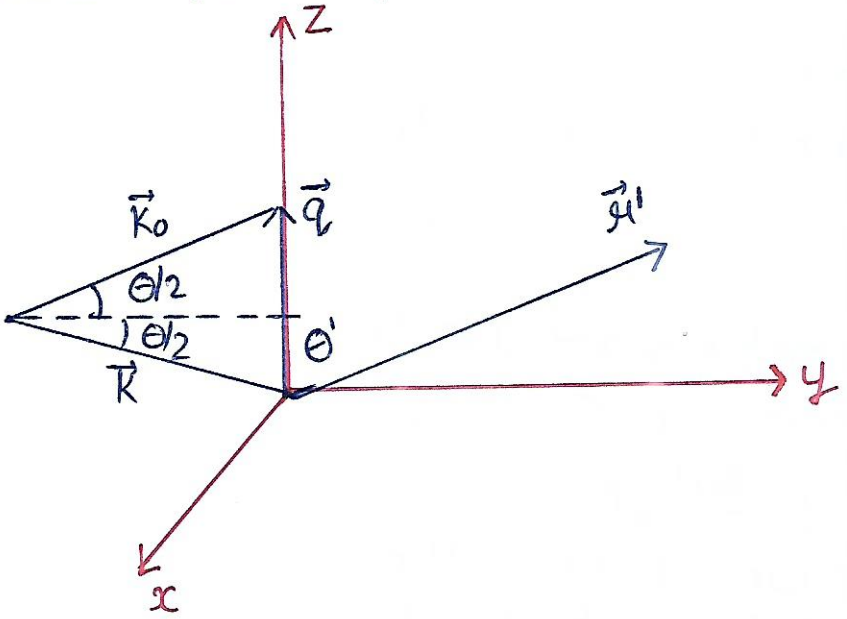
Where $\vec{q} = \vec{K_0} - \vec{K}$

$\hbar q$ = momentum transfer

$\hbar \vec{K_0}$ = Linear incident momenta

$\hbar \vec{K}$ = Linear scattered momenta

In Elastic Scattering, the magnitudes of $\vec{K_0}$ and $\vec{K}$ are equal.



$$q = |\vec{K_0} - \vec{K}| = \sqrt{K_0^2 + K^2 - 2K_0 K\cos\theta}$$

$$q = K\sqrt{2(1-\cos^2\theta)}$$

$$q = 2K\sin\left(\frac{\theta}{2}\right) \quad \ldots\ldots\ldots (4)$$

If the potentially $V(\vec{r'})$ is spherically symmetric, $V(\vec{r'}) = V(r')$ and choosing the z-axis along $\vec{q}$ then $\vec{q}\cdot\vec{r'} = qr\cos\theta'$ and therefore

$$\int e^{iqr'} V(\vec{r'})\, d^3r' = \int_0^\infty r'^2 V(r')\, dr' \int_0^\pi e^{iqr'\cos\theta'} \sin\theta' d\theta' \int_0^{2\pi} d\phi$$

$$= 2\pi \int_0^\infty r'^2 V(r')\, dr' \int_{-1}^{1} e^{iqr'x}\, dx \int_0^\infty r' V(r') \sin(qr')\, dr'$$

$\ldots\ldots\ldots (5)$

inserting eq^n(5) into eq^n(2) and (3), we obtain

$$\boxed{f(\theta) = \frac{-2\mu}{\hbar^2 q}\int_0^\infty r' V(r') \sin(qr')\, dr'}$$

And

$$\boxed{\frac{d\sigma}{d\Omega} = |f(\theta)|^2 = \frac{4\mu^2}{\hbar^4 q^2}\left|\int_0^\infty r' V(r') \sin(qr')\, dr'\right|^2} \quad \ldots\ldots\ldots (6)$$

In summary, we have shown that by solving the schrodinger eq^n

$$\frac{-\hbar^2}{2\mu}\nabla^2 \psi(\vec{r}) + \hat{V}(\vec{r}')\psi(\vec{r}') = E\psi(\vec{r}')$$

to the first order Born Approximation. The diff. cross section is given by eq^n(6) for the spherically symmatric potential.

Validity

The First Born Approximation is valid whenever the wavefunction $\psi(\vec{r})$ is only slightly different from the incident plane wave; that is, whenever the second term in eq^n(1) is very small compared to the first

$$\left|\frac{\mu}{2\pi\hbar^2}\int \frac{e^{iK|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} V(\vec{r}')\psi(r')\, e^{iK_0 r'}\, d^3r'\right| <<< |\phi_{inc}(\vec{r})|^2$$

Since $\phi_{inc} = e^{iK_0\vec{r}}$

$$\left|\frac{\mu}{2\pi\hbar^2}\int \frac{e^{iK|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} V(\vec{r}') e^{iK_0 r'}\, d^3r'\right| <<< 1$$

In elastic scattering

$$K_0 = K$$

And assuming that the scattering potential is larger near $r=0$, we have

$$\left|\frac{\mu}{\hbar^2}\int_0^\infty r' e^{ikr'} V(r')\, dr' \int_0^\pi e^{ikr'\cos\theta'} \sin\theta'\, d\theta'\right| << 1$$

$$\boxed{\frac{\mu}{\hbar^2 K}\left|\int_0^\infty V(r')\left(e^{2iKr'}-1\right)dr'\right| \ll 1} \quad \ldots\ldots\ldots (7)$$

Since the energy of the incident particles is propotional to K, we infer from eqn(7) that the Born Approximation is valid for large incident energies and weak scattered potential. That is, when the avarage interaction energy b/w the incident particle and the scattering potential is much smaller than the Kinetic energy of incident particles, the scattered wave can be considered to be a plane wave.

- **Optical Theorem–**

Total cross section

$$\sigma = \int |f_K(\theta)|^2 d\Omega = \int f_K^*(\theta) f_K(\theta) d\Omega$$

$$\sigma = \frac{4\pi}{K^2}\sum_{ll'}\sqrt{2l+1}\sqrt{2l'+1}\, e^{-i\delta_l}\sin\delta_l\, e^{i\delta_{l'}}\sin\delta_{l'}\int d\Omega Y_{l0}^*(\Omega) Y_{l'0}(\Omega)$$

$\delta_{ll'}$
Normality for spherical harmonic

$$\boxed{\sigma = \frac{4\pi}{K^2}\sum(2l+1)\sin^2\delta_l}$$

It is important to calculate the Phase Shift. The cross section is expressed in terms of Phase Shift. The one more concept i.e. more important, i.e. optical theorem. The Probability of conservation of flux, Its fairly non trivial. When you have an object, the thing you detect, all the deflected objects creates shadow. The light gone through it.

In forward direction,

$$f_K(\theta=0)$$

$$Y_{l0}(\theta) = \sqrt{\frac{2l+1}{4\pi}}\, P_l(\cos\theta)$$

$$Y_{l0}(\theta=0) = \sqrt{\frac{2l+1}{4\pi}}$$

$$\boxed{P_l(x=1) = 1}$$

$$f_k(\theta) = \frac{\sqrt{4\pi}}{k} \sum \sqrt{2l+1} \frac{\sqrt{2l+1}}{\sqrt{4\pi}} e^{i\delta_l} \sin\delta$$

$$\boxed{f_k(\theta) = \frac{1}{k} \sum (2l+1) e^{i\delta_l} \sin\delta}$$

$$\mathrm{Im} f_k(\theta) = \frac{1}{k} \sum (2l+1) \sin^2\delta$$

$$\boxed{\sigma = \frac{4\pi}{k} \mathrm{Im}[f(\theta=0)]}$$

This is the final expression of Optical Theorem. It is context of optics. The shadow contains all the information about the incoming beam.

#Scattering Amplitude-

We are going to show here that we can obtain the diff. cross section in the CM frame from an asymptotic form of the solution of the schrodinger eqn which can be written as

$$(\nabla^2 + k^2)\psi(\vec{r}) = \frac{2\mu}{\hbar^2} V(\vec{r})\psi(\vec{r}) \quad \ldots\ldots (1)$$

The solution of the equation

$$(\nabla^2 + k^2)\psi_{homo}(\vec{r}) = 0 \quad \ldots\ldots\ldots (2)$$

$\psi_{homo}(\vec{r})$ is incident Plane wave.

In terms of green function,

$$\psi(\vec{r}) = \phi_{inc}(\vec{r}) + \frac{2\mu}{\hbar^2} \int G(\vec{r}-\vec{r}') V(\vec{r}') \psi(\vec{r}') \, d^3r' \quad \ldots\ldots (3)$$

$$\phi_{inc}(\vec{r}) = e^{ik_0\vec{r}}$$

$$(\nabla^2+K^2)G(\vec{r}-\vec{r}') = \delta(\vec{r}-\vec{r}') \quad \ldots\ldots (4)$$

Fourier Transformations

$$G(\vec{r}-\vec{r}') = \frac{1}{(2\pi)^3}\int e^{iq(\vec{r}-\vec{r}')} G(\vec{q})\, d^3q \quad \ldots\ldots (5)$$

$$\delta(\vec{r}-\vec{r}') = \frac{1}{(2\pi)^3}\int e^{iq(\vec{r}-\vec{r}')}\, d^3q \quad \ldots\ldots (6)$$

Substitute the value of eq^n(5) & (6) in eq^n(4)

$$(\nabla^2+K^2)\frac{1}{(2\pi)^3}\int e^{iq(\vec{r}-\vec{r}')} G(\vec{q})\, d^3q = \frac{1}{(2\pi)^3}\int e^{iq(\vec{r}-\vec{r}')}\, d^3q$$

$$(-\vec{q}^2+\vec{K}^2)G(\vec{q}) = 1 \Rightarrow G(\vec{q}) = \frac{1}{(\vec{K}^2-\vec{q}^2)} \quad \ldots\ldots (7)$$

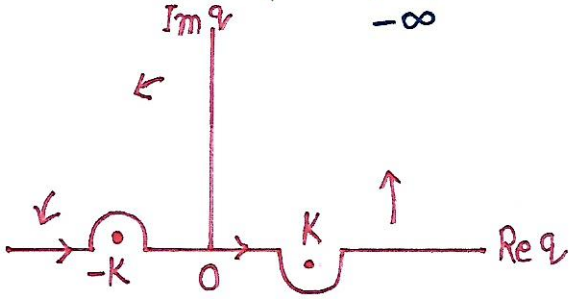
Put the value of $G(\vec{q})$ in eq^n(5)

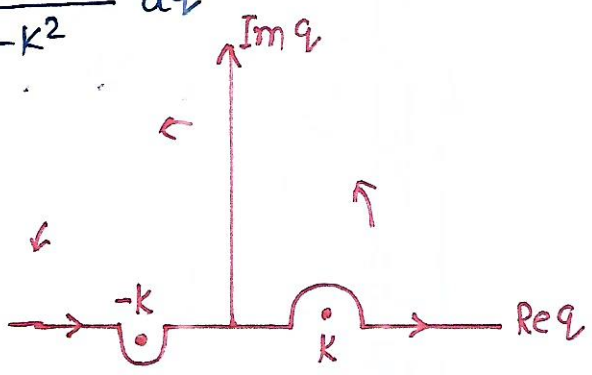
$$G(\vec{r}-\vec{r}') = \frac{1}{(2\pi)^3}\int \frac{e^{iq(\vec{r}-\vec{r}')}}{(\vec{K}^2-\vec{q}^2)}\, d^3q \quad \ldots\ldots (8)$$

To integrate over the angle in

$$G(\vec{r}-\vec{r}') = \frac{1}{(2\pi)^3}\int_0^\infty \frac{q^2 dq}{\vec{K}^2-\vec{q}^2}\int_0^\pi e^{iq|\vec{r}-\vec{r}'|\cos\theta}\sin\theta\, d\theta \int_0^{2\pi} d\varphi$$

$$G(\vec{r}-\vec{r}') = \frac{-1}{4\pi^2 i|\vec{r}-\vec{r}'|}\int_{-\infty}^{+\infty} \frac{q e^{iq|\vec{r}-\vec{r}'|}}{q^2-K^2}\, dq$$





$$G_+(\vec{r}-\vec{r}') = \frac{-1}{4\pi}\frac{e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \quad \ldots\ldots\ldots (8)$$

$$G_-(\vec{r}-\vec{r}') = -\frac{1}{4\pi}\frac{e^{-ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \quad \ldots\ldots\ldots (9)$$

Outgoing spherical wave

Incoming wave

Put the value of eq^n(8) in eq^n(3)

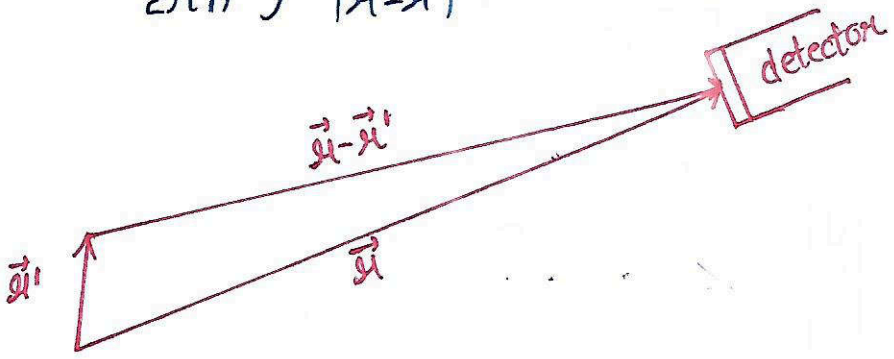
$$\psi(\vec{r}) = \phi_{inc}(\vec{r}) + -\frac{\mu}{2\pi\hbar^2}\int \frac{e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} V(\vec{r}')\,\psi(\vec{r}')\,d^3r'$$

This is an integration equation.

$$\psi_1(\vec{r}) = \phi_{inc}(\vec{r}) - \frac{\mu}{2\pi\hbar^2}\int \frac{e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} V_1(\vec{r}')\,\psi_0(\vec{r}_1)\,d^3r_1$$

$$= \phi_{inc}(\vec{r}) - \frac{\mu}{2\pi\hbar^2}\int \frac{e^{ik|\vec{r}-\vec{r}_1|}}{|\vec{r}-\vec{r}'|} V_1(\vec{r}')\,\phi_{inc}(\vec{r}_1)\,d^3r_1$$

$$\psi_2(\vec{r}) = \phi_{inc}(\vec{r}) - \frac{\mu}{2\pi\hbar^2}\int \frac{e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} V_2(\vec{r}_2)\,\psi_1(r_2)\,d^3r_2$$



$$\psi(\vec{r}) = \phi_{inc}(\vec{r}) - \frac{\mu}{2\pi\hbar^2}\int \frac{e^{ik|\vec{r}-\vec{r}_2|}}{|\vec{r}-\vec{r}_2|} V(\vec{r}_2)\,\phi_{inc}(\vec{r}_2)\,d^3r_2$$

$$+\left(\frac{\mu}{2\pi\hbar^2}\right)^2 \int \frac{e^{ik|\vec{r}-\vec{r}_2|}}{|\vec{r}-\vec{r}_2|} V(\vec{r}_2)\,d^3r_2 \int \frac{e^{ik|\vec{r}_2-\vec{r}_1|}}{|\vec{r}_2-\vec{r}_1|} V(\vec{r}_1)\,\phi_{inc}(\vec{r}_1)\,d^3r_1 \quad \ldots\ldots (10)$$

$$K|(\vec{r}-\vec{r}')| = \frac{1}{r}$$

where

$$f(\theta,\phi) = \frac{-\mu}{2\pi\hbar^2}\int e^{i\vec{K}\cdot\vec{r}}\, V(\vec{r}')\,\psi(\vec{r}')\,d^3r' = \frac{-\mu}{2\pi\hbar^2}\langle\phi|\hat{V}|\psi\rangle$$

Cross section is given by

$$\frac{d\sigma}{d\Omega} = |f(\theta,\phi)|^2 = \frac{\mu^2}{4\pi^2\hbar^4}\left|\int e^{i\vec{K}\vec{r}'}\, V(\vec{r}')\,\psi(\vec{r}')\,d^3r'\right|^2 = \frac{\mu^2}{4\pi^2\hbar^4}\left|\langle\phi|\hat{V}|\psi\rangle\right|^2$$

- L22.4 Identical Particles are exchange degeneracy-

Identical Particles — mass, Spin, Charge, Magnetic moment

If all properties are same hence they are identical particles.

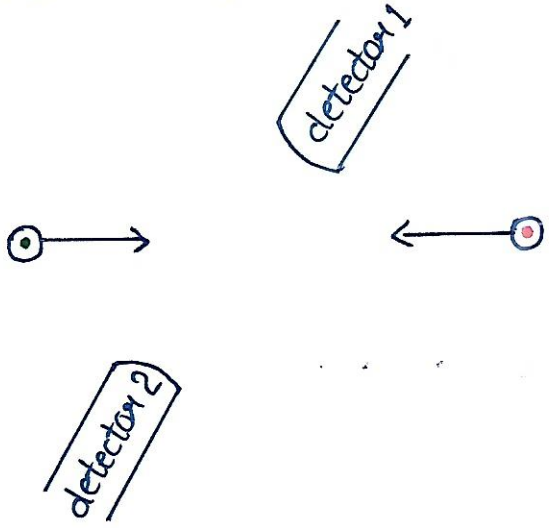
- Isospin in any new direction. This is a quantum philosophy.

Classical Mechanics, Identicle Particles are distingusible.



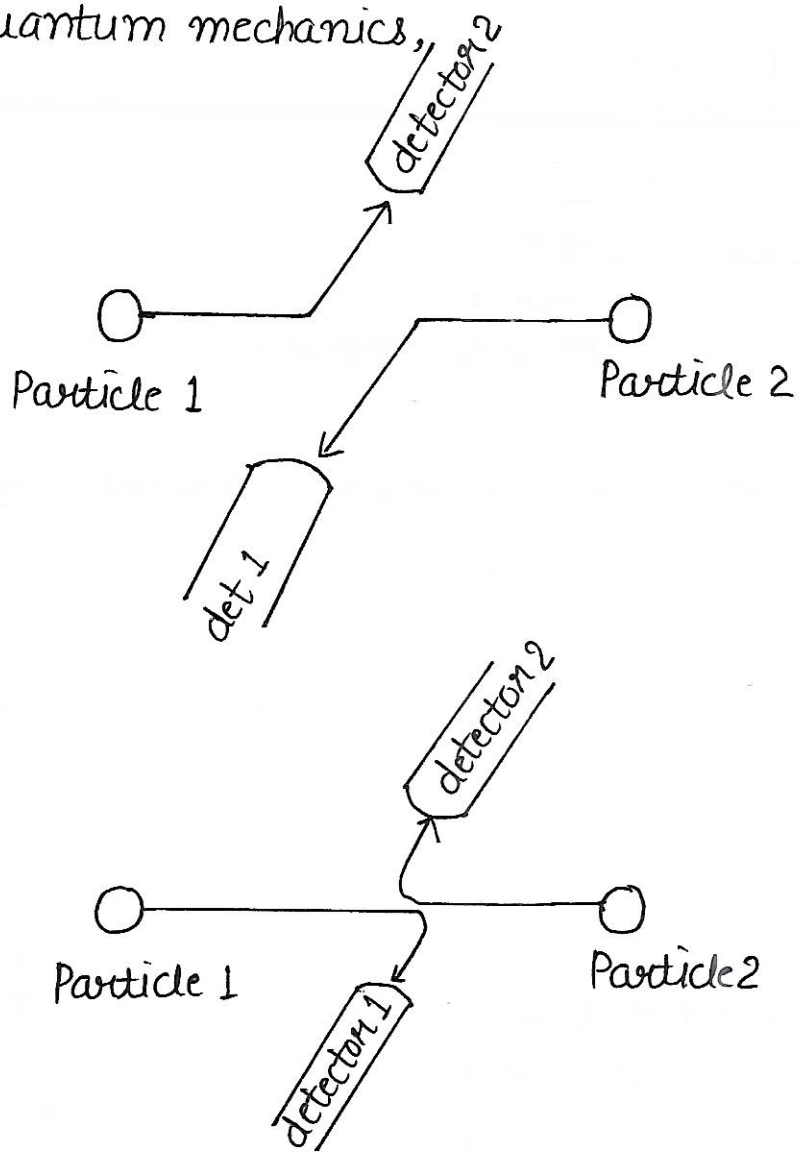
because you know trajactories of the particles. We can tag the particles. So that we can define which particle is which.

$$H(\vec{r}_1, \vec{p}_1 ; \vec{r}_2, \vec{p}_2) = H(\vec{r}_2, \vec{p}_2 ; \vec{r}_1, \vec{p}_1) \Rightarrow \text{Symattery}$$



If I tag that particles, I know which particle is going where.

But in quantum mechanics,



We can't tag particles in quantum Mechanics.

For distinguisable Particles 1 ..... N

$$|\psi_{i_1}\rangle_{(1)} \otimes |\psi_{i_2}\rangle_{(2)} \otimes \ldots\ldots \otimes |\psi_{in}\rangle_{(n)}$$

Suppose we have 2 e^-

$$|+\rangle \quad , \quad |-\rangle$$

$$|+\rangle_{(1)} \otimes |-\rangle_{(2)} \; (\checkmark)$$

$$|-\rangle_{(1)} \otimes |+\rangle_{(2)} \; (\checkmark)$$

How do I describe the state?
Both are equally possible

If the system particles are identical then those two states are equally identical. They should be indistinguisible, identical, physically equivalent. Unfortunately that does not work. It seems so logical.

If equivalent,

$$|\psi(\alpha,\beta)\rangle = \alpha|+\rangle|-\rangle + \beta(|-\rangle|+\rangle)$$
$$|\alpha|^2 + |\beta|^2 = 1$$

Let search the probability

$$|\psi_0\rangle = |+,x\rangle \oplus |+,x\rangle = \frac{1}{2}(|+\rangle+|-\rangle)(|+\rangle+|-\rangle)$$
$$|\psi_0\rangle = \frac{1}{2}(|+\rangle|+\rangle + |+\rangle|-\rangle) + (|-\rangle|+\rangle + |-\rangle|-\rangle)$$

$$|\langle\psi_0||\psi(\alpha,\beta)\rangle|^2 = \left|\frac{1}{2}(\alpha+\beta)\right|^2 = \frac{1}{2}[(|\alpha|^2+|\beta|^2+2\,Re(\alpha\beta^*)] = \frac{1}{2}[+Re(\alpha\beta^*)]$$

So this Hypothesis of two states are equivalent they $|\psi(\alpha,\beta)\rangle$, Are normalized and you would have the probability to find ψ_0 would depend on what values of α,β do you choose so it is a contradiction so we can't solve the problem of degeneracy of Identical particles.

8.04 Alan Adams-

Why diamond is transparent ? Why Cu is not transparent ?

- The Schrodinger equation for a system consisting of Identical particles-

Up to this point, we have dealt mainly with the motion of a single particles. Now we want to examine how to describe system with many particles. We shall focus on system of Identical particles and examine how to construct their wave function.

Many physical system-nucleons, nuclei, atoms, molecules, solids, fluids, gases etc.-involve many particles. They are known as many-particles or many-body systems.

How does one describe the dynamics of a system of one to N particles. This description can be obtained from a generalization of the dynamics of a single particle. The state of a system of N spinless particles is described by a wave function $\psi(\vec{r}_1,\vec{r}_2,\vec{r}_3,\ldots\vec{r}_N,t)$.

where $|\psi(\vec{r}_1, \vec{r}_2, \ldots \vec{r}_N, t)|^2 \, d^3r_1 d^3r_2 \, d^3r_N$ represents the probability at time t of finding particle 1 in the volume element d^3r_1 centered about $\vec{r}_1$, particle 2 in the volume d^3r_2 about $\vec{r}_2$ ..... and particle N in the volume d^3r_N about $\vec{r}_N$.

The normalization condition

$$\int d^3r_1 \int d^3r_2 \ldots \int |\psi(\vec{r}_1, \vec{r}_2, \vec{r}_3 \ldots \vec{r}_N, t)|^2 \, d^3r_N = 1 \quad \ldots (1)$$

Now According to the time depending Schrodinger Eq^n

$$i\hbar \frac{\partial}{\partial t} \psi(\vec{r}_1, \vec{r}_2 \ldots \vec{r}_N, t) = \hat{H} \psi(\vec{r}_1, \vec{r}_2, \ldots \vec{r}_N, t) \quad \ldots (2)$$

Generalization of one particle Hamiltonion to N

$$\hat{H} = \frac{\vec{p}^2}{2m} + V(\vec{r})$$

$$\hat{H} = \sum_{j=1}^{N} \frac{\vec{p}_j^{\,2}}{2m_j} + V(\vec{r}_1, \vec{r}_2 \ldots \vec{r}_N, t) = -\sum_{j=1}^{N} \frac{\hbar^2}{2m_j} \nabla_j^2 + \hat{V}(\vec{r}_1, \vec{r}_2 \ldots \vec{r}_N, t)$$

Mass

Momentum

Operator

The formalism of quantum mechanics for an N-particles system can be, in principle, inferred from that of a single particle.

$$[\hat{X}_j, \hat{P}_{Xk}] = i\hbar \delta_{j,k}$$

$$[\hat{X}_j, \hat{X}_k] = [\hat{P}_{Xj}, \hat{P}_{Xk}] = 0$$

Then stationary state

$$\psi(\vec{r}_1, \vec{r}_2 \ldots \ldots, \vec{r}_N, t) = \psi(\vec{r}_1, \vec{r}_2, \vec{r}_3 \ldots \vec{r}_N) e^{-iEt/\hbar}$$

Total Energy of the system

$$\left[-\sum_{j=1}^{N}\frac{\hbar^2}{2m_j}\vec{\nabla}_j^2 + V(\vec{r}_1 \ldots \vec{r}_N)\right]\Psi(\vec{r}_1, \vec{r}_2 \ldots, \vec{r}_N) = E\Psi(\vec{r}_1 \ldots \vec{r}_N)$$

Probability density

$$\langle\Psi|\hat{A}|\Psi\rangle = \langle\Psi|\hat{A}|\Psi\rangle = \int d^3r_1 \int d^3r_2 \ldots \int \psi^*(\vec{r}_1 \ldots \vec{r}_N)\hat{A}\psi(\vec{r}_1 \ldots r_N)d^3r_N$$

The stationary state is conserved.

As an Illusion, Let us consider an atom with z e^-.

$$\left[\frac{-\hbar^2}{2m_e}\sum^{Z}\nabla_{r_i}^2 - \frac{\hbar^2}{2M}\vec{\nabla}_R^2 - \sum_{i=1}^{Z}\frac{Ze^2}{|r_i - R|} + \sum_{i>j}\frac{e^2}{|r_i - r_j|}\right]\psi(\vec{r}_1 \ldots \vec{r}_Z, \vec{R})$$

$$= E\psi(\vec{r}_1 \ldots \vec{r}_Z, \vec{R})$$

Repulsive columb interaction

Mass of Nucleus

Kinetic energy operator

Attractive columb Interaction

This is the schrodinger equation for all many particle differential Equation.

Symmetric and Anti-symmetric wave Function-

→ Before we understand the role of the exclusion principle of Atomic structure. It is interesting to look into its quantum Mechanical model.

$$\psi(\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots \vec{r}_N)$$

N (non-interacting particles)

$$\psi(\vec{r}_1 \ldots \vec{r}_N) = \psi(\vec{r}_1)\psi(\vec{r}_2) \ldots \psi(\vec{r}_N) \ldots\ldots (1)$$

This equation (1) can be used to describe a system of identical particles.

→ Particles in quantum state ψ_A, ψ_B.

→ Particles are Identical.

$_1e^-$: $\vec{r}_1 \longrightarrow \psi_A$ ← Eigen state

$_2e^-$: $\vec{r}_2 \longrightarrow \psi_B$ ↑ Energy Levels

↑ Probability density $|\psi|^2$

Combined wave function

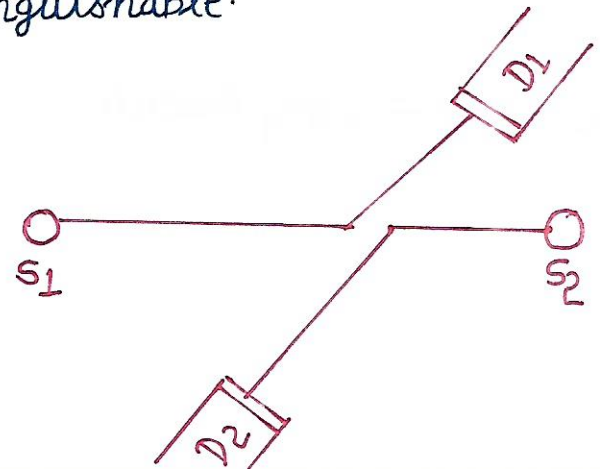
$$\psi(\vec{r}_1, \vec{r}_2) = \psi_A(\vec{r}_1)\, \psi_B(\vec{r}_2)$$

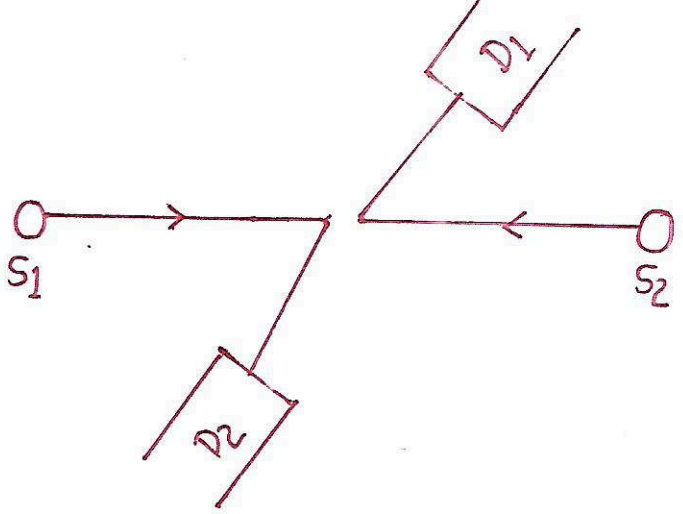
In classical Mechanics,



When a system is made of identical particles, it is possible to identify and distinguish each particle from the others. That is, although all particles have the same physical properties, we can tag each classical particle and follow its motion along a path. For instance, each particle can be coloured differently from the rest; hence we can follow the trajectory of each particle separately at each time. Identical classical particles, therefore do not lose their identity, they are distinguishable.

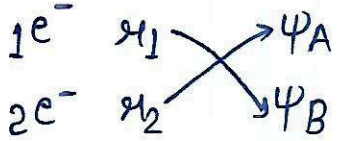
In quantum Mechanics, however, identical particles are truly indistinguishable.





Combined wave function

$\psi(\vec{r}_2, \vec{r}_1) = \psi_A(\vec{r}_2)\psi_B(\vec{r}_1)$



Probability density

$|\psi(\vec{r}_1, \vec{r}_2)|^2 = |\psi(\vec{r}_2, \vec{r}_1)|^2$

$\psi|(\vec{r}_1, \vec{r}_2)| = \pm \psi|\vec{r}_2, \vec{r}_1|$

Symmetric condition-

$\psi_{Sy}(\vec{r}_1, \vec{r}_2) = +\psi(\vec{r}_2, \vec{r}_1)$

Assymmatric condition-

$\psi_{Asy}(\vec{r}_1, \vec{r}_2) = -\psi(\vec{r}_2, \vec{r}_1)$

Normalized wave function,

$$\psi_S = \frac{1}{\sqrt{2}}\left[\psi_A(\vec{r}_1)\psi_B(\vec{r}_2) + \psi_A(\vec{r}_2)\psi_B(\vec{r}_1)\right]$$

$$\psi_A = \frac{1}{\sqrt{2}}\left[\psi_A(\vec{r}_1)\psi_B(\vec{r}_2) - \psi_A(\vec{r}_2)\psi_B(\vec{r}_1)\right]$$

$\frac{1}{\sqrt{2}}$ is normalization factor.

Special case:

$A = B$

$1e^- : \vec{r}_1 \longrightarrow \psi_A$

$2e^- : \vec{r}_2 \longrightarrow \psi_A$

$$\psi(\vec{r}_1, \vec{r}_2) = \psi_A(\vec{r}_1)\,\psi_A(\vec{r}_2)$$
$$\psi(\vec{r}_2, \vec{r}_1) = \psi_A(\vec{r}_2)\,\psi_A(\vec{r}_1)$$

Then

ψ_S = Sum value

$$\boxed{\psi_A = 0}$$

Particles does not exist in reality but reality of maths framework changes. Hence it leads to Pauli Exclusion principle.

Identical Particles

Bosons

→ Symmetric

→ Do not follow pauli Exclusion principle

→ Spin 0, 1

→ Photons, He Atom

→ Bose Einstein Statistics

Fermions

→ Anti-symmetric

→ Follow pauli Exclusin principle

→ Spin $\frac{1}{2}, \frac{3}{2}$

→ e^-, p^+, n^0

→ Fermi Dirac Statistics

Relativistic Wave Equation

#Klein-Gordon Equation-

→ Relativistic Equation
→ Comparison of particle's speed with speed of light
→ Quantized version of Energy momentum Relation
→ Theoritical Revolution of Fermi Dirac Eqn
→ Quantum Model

Schrodinger Eq^n in operator Form

$$\hat{H}\psi = i\hbar\frac{\partial\psi}{\partial t} \quad \ldots\ldots(1)$$

Non-relativistic form of Hamiltonian for free Particles

$$\hat{H} = \frac{\hat{p}^2}{2m} \qquad \left[\because p = i\hbar\nabla\right]$$

$$\hat{H} = \frac{-\hbar^2\nabla^2}{2m} \quad \ldots\ldots(2)$$

From eq^n(1) & (2)

$$\frac{-\hbar^2}{2m}\nabla^2\psi = i\hbar\frac{\partial\psi}{\partial t}(\vec{r},t) \quad \ldots\ldots(3)$$

Eq^n(3) can be rewritten as, Relativistic Hamiltonian

$$\hat{H} = E = \pm\sqrt{p^2c^2+m^2c^4} \Rightarrow \pm\sqrt{p^2c^2+m^2c^4}\,\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\left[-\hbar^2\nabla^2c^2+m^2c^4\right]^{1/2}\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\left[\hat{H}\psi = i\hbar\frac{\partial\psi}{\partial t}\right]\times\hat{H}$$

$$\hat{H}^2\psi = H\left(i\hbar\frac{\partial\psi}{\partial t}\right)$$

$$\hat{H}^2\psi = i\hbar\frac{\partial}{\partial t}(\hat{H}\psi)$$

$$\hat{H}^2\psi = i\hbar\frac{\partial}{\partial t}\left(i\hbar\frac{\partial\psi}{\partial t}\right)$$

$$\boxed{\hat{H}^2\psi = -\hbar^2\frac{\partial^2\psi}{\partial t^2}}$$

This is K-G eqn for relativistic case.

For Free particles:

$$\hat{H}^2 = p^2c^2 + m^2c^4$$

$$\hat{H}^2 = -\hbar^2c^2\nabla^2 + m^2c^4$$

$$(-\hbar^2c^2\nabla^2 + m^2c^4)\psi = -\hbar^2\frac{\partial^2\psi}{\partial t^2}$$

$$\left(\frac{-\hbar^2c^2}{-\hbar^2c^2}\nabla^2\psi + \frac{m^2c^4}{-\hbar^2c^2}\psi + \frac{\hbar^2}{-\hbar^2c^2}\frac{\partial^2\psi}{\partial t^2}\right) = 0$$

$$\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} - \frac{m^2c^2}{\hbar^2}\psi = 0$$

$$\left[\Box^2 - \frac{m^2c^2}{\hbar^2}\right]\psi = 0 \qquad \left[\because \Box^2 = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]$$

De-Alembert Operator

This is second order diff^n eq^n of space and time. This is relativistic version of schrodinger Eq^n. This is the invariance in Lorentz transformation.

Drawbacks:

(i) Describes the particles of zero spin only like π-mesons.

(ii) ψ has only scalar components.

(iii) current density and charge density could not exactly explained.

(iv) Only deal with non-interacting particles.

Charge and current Density -

$$\left(\Box^2 - \frac{m^2c^2}{\hbar^2}\right)\psi = 0$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m^2c^2}{\hbar^2}\right)\psi = 0$$

$$\left(\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} - \frac{m^2c^2}{\hbar^2}\psi = 0\right) \times \psi^* \quad \ldots\ldots\ldots (1)$$

$$\left(\nabla^2\psi^* - \frac{1}{c^2}\frac{\partial^2\psi^*}{\partial t^2} - \frac{m^2c^2}{\hbar^2}\psi^* = 0\right) \times \psi \quad \ldots\ldots\ldots (2)$$

$$\psi^*\nabla^2\psi - \psi^*\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} - \psi^*\frac{m^2c^2}{\hbar^2}\psi = 0$$

$$\psi^*\nabla^2\psi - \frac{1}{c^2}\psi^*\frac{\partial^2\psi}{\partial t^2} - \frac{m^2c^2}{\hbar^2}\psi^*\psi = 0 \quad \ldots\ldots\ldots (3)$$

$$\psi\nabla^2\psi^* - \frac{1}{c^2}\psi\frac{\partial^2\psi^*}{\partial t^2} - \frac{m^2c^2}{\hbar^2}\psi\psi^* = 0 \quad \ldots\ldots\ldots (4)$$

Eq^n(3) - Eq^n(4)

$$\psi^*\nabla^2\psi - \frac{1}{c^2}\psi^*\frac{\partial^2\psi}{\partial t^2} - \cancel{\frac{m^2c^2}{\hbar^2}\psi^*\psi} - \psi\nabla^2\psi^* + \frac{1}{c^2}\psi\frac{\partial^2\psi^*}{\partial t^2} + \cancel{\frac{m^2c^2}{\hbar^2}\psi\psi^*} = 0$$

$$\nabla\left[\psi^*\nabla\psi - \psi\nabla\psi^*\right] - \frac{1}{c^2}\left[\psi^*\frac{\partial^2\psi}{\partial t^2} \pm \psi\frac{\partial^2\psi^*}{\partial t^2}\right] = 0 \times \frac{\hbar}{2im}$$

$$\nabla\left[\frac{\hbar}{2im}(\psi^*\nabla\psi - \psi\nabla\psi^*)\right] + \frac{\partial}{\partial t}\left[\frac{\hbar}{2imc^2}\left(\psi^*\frac{\partial\psi}{\partial t} \pm \psi\frac{\partial\psi^*}{\partial t}\right)\right] = 0$$

$\Downarrow$ $S(\vec{r},t)$

$\Downarrow$ $P(\vec{r},t)$

$$\nabla S(\vec{r},t) + \frac{\partial \rho}{\partial t}(\vec{r},t) = 0$$ → Equation of continuity

S → current density

ρ → Can't be interpreted as position probability density

$$\rho(\vec{r},t) = \frac{\hbar}{2imc^2}\left(\psi\frac{\partial\psi^*}{\partial t} - \psi^*\frac{\partial\psi}{\partial t}\right)$$

$$\rho(\vec{r},t) = \frac{1}{2mc^2}\left[\left(-i\hbar\frac{\partial\psi^*}{\partial t}\right)\psi + \psi^*\left(i\hbar\frac{\partial\psi}{\partial t}\right)\right]$$

$$H\psi = i\hbar\frac{\partial\psi}{\partial t} = E\psi$$

$$H^*\psi^* = -i\hbar\frac{\partial\psi^*}{\partial t} = E^*\psi^*$$

$$\rho(\vec{r},t) = \frac{1}{2mc^2}\left[(E\psi^*)\psi + \psi^*(E\psi)\right]$$

$$\rho(\vec{r},t) = \frac{1}{2mc^2}\left[2E\psi\psi^*\right]$$

$$\boxed{\rho(\vec{r},t) = \frac{E}{mc^2}[\psi\psi^*]}$$

$$E = \pm\sqrt{p^2c^2 + m^2c^4}$$

$e\rho$ → charge density

eS → current density

#Klein-Gordon Equation for EM Field-

Vector Potential → A
Scalar Potential → ϕ

An Electromagnetic field can be uniquely represented by a vector potential A and scalar potential ϕ.

If e is charge of particle

$$\left.\begin{aligned} P &\longrightarrow P - \frac{eA}{C} \\ E &\longrightarrow E - e\phi \end{aligned}\right] \ldots\ldots\ldots\ldots (1)$$

$$E = \pm\sqrt{p^2c^2 + m^2c^4}$$

$$E^2 = p^2c^2 + m^2c^4$$

Put the values of E and P from eq^n(1)

$$(E - e\phi)^2 = \left(P - \frac{eA}{C}\right)^2 c^2 + m^2c^4$$

$$(E - e\phi)^2 = \left(\frac{CP - eA}{C}\right)^2 C^2 + m^2c^4$$

$$E = i\hbar\frac{\partial}{\partial t} \quad \& \quad P = -i\hbar\nabla$$

We get, K-G eq^n as

$$\left(i\hbar\frac{\partial}{\partial t} - e\phi\right)^2\psi = \left[\left\{-i\hbar\nabla C - eA\right\}^2 + m^2c^4\right]\psi \ldots\ldots\ldots (2)$$

$$(A-B)^2 = A^2 - AB - BA + B^2$$

$$\left(i\hbar\frac{\partial}{\partial t} - e\phi\right)^2\psi = \left[i^2\hbar^2\frac{\partial^2}{\partial t^2} - i\hbar\frac{\partial}{\partial t}e\phi - e\phi\, i\hbar\frac{\partial}{\partial t} + (e\phi)^2\right]\psi$$

$$\left(i\hbar\frac{\partial}{\partial t} - e\phi\right)^2\psi = -\hbar^2\frac{\partial^2\psi}{\partial t^2} - i\hbar e\frac{\partial}{\partial t}\phi\psi - e\phi\, i\hbar\frac{\partial\psi}{\partial t} + e^2\phi^2\psi$$

$$\left(i\hbar\frac{\partial}{\partial t}-e\phi\right)^2\psi=-\hbar^2\frac{\partial^2\psi}{\partial t^2}-i\hbar e\left[\phi\frac{\partial\psi}{\partial t}+\psi\frac{\partial\phi}{\partial t}\right]-e\phi\, i\hbar\frac{\partial\psi}{\partial t}+e^2\phi^2\psi$$

$$=-\hbar^2\frac{\partial^2\psi}{\partial t^2}-2i\hbar e\phi\frac{\partial\psi}{\partial t}-i\hbar e\psi\frac{\partial\phi}{\partial t}+e^2\phi^2\psi \quad \ldots\ldots(3)$$

$$(-i\hbar c\nabla-eA)^2\psi=\left[-\hbar^2c^2\nabla^2+ie\hbar c\nabla A+2ie\hbar cA\nabla+e^2A^2\right]\psi \quad \ldots\ldots(4)$$

Put the values from eq^n(3) & (4) to eq^n(2)

$$\left(-\hbar^2\frac{\partial^2}{\partial t^2}-i\hbar e\phi\frac{\partial}{\partial t}-i\hbar e\frac{\partial\phi}{\partial t}-e\phi\, i\hbar\frac{\partial}{\partial t}+e^2\phi^2\right)\psi=$$

$$\left[-\hbar^2c^2\nabla^2+ie\hbar c\nabla A+2ie\hbar cA\nabla+e^2A^2+m^2c^4\right]\psi \quad \ldots\ldots(5)$$

Let us make substitution taking mc^2 as rest mass

$$\psi(\vec{r},t)=\psi'(\vec{r},t)e^{-imc^2t/\hbar}$$

$$\frac{\partial\psi}{\partial t}=\left(\frac{\partial\psi'}{\partial t}-\frac{imc^2\psi'}{\hbar}\right)e^{-imc^2t/\hbar}$$

$$\frac{\partial^2\psi}{\partial t^2}=\left[\frac{\partial^2\psi'}{\partial t^2}-\frac{2imc^2}{\hbar}\frac{\partial\psi'}{\partial t}-\frac{m^2c^4}{\hbar}\psi'\right]e^{-imc^2t/\hbar}$$

Put all the values in eq^n(5)

$$\left[-\hbar^2\frac{\partial^2\psi'}{\partial t^2}-\frac{2imc^2}{\hbar}\frac{\partial\psi'}{\partial t}+\frac{m^2c^4}{\hbar}\psi'-i\hbar e\frac{\partial\phi}{\partial t}\psi'-2i\hbar e\phi\frac{\partial\psi'}{\partial t}-2mc^2e\phi\psi'\right.$$

$$\left.+e^2\phi^2\psi'\right]e^{-imc^2t/\hbar}=\left(-\hbar^2c^2\nabla^2+ie\hbar c\nabla A+2ie\hbar cA\nabla+e^2A^2+m^2c^4\right)\psi'e^{-imc^2t/\hbar}$$

cancelling the common terms then devide by $2mc^2$

$$\frac{-\hbar^2}{2mc^2}\frac{\partial^2\psi'}{\partial t^2}+i\hbar\frac{\partial\psi'}{\partial t}-\frac{ie\hbar}{2mc^2}\frac{\partial\phi}{\partial t}\psi'-\frac{ic\hbar\phi}{2mc^2}\frac{\partial\psi'}{\partial t}-e\phi\psi'+\frac{e^2\phi}{2mc^2}\psi'$$

$$=\left[\frac{\hbar^2}{2m}\nabla^2+\frac{ie\hbar}{2mc}\nabla A+\frac{ie\hbar}{mc}A\cdot\nabla+\frac{e^2A^2}{2mc^2}\right]\psi'$$

Non-relativistic case

$mc^2 >>>> E$

$mc^2 >>>> e\phi$

$\frac{1}{mc^2}$ → Neglect this term

$$i\hbar\frac{\partial\psi'}{\partial t} = \left[\frac{-\hbar^2}{2m}\nabla^2 + \frac{ie\hbar}{2mc}\nabla A + \frac{ie\hbar}{mc}A\nabla + \frac{e^2A^2}{2mc^2} + e\phi\right]\psi'$$

This equation is non- relativistic schrodinger eq^n in Electromagnetic field.

#Dirac Relativistic Equation-

Dirac Equation is a relativistic wave equation derived by a British Physicist Dirac. It is consistent with both the principle of Quantum Mechanics and theory of relativity and was the first theory to account the special theory in context of Quantum Mechanics.

Dirac Formulated an eq^n to avoid the difficulty arrising in K-G Eq^n on the basis of Lorentz Invariance which demands that an eq^n which is linear in H and Hence in E must be linear in P.

K-G Eq^n is also the relativistic version of schrodinger eq^n, how-ever K-G eq^n is second order time derivative. K-G eq^n corresponds to quantum spinless particles. Whereas the Dirac eq^n is First order in time derivatives. Dirac eq^n also describes the quantum Half spin particles.

Dirac approaches the problem by finding a relativistic wave eq^n from

$$\hat{H}\psi = i\hbar\frac{\partial\psi}{\partial t} \quad \ldots\ldots\ldots (1)$$

Linearization of Hamiltonian, Dirac assumed that the Hamiltonian

$\hat{H}$ is linear in energy and momentum. So simplest linearization of Hamiltonian

$$\hat{H} = c\vec{\alpha}\cdot P + \beta mc^2 \quad \ldots\ldots\ldots\ldots (2)$$

where β and α are undetermined.

From eqn (1) & (2)

$$\left[c\vec{\alpha}\cdot P + \beta mc^2\right]\psi(\vec{r},t) = i\hbar\frac{\partial\psi}{\partial t}(\vec{r},t) \quad \ldots\ldots\ldots (3)$$

$$P = -i\hbar\nabla$$

$$\left[c\vec{\alpha}(-i\hbar\nabla) + \beta mc^2\right]\psi - i\hbar\frac{\partial\psi}{\partial t} = 0$$

$$i\hbar\frac{\partial\psi}{\partial t} + i\hbar c\alpha\nabla\psi - \beta mc^2\psi = 0 \quad \ldots\ldots\ldots\ldots (4)$$

Put $E \longrightarrow i\hbar\frac{\partial}{\partial t}$

$P = -i\hbar\nabla$ → Put

$$\left(E - c\alpha P - \beta mc^2\right)\psi = 0 \quad \ldots\ldots\ldots\ldots (5)$$

Operating above eqn by $(E + c\alpha\vec{P} + \beta mc^2)$ from left to right.

$$\Rightarrow (E + c\alpha\cdot\vec{P} + \beta mc^2)(E - c\vec{\alpha}P - \beta mc^2)\psi(\vec{r},t) = 0$$

$$\Rightarrow E^2\psi - Ec\vec{\alpha}\vec{P}\psi - \beta Emc^2\psi + Ec\alpha\vec{P}\psi - c^2(\vec{\alpha}\cdot P)^2\psi - c\alpha P\cdot\beta mc^2\psi$$
$$+ E\beta mc^2\psi - \beta mc^3\alpha P\psi - \beta^2 m^2c^4\psi = 0$$

$$\Rightarrow \left\{E^2 - c^2(\vec{\alpha}\cdot P)^2 - \beta^2m^2c^4 - mc^3(\alpha\cdot P)\beta - mc^3\beta\vec{\alpha}\cdot P\right\}\psi = 0 \quad \ldots\ldots (6)$$

$$\alpha = i\alpha_x + j\alpha_y + k\alpha_z$$

$$P = iP_x + jP_y + kP_z$$

$$\Rightarrow \left[E^2 - c^2(\alpha_x p_x + \alpha_y p_y + \alpha_z p_z)^2 - \beta^2 m^2 c^4 - mc^3(\alpha_x p_x + \alpha_y p_y + \alpha_z p_z)\beta - mc^3\beta(\alpha_x p_x + \alpha_y p_y + \alpha_z p_z)\right]\psi = 0 \quad \ldots\ldots\ldots\ldots (7)$$

$$\Rightarrow \left[E^2 - c^2(\alpha_x^2 p_x^2 + \alpha_y^2 p_y^2 + \alpha_z^2 p_z^2 + (\alpha_x\alpha_y + \alpha_y\alpha_x)p_x p_y + (\alpha_y\alpha_z + \alpha_z\alpha_y) p_y p_z + (\alpha_z\alpha_x + \alpha_x\alpha_y) p_z p_x\right] - \beta^2 m^2 c^4 - mc^3\left[(\alpha_x\beta + \beta\alpha_x)p_x + (\alpha_y\beta + \beta\alpha_y)p_y + (\alpha_z\beta + \beta\alpha_z)p_z\right] - mc^3\beta = 0 \quad \ldots\ldots\ldots (8)$$

$\left(i\hbar\frac{\partial}{\partial t}$ for E & $i\hbar\nabla$ for P$\right)$

$$\left[E^2 - c^2(p_x^2 + p_y^2 + p_z^2) - m^2c^4\right]\psi = 0 \quad \ldots\ldots\ldots (9)$$

Comparing eq^n(8) and (9)

$\Rightarrow \alpha_x^2 = \alpha_y^2 = \alpha_z^2 = \beta^2 = 1$

$\Rightarrow \alpha_x\alpha_y = \alpha_y\alpha_x = \alpha_y\alpha_z = \alpha_z\alpha_y = \alpha_z\alpha_x = \alpha_x\alpha_z = 0$

$\Rightarrow \alpha_x\beta + \beta\alpha_x = 0$

$\Rightarrow \alpha_y\beta + \beta\alpha_y = 0$

$\Rightarrow \alpha_z\beta + \beta\alpha_z = 0$

These four quantities $\alpha_x, \alpha_y, \alpha_z, \beta$ have following properties.

1. Their square roots are unity.
2. They commute with one another in pair. Since α, β anticommute rather than commute with each other, they can't be numbers, moreover the quantities of this type can be expressed in terms of Matrices & It is convinient to find matrix representation of them.

Now the square of all four matrices are unity so their eigenvalues are +1 and −1.

Let us arbitrary choose β as the matrix that is to be diagonalise.

$$\beta = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\beta = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Also we have already three well known matrices called Pauli Spin Matrices which satisfy above properties.

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\sigma_x = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\sigma_y = \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{bmatrix}$$

$\sigma_z = ?$

If the dirac operators are to involve in 4X4 matrices, then the dirac operators must have four components.

$$\psi = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{bmatrix} \Rightarrow H\psi = i\hbar \frac{\partial \psi}{\partial t}$$

$$\Rightarrow \frac{\partial \psi}{\partial t} = \frac{\partial}{\partial t} \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{bmatrix}$$

$$\Rightarrow \frac{\partial}{\partial t}\begin{bmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \psi_4\end{bmatrix} = \begin{bmatrix}\frac{\partial \psi_1}{\partial t}\\ \frac{\partial \psi_2}{\partial t}\\ \frac{\partial \psi_3}{\partial t}\\ \frac{\partial \psi_4}{\partial t}\end{bmatrix}$$

$$\boxed{H\psi = i\hbar \begin{bmatrix}\partial\psi_1/\partial t\\ \partial\psi_2/\partial t\\ \partial\psi_3/\partial t\\ \partial\psi_4/\partial t\end{bmatrix}}$$

This is linearization of Dirac or Hamiltonian. This is also schrodinger eqn in matrics form.

Negative Energy State-

We have Dirac Equation in Electromagnetic field as:

$$\left[E - e\phi - \vec{\alpha}(cP - eA) - \beta mc^2\right]\psi(\vec{r},t) = 0 \quad \ldots\ldots\ldots (1)$$

This eqn may be written as four equivalent eqn's, two corresponding to positive energy and remaining two corresponding to negative energy state of a particle.

According to classical Mechanics, negative energy state is not considered but according to Quantum Mechanics, it is considered because energy change take place discontinuously and the transition from positive to negative energy state and vice versa are possible.

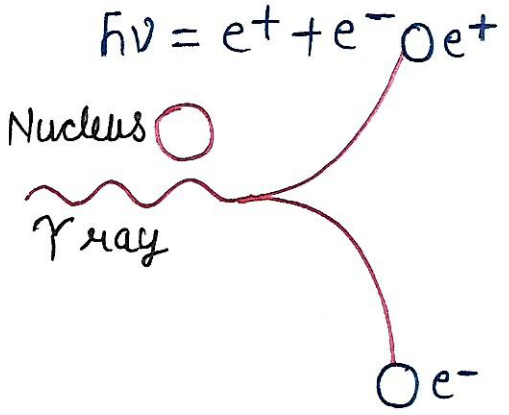
$$E = i\hbar\frac{\partial}{\partial t} \text{ and } P = -i\hbar\nabla$$

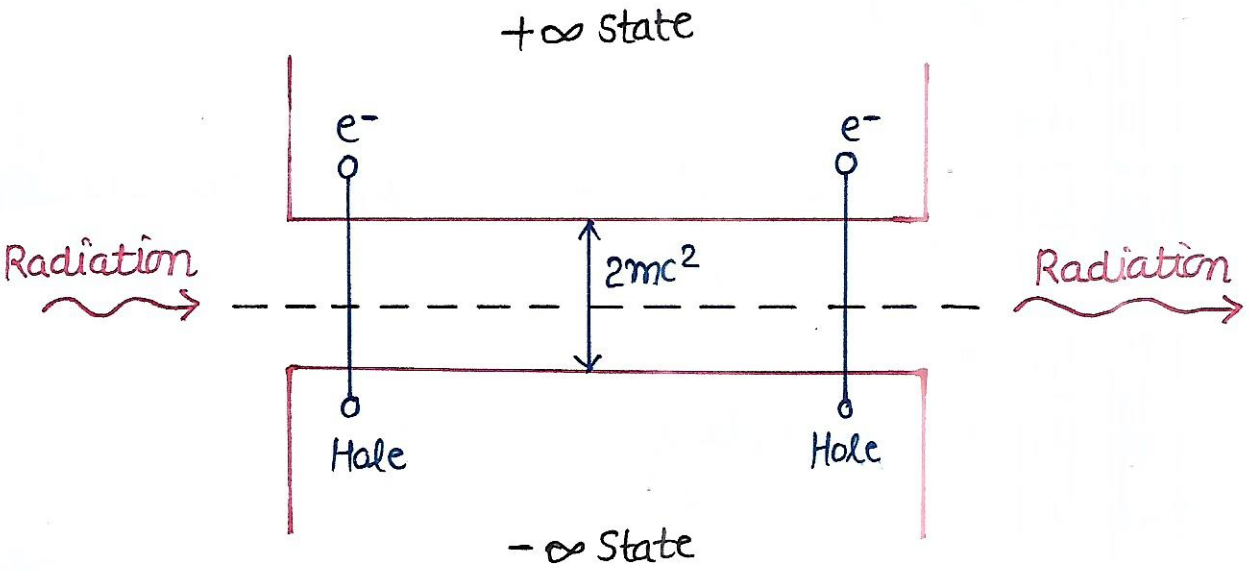
We take complex conjugate of eqn(1)

$$\left[E + e\phi - \vec{\alpha}(cP + eA) - \beta mc^2\right]\psi^* = 0 \quad \ldots\ldots\ldots\ldots (2)$$

The negative energy solutions of eqn(1) refers to the motion of a new kind of particle having the mass of an e^- and opposite charge. Such particles have been observed experimentally and called positron.

Dirac picture of +ve and -ve energy states of an e^-.





#Dirac proposed that nearly all the negative energy states are occupied with 1 e^- in each state according to Pauli Exclusion principle and there are no positive energy e^-.

#The energy diff. b/w +ve and -ve energy states is separated by a gap = $2mc^2$

#Unoccupied (-ve) energy states are called holes or positrons.

#Thus A/c to the Hole theory, to each particle, there is an Anti-particle. So the existance of e^- implies the existance of e^+.

UNIT-4

Quantization of Wave(like) Fields

#Quantization of Non-relativistic schrodinger Equation-

The theory of quantum mechanics has dealt with system that, in the classical limit, consist of material particles. We wish to extend the theory so that it can be applied to the electromagnetic field and thus provided a consistent basis for the quantum theory of radiation. The quantization of a wave field imparts to it some particle properties; in the case of electromagnetic field, a case of light quanta results.

The Field quantaization technique can also be applied to a ψ field, such as that described by the non relativistic schrodinger eqn...

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V(x,t)\psi = i\hbar\frac{\partial\psi}{\partial t} \quad \ldots\ldots\ldots (1)$$

→ Why schrodinger go through with a non-relativistic quantum Mechanics when special theory of relativity was published.

or by one of the relativistic equation

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} = -\hbar^2c^2\nabla^2\psi + m^2c^4\psi$$

As well, we shall see in the non relativistic case, it then converts a one-particle theory into a many-particle theory. It might seem that the quantization of ψ field merely provides another formal approach to the many-particle problem.

As a first example of the application of the field quantization technique developed in the proceding section, we consider here the quantization of the non-relativistic schrodinger Equation (1).

Two Approach

Classical Theory
↓
Motion of some kind of material fluid.

Quantum Theory
↓
Resulting quantized Field Theory
↓
Many Particle schrodinger Eq^n

Second Quantization

This term implies that the transition from classical particle mechanics to eq^n

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$

constitutes the first quantization.

→ CLASSICAL LAGRANGIAN AND HAMILTONIAN EQUATION-

The Lagrangian density

$$L = i\hbar\psi^*\psi - \frac{\hbar^2}{2m}\nabla\psi^*\cdot\nabla\psi - V\psi^*\psi$$

ψ and ψ^* can be varied seperately to obtain the Lagrangian eq^n of motion.

$$-i\hbar\psi^* = -\frac{\hbar^2}{2m}\nabla^2\psi^* + V\psi^*$$

And

$$i\hbar\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi$$

The momentum canonically conjugate to ψ is

$$i\hbar\psi^* = \frac{\partial L}{\partial\psi} = \pi$$

However ψ^* does not appear in the Lagrangian density, so that $\tilde{\pi}$ is identically zero. It is therefore impossible to satisfy the commutation relation.
Then they can easily be eliminated from the Hamiltonian since π^* never appears and $\pi = \frac{\partial L}{\partial \dot{\psi}} = i\hbar\psi^*$ gives ψ^* in terms of π^{-1}.

The Hamiltonian density is

$$H = \pi\dot{\psi} - L = -\frac{i\hbar}{2m}\nabla\psi\cdot\nabla\psi - i\hbar V\pi\psi$$

$$\downarrow \pi$$

$$H = \pi\dot{\psi} - L = -\frac{i\hbar}{2m}\nabla\psi\cdot\nabla\pi - i\hbar V\pi\psi$$

Then Hamiltonian equations of motion obtained from $\dot{\psi} = \frac{\partial H}{\partial \pi}$, $\dot{\pi} = -\frac{\partial H}{\partial \psi}$,

with the help

$$\dot{\psi} = -\frac{i}{\hbar}V\psi + \frac{i\hbar}{2m}\nabla^2\psi$$

$$\dot{\pi} = \frac{i}{\hbar}V\psi - \frac{i\hbar}{2m}\nabla^2\pi$$

Thus this shown the classical field Theory.

→ QUANTUM EQUATIONS-

The quantum equations are obtained by adopting the volume integral of $H = \pi\dot{\psi} - L$ as the Hamiltonian, $\frac{dF}{dt}$ as the equation of motion, and the first of eqn as the quantum condition of the wave field.

The Hamiltonian is conveniently written with replacemnt of π by $i\hbar\psi^\dagger$ and becomes

$$H = \int \left(\frac{\hbar^2}{2m}\nabla\psi^\dagger\cdot\nabla\psi + V\psi^\dagger\psi\right) d^3r$$

H is Hermitian.

The quantized hamiltonian gives in H is the operator that represents the total energy of the field. We has as yet given no explicit representation for the new operator ψ and H.

The physical interpretation of the formalism –

$$[\psi(x),\psi(x')] = [\psi^\dagger(x),\psi^\dagger(x')] = 0$$

$$[\psi(x),\psi^\dagger(x')] = \delta^3(x-x')$$

The equation of motion for ψ is

$$i\hbar\dot{\psi} = [\psi, H]$$

$$i\hbar\dot{\psi} = \left[\psi, \int \frac{\hbar^2}{2m}\nabla'\psi^{\dagger\prime}, \nabla'\psi' d^3x'\right] + \left[\psi, \int V'\psi^{\dagger\prime}\psi' d^3x'\right]$$

Evaluation of $\left[\psi, \int V'\psi^{\dagger\prime}\psi' d^3x'\right]$ with the help of above

$$\int V'(\psi\psi^{\dagger\prime}\psi' - \psi^{\dagger\prime}\psi'\psi) d^3x' = \int V'(\psi\psi^{\dagger\prime} - \psi^{\dagger\prime}\psi)\psi' d^3x'$$

$$= \int V'\psi'(\delta^3(x-x')) d^3x'$$

$$\left[\psi, \int \nabla'\psi^{\dagger\prime}.\nabla'\psi' d^3x'\right] = -\left[\psi, \int \psi^{\dagger\prime}\nabla'^2\psi' d^3x'\right]$$

$$= -\int [\psi\psi^{\dagger\prime}]\nabla'^2\psi' d^3x'$$

$$= -\int (\nabla'^2\psi')\delta^3(x-x') d^3x' = -\nabla^2\psi$$

Another operator

$$N = \int \psi^\dagger\psi d^3x$$

→ N is Hermitian and its time derivative.

$$i\hbar\dot{N} = [N, H]$$

$$= \left[\int \psi^\dagger\psi d^3x, \int \left(\frac{\hbar^2}{2m}\nabla'\psi^{\dagger\prime}.\nabla'\psi' + V\psi^{\dagger\prime}\psi'\right) d^3x\right]$$

The commutator of N with the V part of H can be written as

$$\iint V'\underbrace{(\psi^\dagger\psi\psi^{\dagger\prime}\psi' - \psi^{\dagger\prime}\psi'\psi^\dagger\psi)}_{\Downarrow\ 0}\, d^3r\, d^3r'$$

$$= \psi^\dagger[\psi^{\dagger\prime}\psi + \delta^3(r-r')]\psi' - \psi^{\dagger\prime}\psi'\psi^\dagger\psi$$
$$= \psi^{\dagger\prime}\psi^\dagger\psi'\psi + \psi^\dagger\psi'\delta^3(r-r') - \psi^{\dagger\prime}\psi'\psi^\dagger\psi$$
$$= \psi^{\dagger\prime}[\psi'\psi^\dagger - \delta^3(r-r')]\psi + \psi^\dagger\psi'\delta^3(r-r') - \psi^{\dagger\prime}\psi'\psi^\dagger\psi$$
$$= 0$$

A similar but slightly more complicated calculation shows that

$$[\psi^\dagger\psi, \nabla'\psi^{\dagger\prime}\cdot\nabla'\psi'] = [\psi^\dagger\nabla'\psi' - (\nabla'\psi^{\dagger\prime})\psi]\cdot\nabla'\delta^3(r-r')$$

The double integral of this over r and r' is zero. Thus [N,H] shows that N is a constant of motion. so that these equations are always valid if they are at particular time.

Second Quantization -

Consider a one-particle system characterized by the orthnormal set of basis vector $\phi_i \Rightarrow (i = 1, 2, \ldots N)$.
Then the state of a system is given by

$$\psi = \sum_i a_i \phi_i \quad \ldots\ldots\ldots\ldots (1)$$

The co-efficients a_i satisfy the normalization condition.

$$\sum_i a_i^* a_i = 1 \quad \ldots\ldots\ldots (2)$$

↑ simple number

The formation which treats the cofficients a_i's as operators is called the second Quantization.
The name second Quantization is justified since the assumption of a_i as operator implies the quantization of probability Amplitude along with the observables of the system. The basic vectors may be disorete or continuous to describe the quantized or continum state respectively.

Let first review the Hamiltonian formulation in brief. If a system is described by generalized coordinates q_K and generalized momentum P_K and time t [i.e. $(2n+1)$] independent variable in phase space, then Hamiltonians are :-

$$\dot{q}_K = \frac{\partial H}{\partial P_K}$$

$$\dot{P}_K = -\frac{\partial H}{\partial q_K} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

And classical Eq^n of motion of function F in Poission's Bracket notation is :-

$$\frac{dF}{dt} = [F,H]_{q,p} + \frac{\partial F}{\partial t} \quad \ldots\ldots\ldots (4)$$

Eq^n(4) immediately gives eq^n(3) if F is replaced by q_K and P_K respectively. The quantum mechanical description of system is obtained by replacing its variables q_K and P_K by operators. Then the poission brackets are replaced by commutators s.t.

$$[F,H] \longrightarrow \frac{1}{i\hbar}[F,H] \quad \ldots\ldots\ldots (5)$$

Further the diff. states of system are described by state vectors of Hilbert Space and the time dependent state vector $\psi(\vec{r},t)$ can be expressed by time depending cofficients $a_i(t)$ i.e.

$$\psi(\vec{r},t) = \sum_i a_i(t)\,\phi_i(r)$$

When the basic vectors $\phi_i(r)$ are time dependent, thus the schrodinger eq^n,

$$E\psi = H\psi$$

$$i\hbar\frac{\partial \psi}{\partial t} = H\psi$$

becomes an eqn describing the time derivatives of the cofficients $a_i(t)$ as follows:

$$i\hbar \frac{\partial a_i}{\partial t} = \sum_i H_{ij} a_j \quad \ldots\ldots\ldots\ldots (6)$$

H_{ij} being the matrix element of H in the basis ϕ_i's.
Now, in order to quantize the cofficients a_i's, the eqn of motion (6) must be quantized for them.

ELECTRONICS-A

CONTENT

1. Number System, Boolean Algebra & Gates
2. Logic gates & Combinational Circuits
3. Sequential Circuit
4. Memory Devices & IC-Technology

#De-Morgan's Theorem-

There is one particular theorem of Boolean Algebra that is so useful that we are going to discuss. The theorem is due to English mathematician and logician, Augustus DeMorgan who was professor.

De-Morgan's Theorem allows a function to be complemented and therefore appears in two forms which are complements of each other.

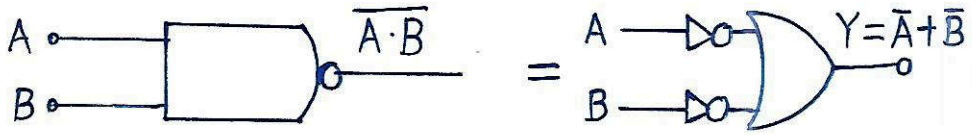
In the formalism of Boolean Algebra,

$$\overline{A+B} = \overline{A \cdot B} \approx \bar{A} \cdot \bar{B}$$

$$\bar{A}+\bar{B} = \overline{A \cdot B}$$

- Law 1-

$$\overline{A \cdot B} = \bar{A} + \bar{B}$$

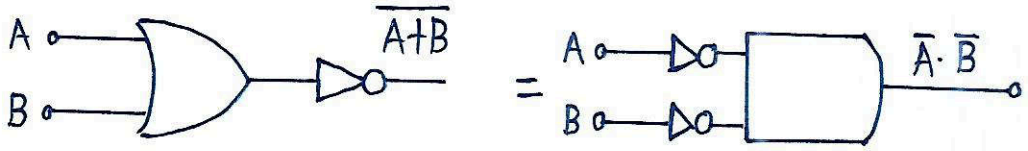


It shows that the NAND gate is equivalent to a bubbled OR gate. This is shown in truth table.

A	B	$\overline{A \cdot B}$	$\bar{A}$	$\bar{B}$	$\bar{A}+\bar{B}$
0	0	1	1	1	1
0	1	1	1	0	1
1	0	1	0	1	1
1	1	0	0	0	0

- Law 2-

$$\overline{A+B} = \bar{A} \cdot \bar{B}$$



A	B	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$
0	0	1	1	1	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	0

Hence the truth tables are verified.

Boolean Postulates and Laws-

The Basic Postulates of Boolean Algebra are:

- Postulates 1. Commutation operations

$$A+B = B+A$$
$$A\cdot B = B\cdot A$$

- Postulate 2. Distribution operations

$$A+(B\cdot C) = (A+B)\cdot(A+C)$$
$$A\cdot(B+C) = (A\cdot B)+(A\cdot C)$$

- Postulate 3. Identity Element Exist

$$A+0 = A$$
$$A\cdot 1 = A$$

- Postulate 4. Complements Exist

$$A+\overline{A} = 1$$
$$A\cdot\overline{A} = 0$$

While these are the postulates of Boolean Algebra and form the starting point for the subject, the reasonableness of these postulates can be illustrated by enumeration. For instance if we take postulate P_4, $A+\overline{A}=1$, the possible value of A are 0 or 1 so we can prepare a truth table which enumeration all possiblities and demonstriate the postulate.

A	$\overline{A}$	$A+\overline{A}$
0	1	1
1	0	1

Similarly other postulates can be illustrated.

And some useful theorems are;

T_1 :	$A+A=A$	$A\cdot A=A$
T_2 :	$A+1=1$	$A\cdot 0=0$
T_3 :	$A+A\cdot B=A$	$A\cdot(A+B)=A$
T_4 :	$A+\bar{A}\cdot B=A+B$	$A\cdot(\bar{A}+B)=A\cdot B$

Let us now prove some of these theorems from postulates.

$$\begin{aligned} A+A &= (A+A)\cdot 1 && \text{by } P_3 \\ &= (A+A)(A+\bar{A}) && \text{by } P_4 \\ &= A+A\bar{A} && \text{by } P_2 \\ &= A+0 && \text{by } P_4 \\ &= A && \text{by } P_3 \end{aligned}$$

Let us proof the second form of T3,

$$\begin{aligned} A\cdot(A+B) &= A \\ &= (A+0)(A+B) && \text{by } P_3 \\ &= A+0\cdot B && \text{by } P_2 \\ &= A+0 && \text{by } T_2 \\ &= A && \text{by } P_3 \end{aligned}$$

Boolean Expression and Function-

Boolean algebra deals with binary variables and logic operation. A Boolean Function is described by an algebraic expression called Boolean expression which consists of binary variables, the constant 0 and 1, and the logic binary symbols.

e.g.

$F(A,B,C,D) = A+B\bar{C}+ADC$

- $F(A,B,C,D)$ → Boolean Function
- $A+B\bar{C}+ADC$ → Boolean Expression
- F → This represents output Y.

$$Y = A+B\bar{C}+ADC$$

- Truth Table Formation-

A truth table represents a table having all combinations of inputs and their corresponding result.
It is possible to convert the switching eqn into a truth table.
e.g.

$F(A,B,C) = A+BC$

Input → Output

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

- Methods to simplyfy the boolean Function
  - K-Map.
  - NAND Gate method.

Karnaugh Maps (K-Map) and Minimization-

The Boolean theorems and the De-Morgan Theorems are useful in manipulating the logic expression. We can realize the logical expression using gates. The number of logic gates required for the realization of a logical expression should be reduced to a minimum possible value of K-map method. The method can be done in two different ways, as discussed below.

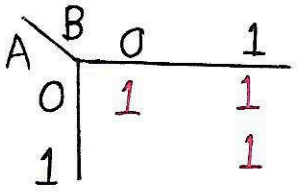
- Sum of products (SOP)-

The sum of products, in the form of three terms AB, AC, BC with each individual term is a product of two variables, say A·B or A·C etc. Therefore such expressions are known as expression in SOP form. The sum and products in SOP form are not the actual addition or multiplication. In fact they are the OR And AND functions. In SOP form, 0 represents a bar and 1 represents an unbar SOP form is represented by

Σ

e.g.
In SOP form

$\bar{A}\bar{B} + \bar{A}B + A\bar{B}$
↓↓ ↓↓ ↓↓
00 01 11



$\sum m(0,1,3)$

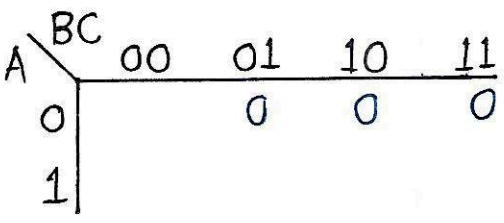
Answer: $\bar{A}\bar{B} + \bar{A}B + AB = \bar{A} + B$

- Products of sums (POS Form)-
  It is in the form of product of three terms $A\bar{B}+B$, $B\bar{C}+C$ or $A\bar{C}+C$ with each term is in the form of a sum of two variables. Such expressions are said to be in the product of sum POS form. In POS form, 0 represents an unbar and 1 represents a bar, POS form is represented by,

$\prod$

e.g.
In POS form

$(B+\bar{C})(\bar{A}+\bar{B})(\bar{B}+C)$
↓ ↓ ↓ ↓ ↓ ↓
0 1 1 1 1 0



$\prod m(1,3,2)$

Answer: $(A+\bar{C})(A+\bar{B})$

Logic Gates -

1. AND Gates
2. OR Gates
3. NOT Gates

→ Basic gates

4. NAND Gates
5. NOR Gates

→ Universal gates

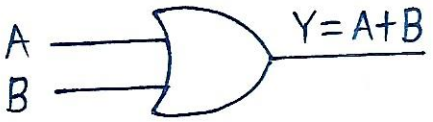
6. XOR Gates
7. XNOR Gates

→ Non-Universal gates / Exclusive gates

A logic gate is an electronic or digital circuit which gives simple output for more than one input. The output of these circuits can be obtained for different combination of input signals.
There are just three basic gates - AND, OR and NOT. Inputs and outputs of logic gates can occur only in two levels. These two levels are termed High and Low or True and False or ON and OFF or simply 1 and 0 and 1.
A table which lists all the possible combination of input variables and the corresponding outputs are called a truth table.

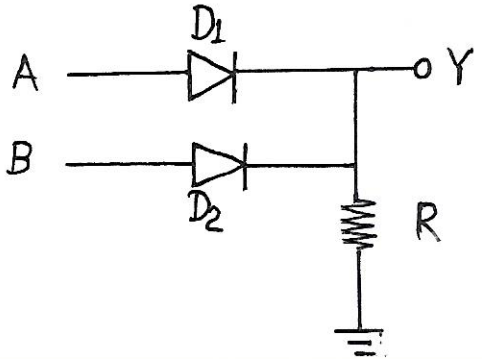
1. OR Gate -

An OR gate may have two or more inputs but only one output.



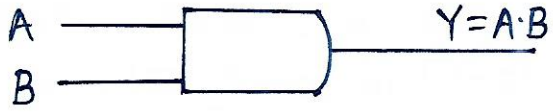
A	B	Y=A+B
0	0	0
0	1	1
1	0	1
1	1	1

If OR gate have two inputs as shown in figure then it gives output for applied two inputs.



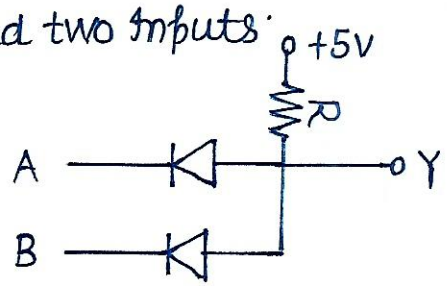
2. AND Gate-

An AND gate has two or more inputs but only one output.



A	B	Y=A·B
0	0	0
0	1	0
1	0	0
1	1	1

If AND gate has two input as shown in figure then it gives output for applied two inputs.



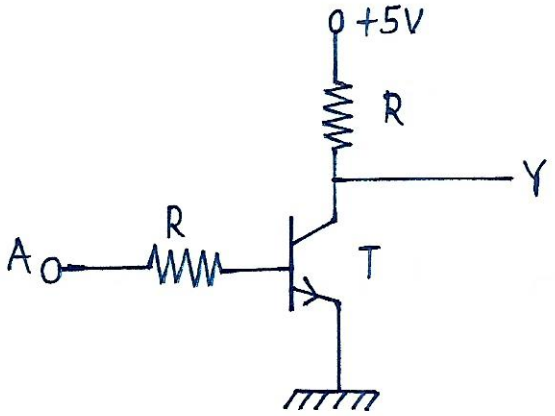
3. NOT Gate-

A NOT gate, also called an inverter, has only one input and of course only one output. It is a device whose output is always the complement of its input.



A	Y=Ā
0	1
1	0

It is also called inverse gate. It gives only one output for one applied input.

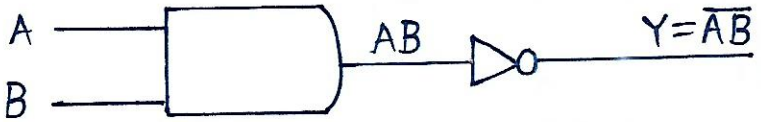


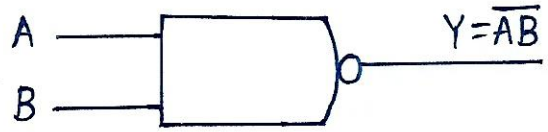
• The Universal Gates-

Through logic circuits of any complexity can be realized by using only three basic gates (AND, OR and NOT), there are two universal gates (NAND and NOR), each of which can also realize logic circuits single-handedly. The NAND and NOR gates are therefore, called universal building blocks.

4. NAND gate-

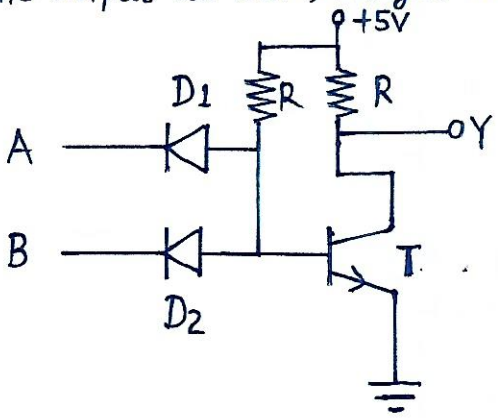
A NAND gate is a combination of an AND gate and a NOT gate.





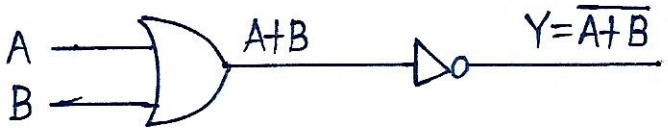
A	B	AB	$\overline{AB}$	Y
0	0	0	1	1
0	1	0	1	1
1	0	0	1	1
1	1	1	0	0

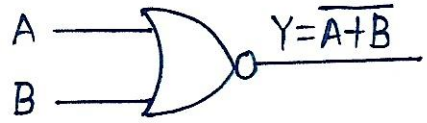
If any of the input is low, we get the high output.



5. NOR gate-

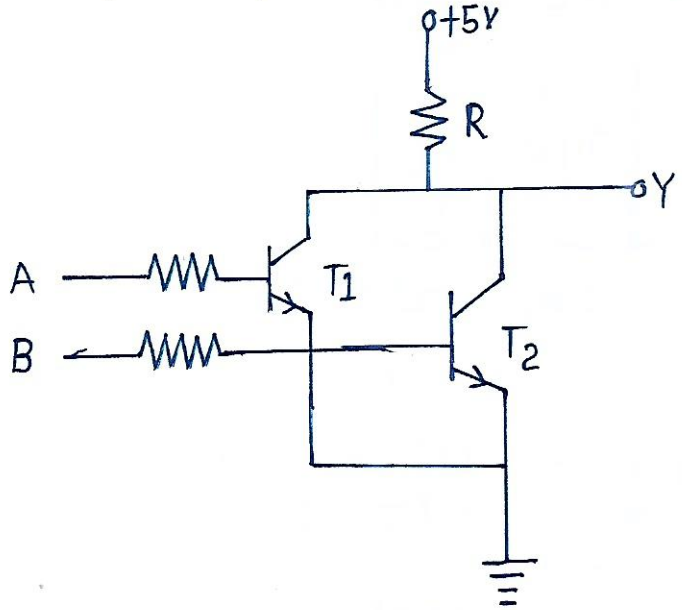
A NOR gate is a combinational circuit of a OR gate and a NOT gate.





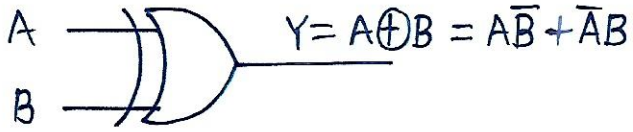
A	B	A+B	Y=$\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

When both of the inputs are low then we get the high output.



6. XOR gate-

An XOR gate is a two input, one output logic circuits whose output assumes a logic 1 state when one and only one of its two inputs assumes a logic 1 state. Under the state when both the inputs assumes the logic 0 state, or when both the inputs assumes the logic state one(1), the output assumes a logic 0 state.

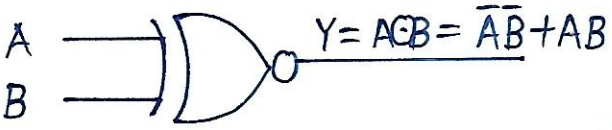


A	B	$\bar{A}$	$\bar{B}$	$\bar{A}B$	$A\bar{B}$	$Y=\bar{A}B+A\bar{B}$
0	0	1	1	0	0	0
0	1	1	0	1	0	1
1	0	0	1	0	1	1
1	1	0	0	0	0	0

If both input are different, we get high output.

7. XNOR gate-

An XNOR gate is a combination of an X-OR gate and a NOT gate.



A	B	$\bar{A}$	$\bar{B}$	$\bar{A}\bar{B}$	AB	Y
0	0	1	1	1	0	1
0	1	1	0	0	0	0
1	0	0	1	0	0	0
1	1	0	0	0	1	1

If both inputs are same, we get high output.

TTL Logic-

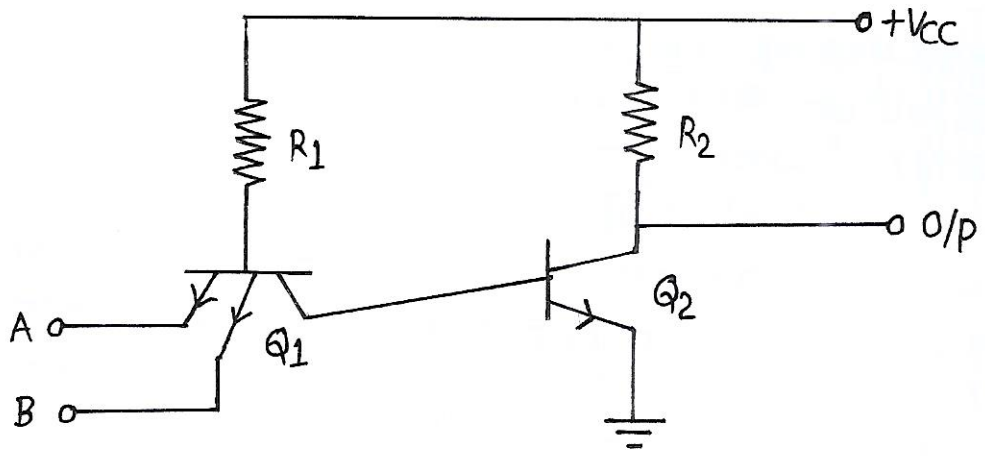
The TTL or T^2L family is so named because of its dependance on transistors alone to perform basic logic operation.

TTL was introduced in the early 1970.

It is a saturated logic. It is the most widely used circuit line since 1970s because of its speed, good fan-in and fan-out and easy interface with other digital circuitry. The unique feature of this circuit is that it uses multiple-emitter transistor at the input which replaces the input diode of the DTL. The no. of emitter is equal to the desired fan-in of the circuit. Since the multi-emitter transistor is smaller in area (that) than the diode it replaces, the yield from a wafer is increased.

- Basic circuit-

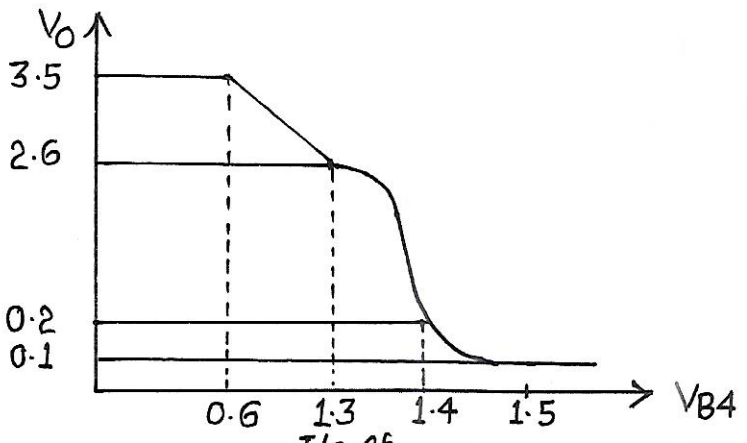
The basic circuit of the TTL family is the NAND gate cell shown in figure However at present, NOR, OR and AND gate configuration have been added to the series.



- Circuit Operation-
  1. If both Input A and B are high, E/B junction of Q_1 is reversed biased so that it has no emitter current. Hence Q_1 is OFF. However, its C/B junction is forward biased supplying base current to Q_2 from V_{CC} via R_1. As a result, transistor Q_2 is turned fully ON (i.e. it becomes saturated) driving point N to 0V. Hence, output is at logic 0.
  2. When either or both inputs are at (level 0) 0V, the associated E/B junction becomes forward baised. The value of R_1 is so selected as to ensure that Q_1 is turned fully ON. The voltage at point M falls to 0V with the result that base current for Q_2 is reduced to zero. Hence Q_2 is cut OFF driving point N and the output to logic 1.

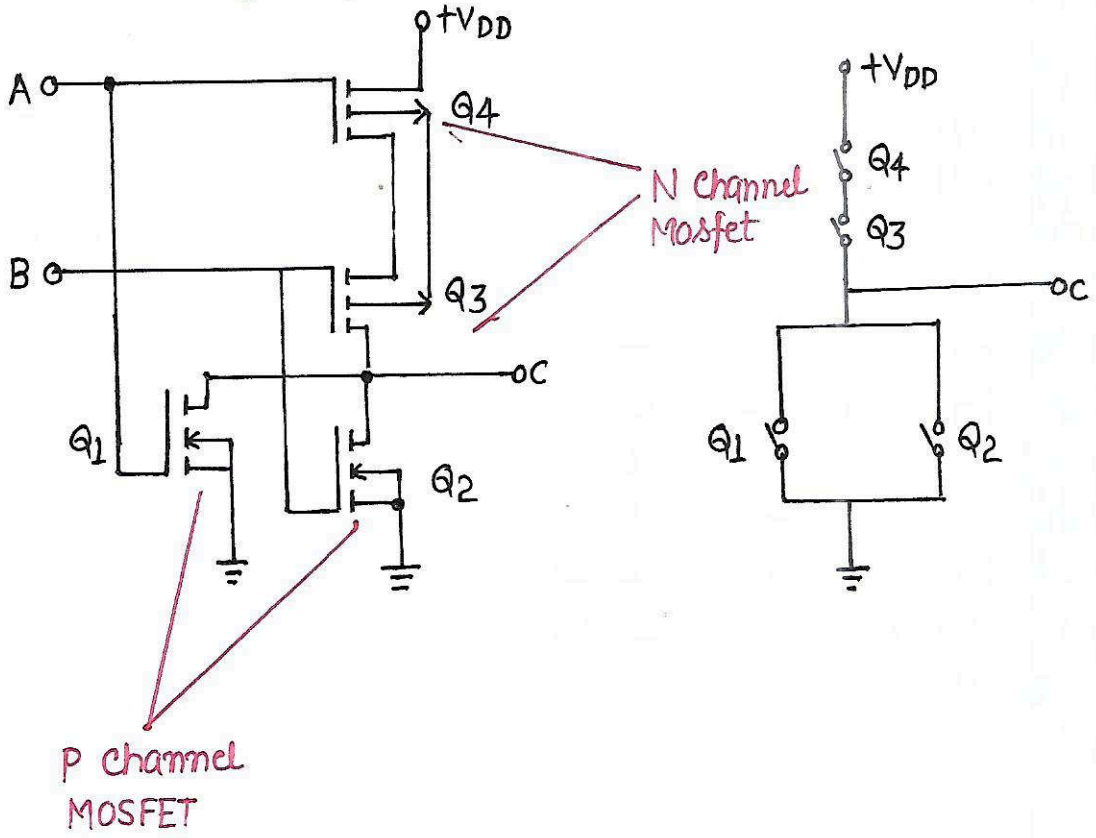
- Advantages-
  1. Greater speed than DTL.
  2. less noise immunity (0.4V)
  3. Avarage propagation delay per gate of 9ns.
  4. Avarage power dissipation of 10mW.
  5. A fan out of 10.
  6. A fan in of 6.

- Characteristics-



#CMOS Logic Circuit-

CMOS refers to Complementry Metal Oxide Semiconductor Logic circuits. CMOS logic circuits both use PMOS and NMOS devices in the same circuit. It gives the advantage of drastic decrease in power dissipation (12 nW per gate) and increase in speed of operation. In fact, it has the lowest power dissipation amongest different logic families. It has very high packing density i.e. large number of circuits can be placed in a single chip. As a result, it is extensively used in VLSI circuits such as one-chip computers and memory systems.



- Circuit Operation-

1. Let A = B = logic 1 i.e. have +ve Voltage.

Q1, Q2 } ON

Q3, Q4 } OFF

Hence output C is logic 0.

2. If either A or B at logic 1

Q1, Q2 } ON

Q3, Q4 } OFF

C is logic 0.

3. When A and B are 0.

Q_3, Q_4 } ON

Q_2, Q_1 } OFF

Then C would be at logic 1.

A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

Parity-Checker-

It is a logic circuit that checks for possible error in the data transmission. This circuit can be an even parity checker or odd parity checker depending on the type of parity generated at the transmission end.

- Even Parity Checker-

A	B	C	D	O/P	Decimal
0	0	0	0	0	0
0	0	0	1	1	1
0	0	1	0	1	2
0	0	1	1	0	3
0	1	0	0	1	4
0	1	0	1	0	5
0	1	1	0	0	6
0	1	1	1	1	7
1	0	0	0	1	8
1	0	0	1	0	9
1	0	1	0	0	10
1	0	1	1	1	11
1	1	0	0	0	12
1	1	0	1	1	13
1	1	1	0	1	14
1	1	1	1	0	15

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD
$\bar{A}\bar{B}$	0	[1] 1	3	[1] 2
$\bar{A}B$	[1] 4	5	[1] 7	6
$A\bar{B}$	12	[1] 13	15	[1] 14
AB	[1] 8	9	[1] 11	10

$PEC = M_1 + M_2 + M_4 + M_7 + M_8 + M_{11} + M_{13} + M_{14}$

$PEC = \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}C\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}BCD + A\bar{B}\bar{C}\bar{D} + A\bar{B}CD + AB\bar{C}D + ABC\bar{D}$

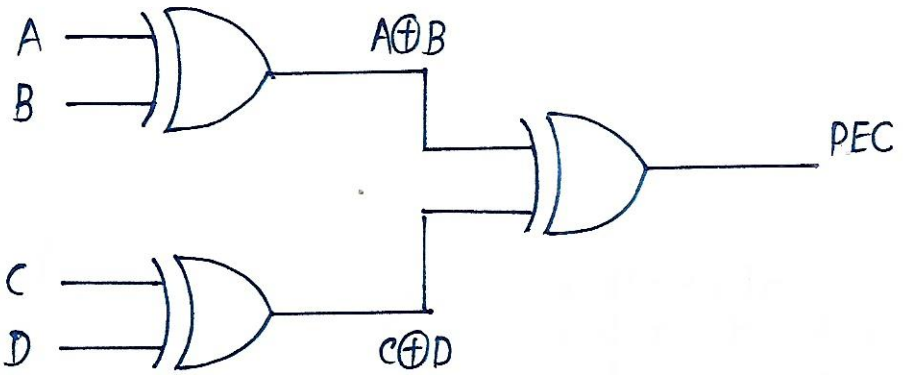
$PEC = \bar{A}\bar{B}(\bar{C}D + C\bar{D}) + \bar{A}B(\bar{C}\bar{D} + CD) + A\bar{B}(\bar{C}\bar{D} + CD) + AB(\bar{C}D + C\bar{D})$

$PEC = \bar{A}\bar{B}(C \oplus D) + \bar{A}B(\overline{C \oplus D}) + A\bar{B}(\overline{C \oplus D}) + AB(C \oplus D)$

$PEC = [(C \oplus D)(\bar{A}\bar{B} + AB) + \overline{C \oplus D}(\bar{A}B + A\bar{B})]$

$PEC = (C \oplus D)(\overline{A \oplus B}) + (\overline{C \oplus D})(A \oplus B)$

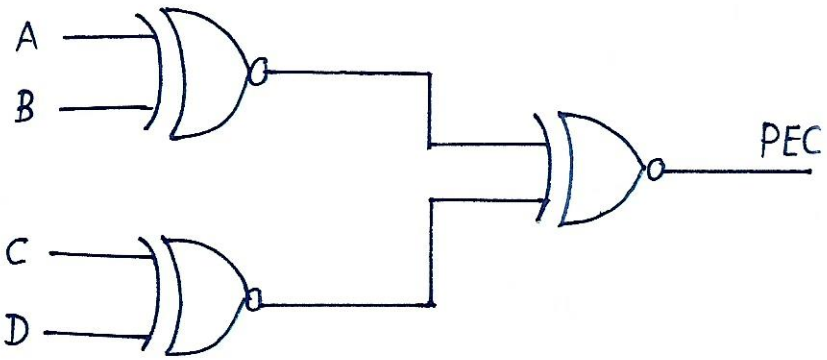
$\boxed{PEC = (C \oplus D) \oplus (A \oplus B)}$



• Odd Parity Checker-

A	B	C	D	O/P	Decimal
0	0	0	0	1	0
0	0	0	1	0	1
0	0	1	0	0	2
0	0	1	1	1	3
0	1	0	0	0	4
0	1	0	1	1	5
0	1	1	0	1	6
0	1	1	1	0	7
1	0	0	0	0	8
1	0	0	1	1	9
1	0	1	0	1	10
1	0	1	1	0	11
1	1	0	0	1	12
1	1	0	1	0	13
1	1	1	0	0	14
1	1	1	1	1	15

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	[1] 0	1	[1] 3	2
$\bar{A}B$	4	[1] 5	7	[1] 6
AB	[1] 12	13	[1] 15	14
$A\bar{B}$	8	[1] 9	11	[1] 10



$(A \odot B) \odot (C \odot D)$

or

$\overline{(A \oplus B) \oplus (C \oplus D)}$

#Multiplexer-

Multiplexering means sharing. A Sharing occur when many devices share a single line to communicate with a computer.

A multiplexer(Mux) is also known as data selector. It is a logic circuit that accepts several data inputs and allows one at a time to output.

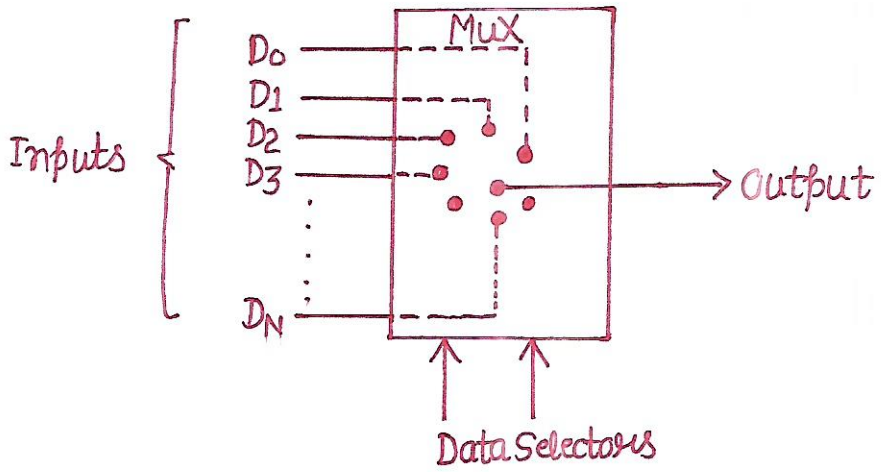
Normally,

Input lines $\rightarrow 2^n$

Select lines $\rightarrow n$

↓

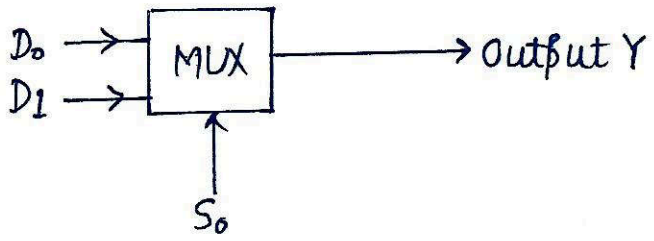
It determines which Input is selected.



- Types -
  (1) 2:1
  (2) 4:1
  (3) 8:1
  (4) 16:1

(1) 2:1

Functional diagram

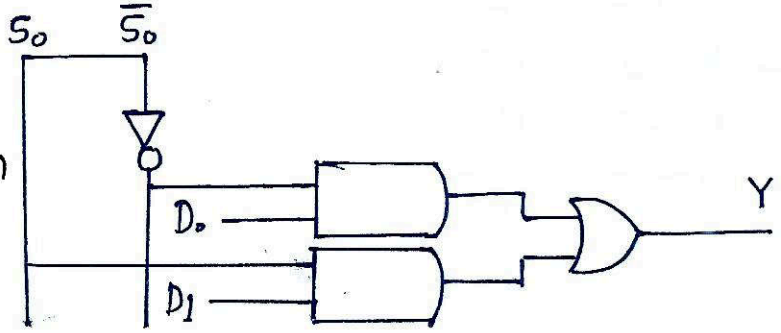


Function Table

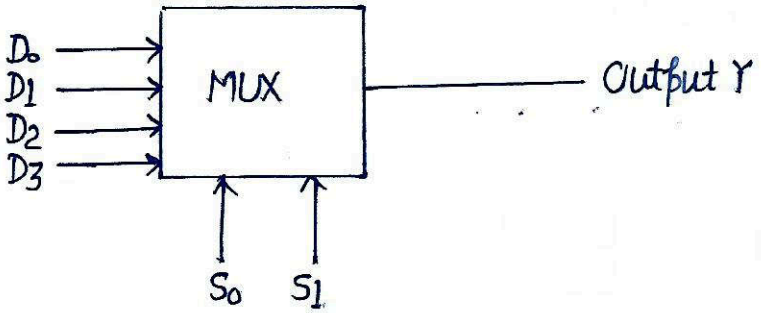
S_0	Y
0	D_0
1	D_1

Output $\boxed{Y = D_0\bar{S}_0 + D_1S_0}$

Logic Diagram

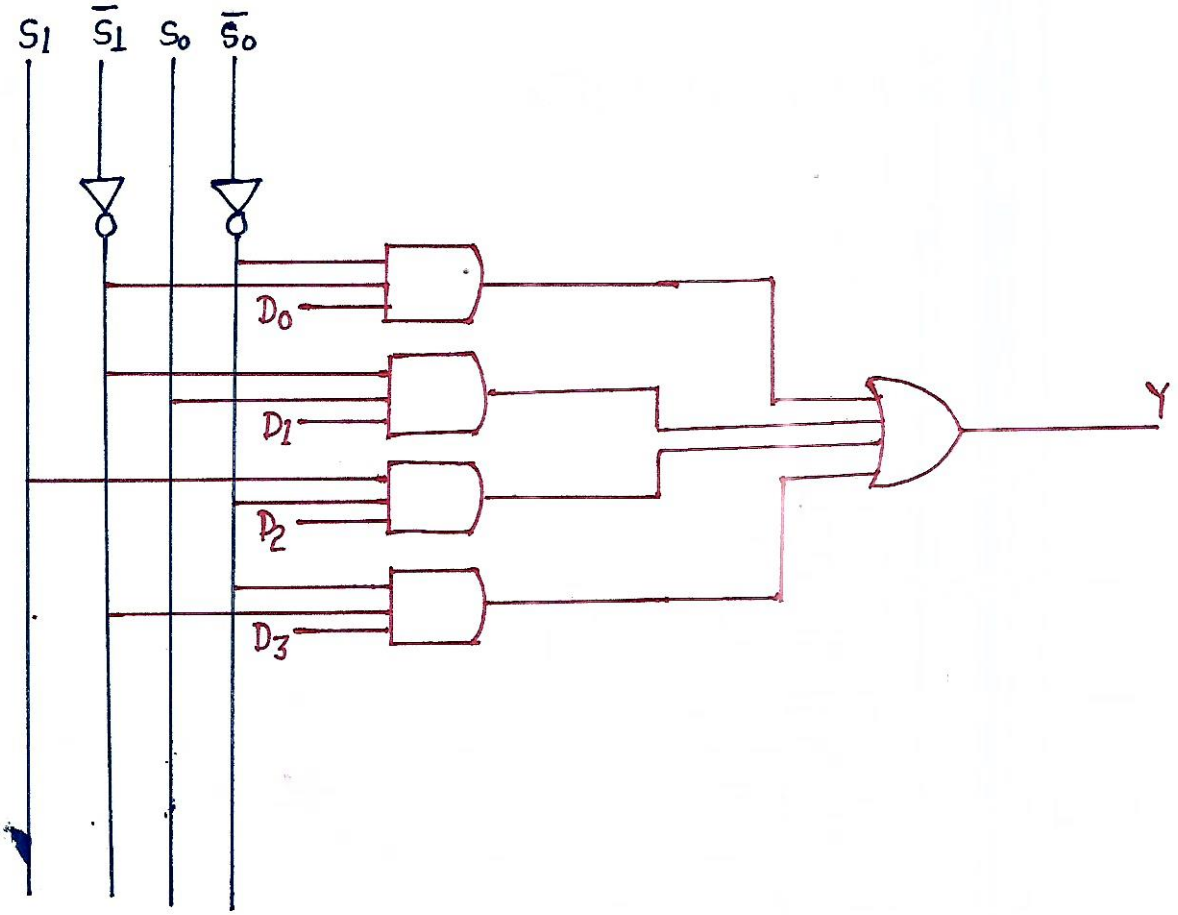


(2) 4:1

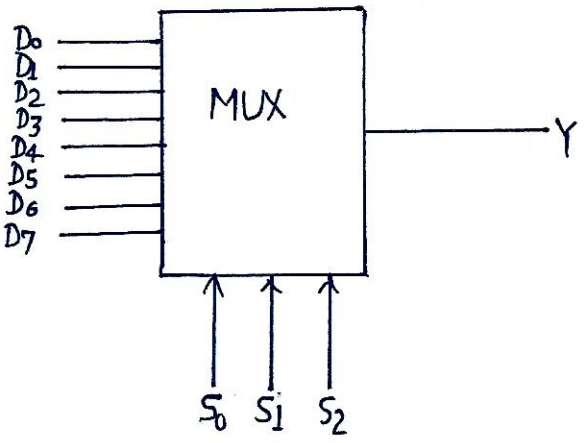


S_1	S_0	Y
0	0	D_0
0	1	D_1
1	0	D_2
1	1	D_3

$$Y = \bar{S}_0\bar{S}_1 D_0 + \bar{S}_1 S_0 D_1 + S_1\bar{S}_0 D_2 + \bar{S}_1\bar{S}_0 D_3$$

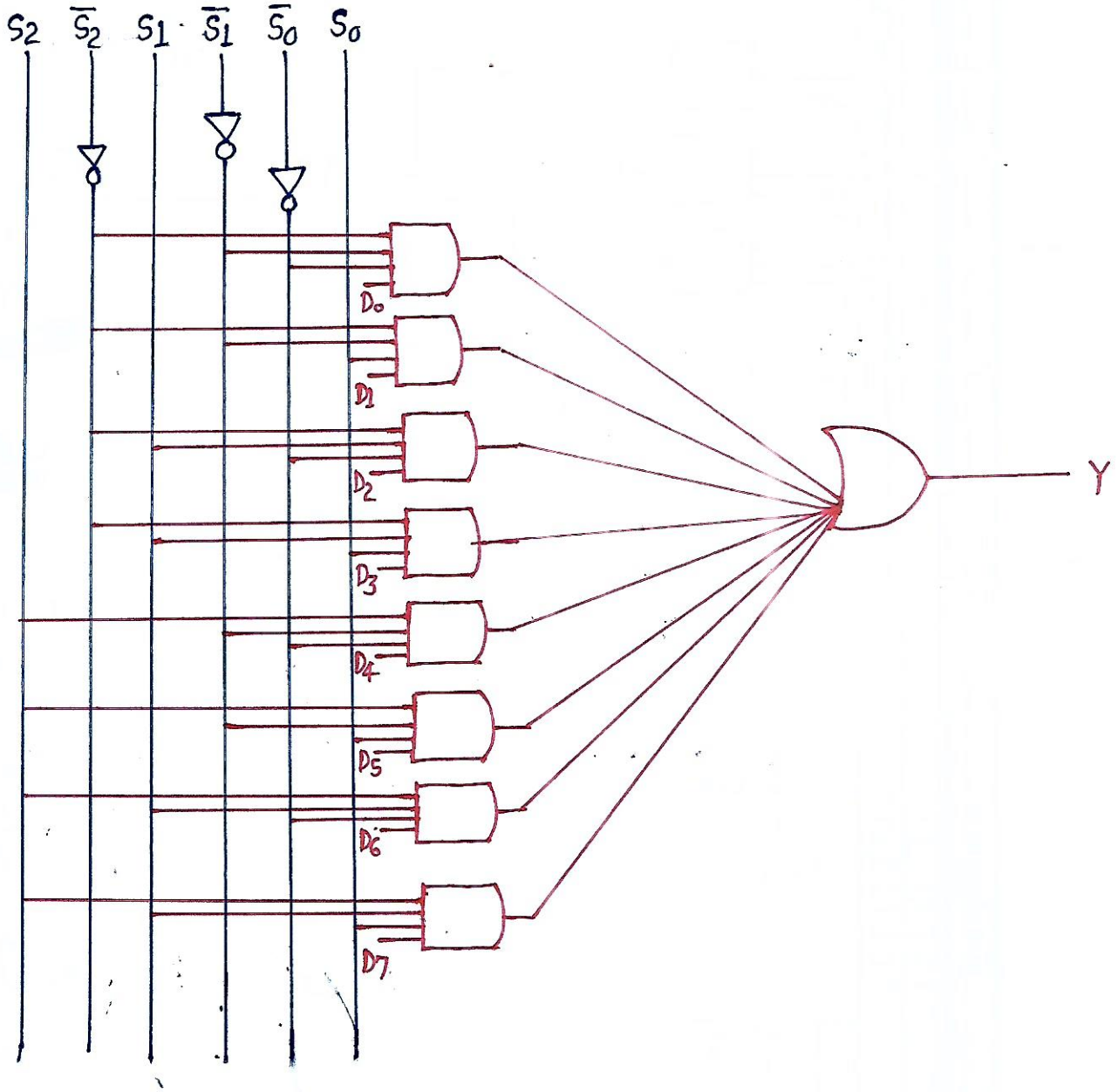


(3) 8:1



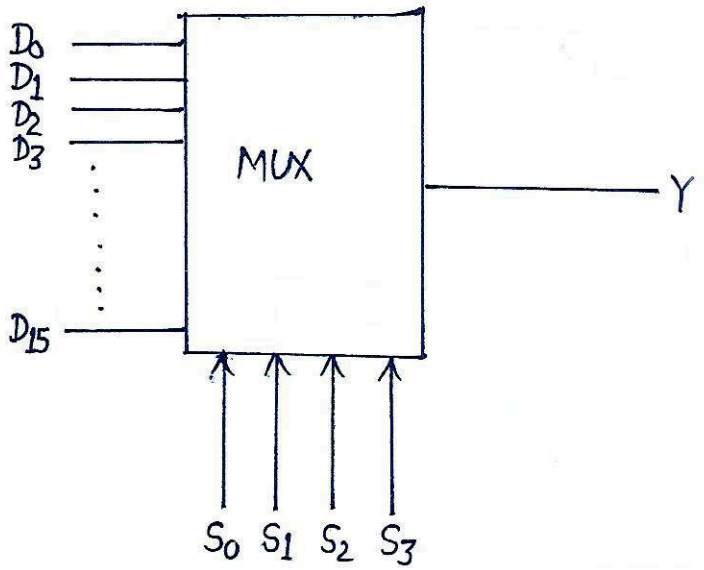
S_2	S_1	S_0	Y
0	0	0	D_0
0	0	1	D_1
0	1	0	D_2
0	1	1	D_3
1	0	0	D_4
1	0	1	D_5
1	1	0	D_6
1	1	1	D_7

$$Y = \bar{S}_2\bar{S}_1\bar{S}_0 D_0 + \bar{S}_2\bar{S}_1 S_0 D_1 + \bar{S}_2 S_1 \bar{S}_0 D_2 + \bar{S}_2 S_1 S_0 D_3 + S_2\bar{S}_1\bar{S}_0 D_4 + S_2\bar{S}_1 S_0 D_5 + S_2 S_1\bar{S}_0 D_6 + S_2 S_1 S_0 D_7$$



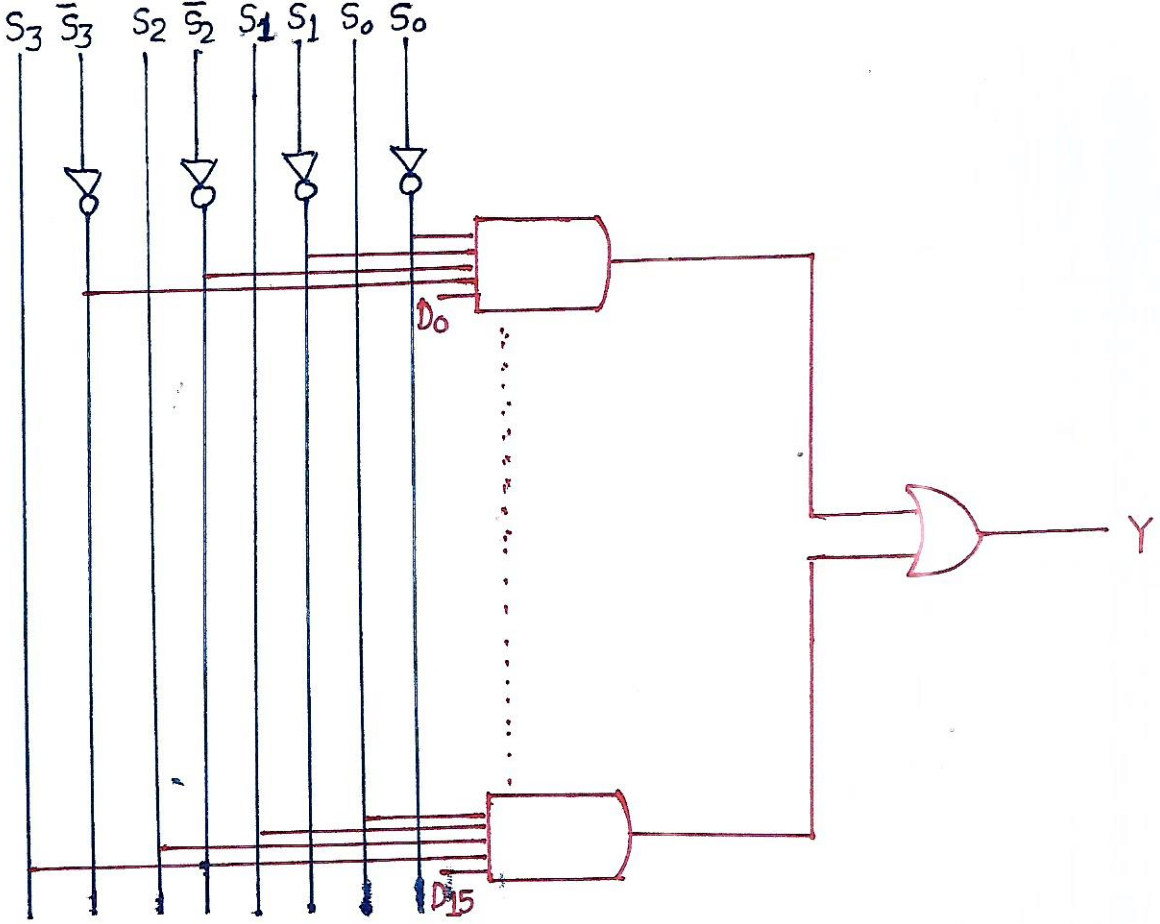
(4) 16:1

$2^4:1$



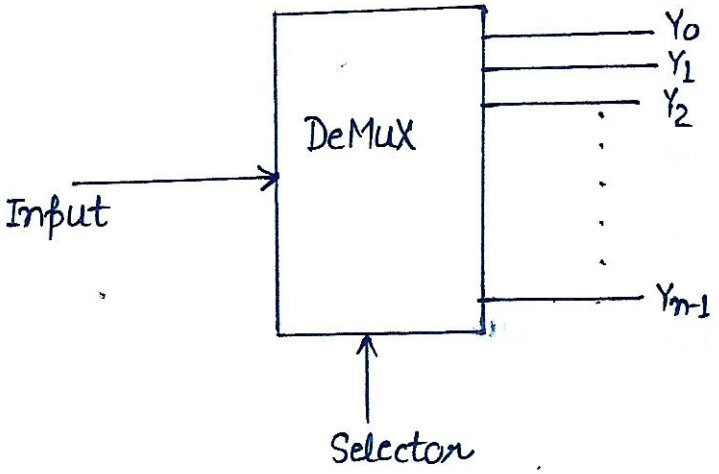
S_3	S_2	S_1	S_0	Y
0	0	0	0	D_0
0	0	0	1	D_1
0	0	1	0	D_2
0	0	1	1	D_3
0	1	0	0	D_4
0	1	0	1	D_5
0	1	1	0	D_6
0	1	1	1	D_7
1	0	0	0	D_8
1	0	0	1	D_9
1	0	1	0	D_{10}
1	0	1	1	D_{11}
1	1	0	0	D_{12}
1	1	0	1	D_{13}
1	1	1	0	D_{14}
1	1	1	1	D_{15}

$$Y = \bar{S}_3\bar{S}_2\bar{S}_1D_0\bar{S}_0 + \bar{S}_3\bar{S}_2\bar{S}_1S_0D_1 + \bar{S}_3\bar{S}_2S_1\bar{S}_0D_2 + \bar{S}_3\bar{S}_2S_1S_0D_3 + \bar{S}_3S_2\bar{S}_1\bar{S}_0D_4 + \bar{S}_3S_2\bar{S}_1S_0D_5 + \bar{S}_3S_2S_1\bar{S}_0D_6 + \bar{S}_3S_2S_1S_0D_7 + S_3\bar{S}_2\bar{S}_1\bar{S}_0D_8 + S_3\bar{S}_2\bar{S}_1S_0D_9 + S_3\bar{S}_2S_1\bar{S}_0D_{10} + S_3\bar{S}_2S_1S_0D_{11} + S_3S_2\bar{S}_1\bar{S}_0D_{11} + S_3S_2\bar{S}_1\bar{S}_0D_{12} + S_3S_2\bar{S}_1S_0D_{12} + S_3S_2\bar{S}_1S_0D_{13} + S_3S_2S_1\bar{S}_0D_{14} + S_3S_2S_1S_0D_{15}$$



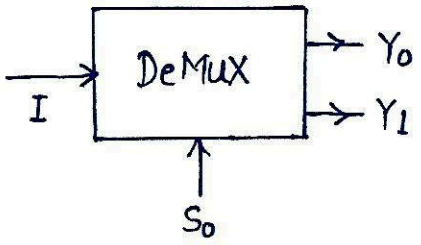
Demultiplexer-

A demultiplexer performs the reverse operation of multiplexer. It takes a single Input and distributes over several outputs. So a demultiplexer can be thought of as a distributor since it transmits the same data to different destinations.



- Types-
  (1) 1:2
  (2) 1:4
  (3) 1:8
  (4) 1:16

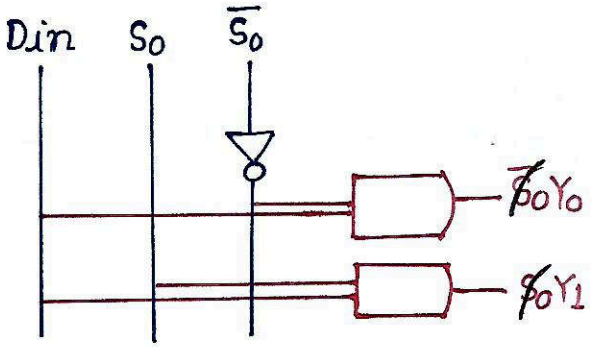
(1) 1:2



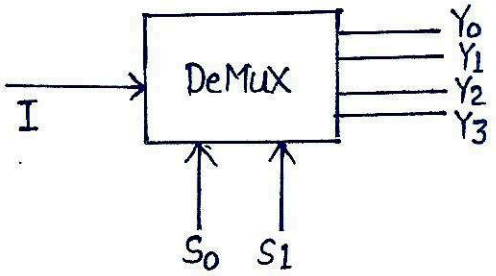
S_0	Y_1	Y_0
0	0	D_{in}
1	D_{in}	0

$Y_0 = \overline{S_0} D_{in}$

$Y_1 = S_0 D_{in}$



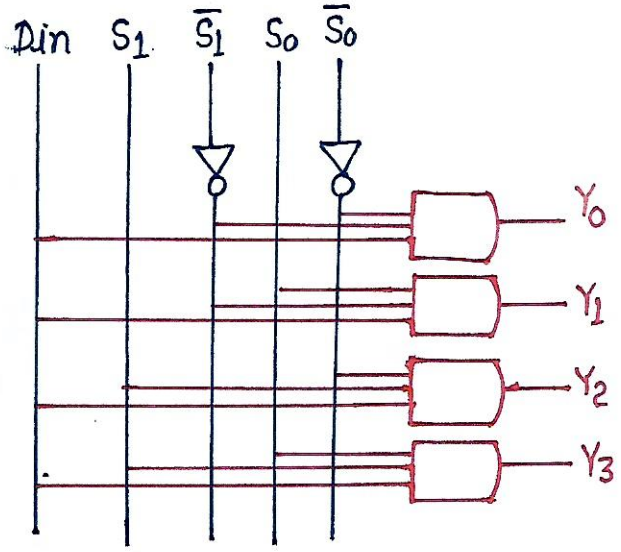
(2) 1:4



S_1	S_0	Y_0	Y_1	Y_2	Y_3
0	0	0	0	0	D_{in}
0	1	0	0	D_{in}	0
1	0	0	D_{in}	0	0
1	1	D_{in}	0	0	0

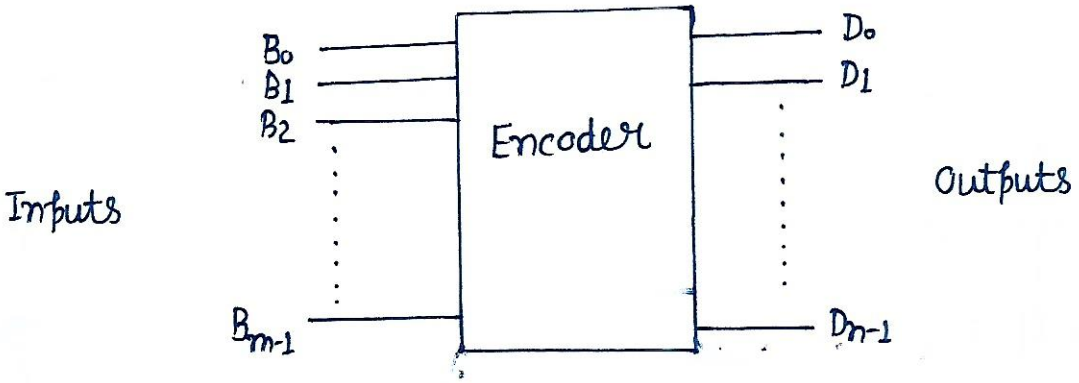
Boolean Expression

$$Y_0 = \bar{S}_1 \bar{S}_0 D_{in}$$
$$Y_1 = \bar{S}_1 S_0 D_{in}$$
$$Y_2 = \bar{S}_0 S_1 D_{in}$$
$$Y_3 = S_1 S_0 D_{in}$$



Encoder-

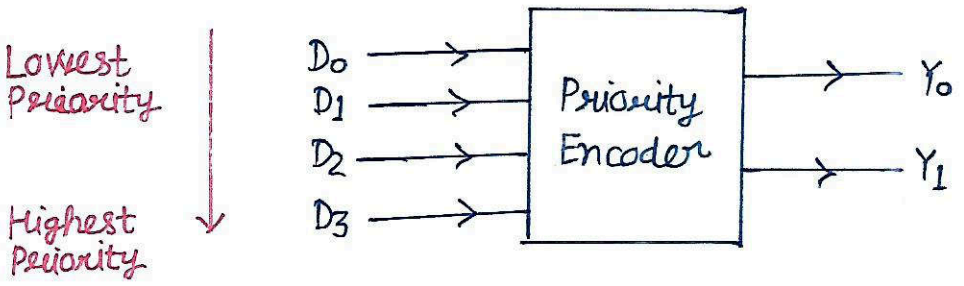
Encoder is a logic circuit. It has many input lines but the output is a coded pattern which identify each of the input & Input signal. The Encoding process transferred the desired signal into binary words.



- classification-
  - (1) Priority Encoder
  - (2) Decimal to BCD
  - (3) Octal to Binary
  - (4) Hexadecimal to Binary

(1) Priority Encoder-

A priority Encoder is a combinational logic circuit that performs similar to an Encoder. A priority Encoder has a number of input lines and more than 1 input lines can be active at a given time and produce an n-bit output code, depending the highest input which is activated



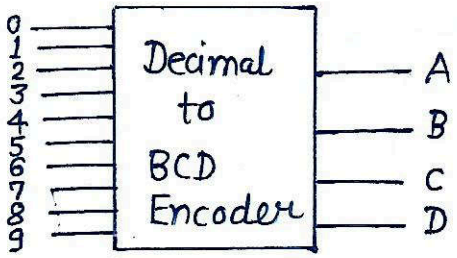
D_3	D_2	D_1	D_0	Y_1	Y_0	Yvalid
0	0	0	0	0	0	0
0	0	0	1	0	0	1
0	0	1	0	0	1	1
0	0	1	1	0	1	1
0	1	0	0	1	0	1
0	1	0	1	1	0	1
0	1	1	0	1	0	1
0	1	1	1	1	0	1
1	0	0	0	1	1	1
1	0	0	1	1	1	1
1	0	1	0	1	1	1
1	0	1	1	1	1	1
1	1	0	0	1	1	1
1	1	0	1	1	1	1
1	1	1	0	1	1	1
1	1	1	1	1	1	1

$$Y_1 = \bar{D}_0\bar{D}_1\bar{D}_2\bar{D}_3 + D_0\bar{D}_1\bar{D}_2\bar{D}_3 + \bar{D}_0D_1\bar{D}_2\bar{D}_3 + D_0D_1\bar{D}_2\bar{D}_3 + \bar{D}_0\bar{D}_1D_2\bar{D}_3 + D_0\bar{D}_1D_2\bar{D}_3 + D_0D_1D_2\bar{D}_3 + \bar{D}_0\bar{D}_1\bar{D}_2D_3 + D_0\bar{D}_1\bar{D}_2D_3 + \bar{D}_0D_1\bar{D}_2D_3 + D_0D_1\bar{D}_2D_3 + \bar{D}_0\bar{D}_1D_2D_3 + D_0\bar{D}_1D_2D_3 + \bar{D}_0D_1D_2D_3 + D_0D_1D_2D_3 + D_0D_1D_2D_3$$

$Y_0 = ?$

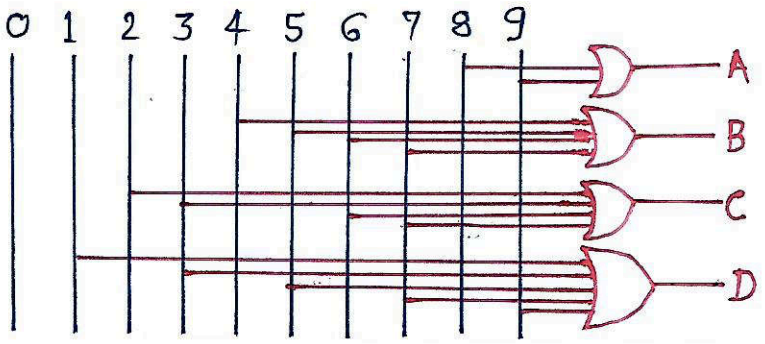
$Y_{yield} = ?$

(2) Decimal to BCD-

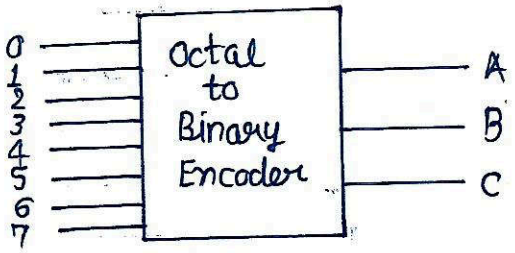


	8	4	2	1
Input	A	B	C	D
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1

A = 8+9
B = 4+5+6+7
C = 2+3+6+7
D = 1+3+5+7+9



(3) Octal to Binary-

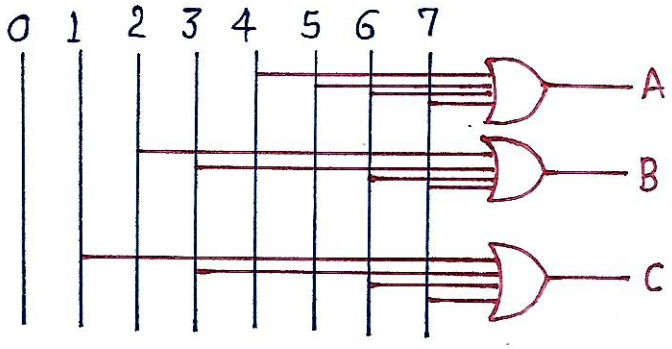


Input	4 A	2 B	1 C
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1

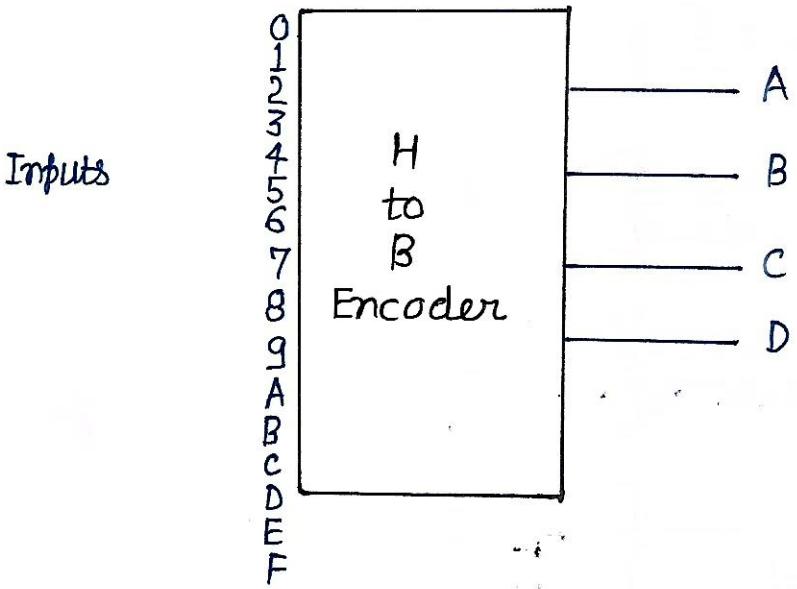
$A = 4+5+6+7$

$B = 2+3+6+7$

$C = 1+3+5+7$



(4) Hexadecimal to Binary-



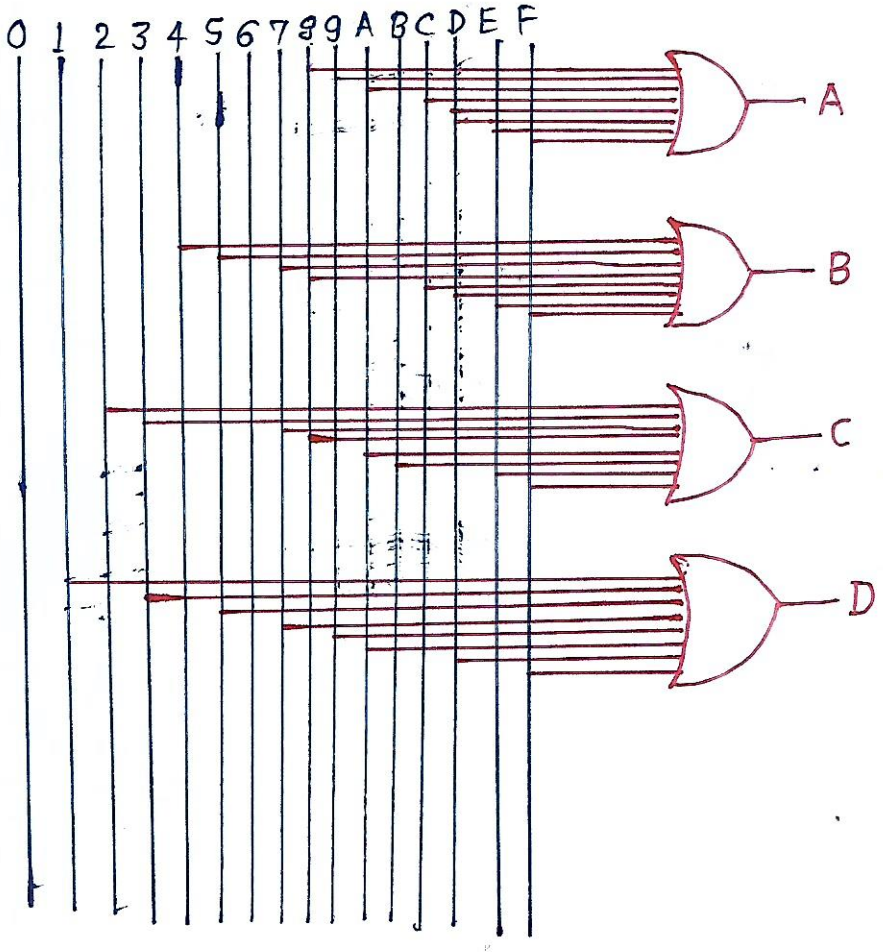
Input	8 A	4 B	2 C	1 D
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
A	1	0	1	0
B	1	0	1	1
C	1	1	0	0
D	1	1	0	1
E	1	1	1	0
F	1	1	1	1

A = 8+9+A+B+C+D+E+F

B = 4+5+6+7+C+D+E+F

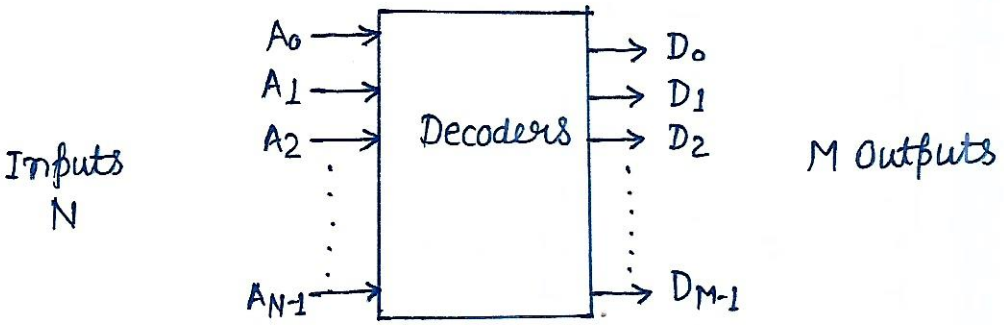
C = 2+3+6+7+A+B+E+F

D = 1+3+5+7+9+B+D+F



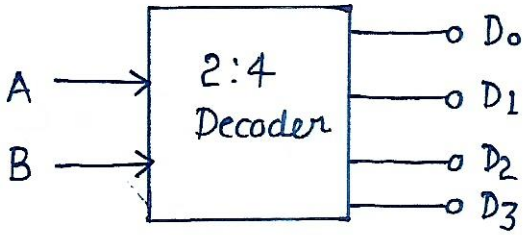
Decoder-

A decoder is a logic circuit that converts an N-bit binary input code into M output such that only one output line is activated for such one of the possible combinations of inputs. In other words, we can say that a decoder indentifies or recoznise or detects a particular code.



- Classification-
  - (1) 2 to 4 Decoder
  - (2) 3 to 8 Decoder
  - (3) 4 to 16 Decoder

(1) 2 to 4 Decoder-



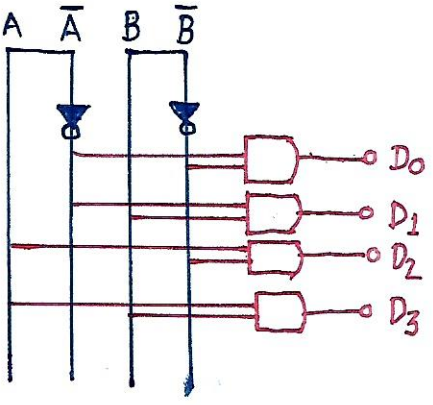
A	B	D_0	D_1	D_2	D_3
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

$D_0 = \bar{A}\bar{B}$

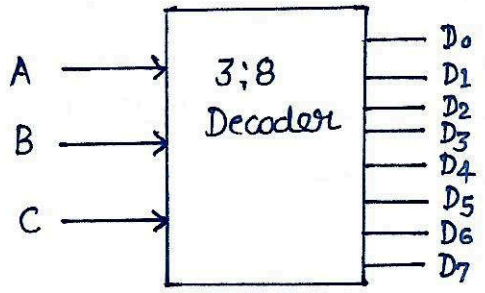
$D_1 = \bar{A}B$

$D_2 = A\bar{B}$

$D_3 = AB$



(2) 3:8 Decoder-



A	B	C	D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

$D_0 = \bar{A}\bar{B}\bar{C}$

$D_1 = \bar{A}\bar{B}C$

$D_2 = \bar{A}B\bar{C}$

$D_3 = \bar{A}BC$

$D_4 = A\bar{B}\bar{C}$

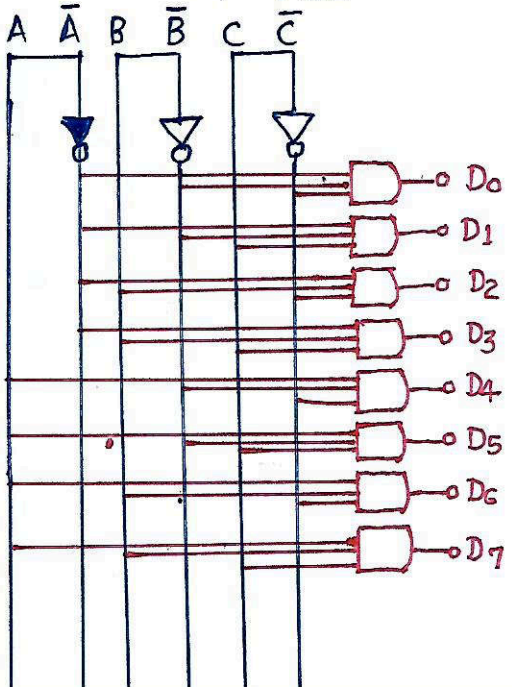
$D_5 = A\bar{B}C$

$D_6 = AB\bar{C}$

$D_7 = ABC$

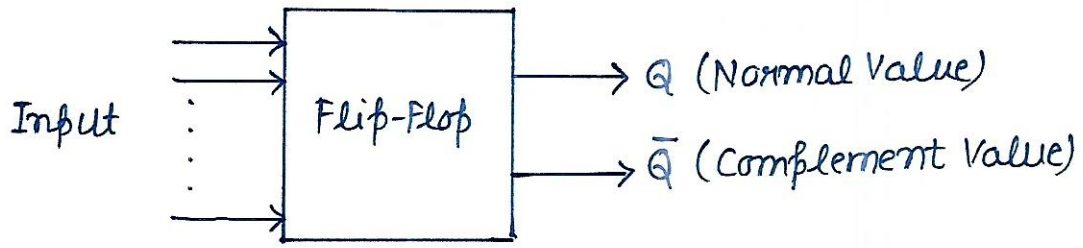
(3) 4:16 Decoder

Same as others



• Flip Flops :→

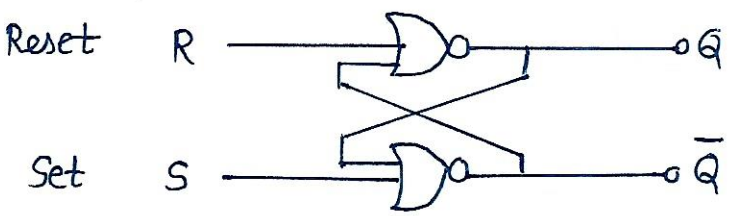
- Flip Flop is a memory element which is capable of storing 1 bit of information and it is used in clocked sequential circuits.
- A flip flop has two outputs, one for normal and other for complement value of the bit stored in it.
- A flip flop can maintain a binary state indefinitely (as long as power is delivered to the circuit) until directed by an input signal to switch state.
- A flip flop is also known as bistable multivibertor.



- Flip-flops are different types, depending on how their inputs and clock pulses cause transition b/w states.
- There are four basic flip flops :
  - Latch or RS Flip Flop
  - JK Flip Flop
  - T Flip Flop
  - D Flip Flop

→ Latch-

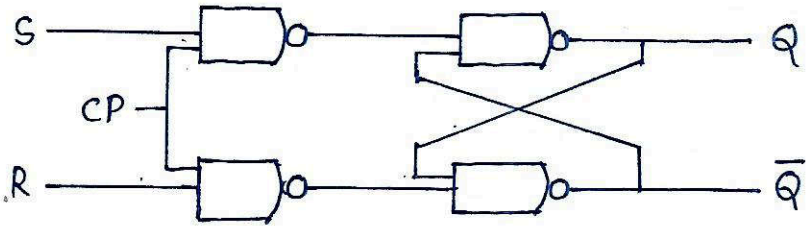
A basic flip-flop or Latch can be constructed using two NAND gates or two NOR gates. These are unclocked and Asynchronous Flip Flops. The operation is much faster in Latches than basic Flip Flops.

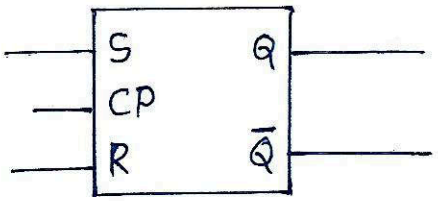


S	R	Q	$\bar{Q}$
1	0	1	0
0	0	1	0
0	1	0	1
0	0	0	1
1	1	0	0 ← n/A

(1) RS Flip Flop-

- A clocked SR Flip Flop consists of two basic NAND Latch and two other NAND gates to provide clock pulse as shown in Figure.
- Clock pulse is used for the synchronization and act as an additional control input.





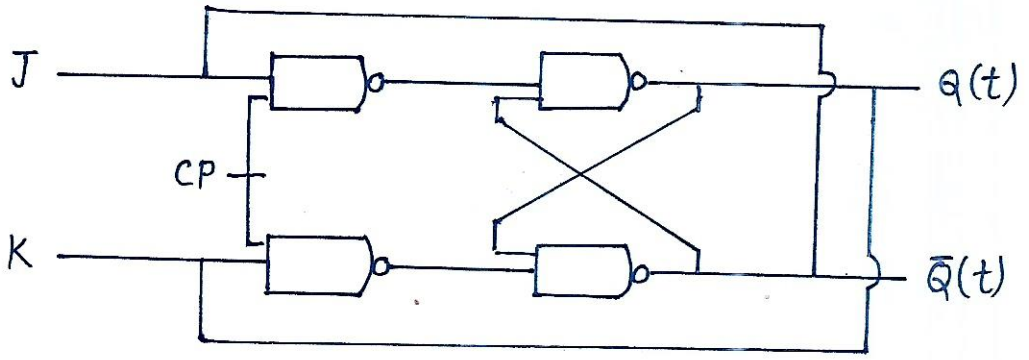
Q	S	R	Q(t+1)
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	X
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	X

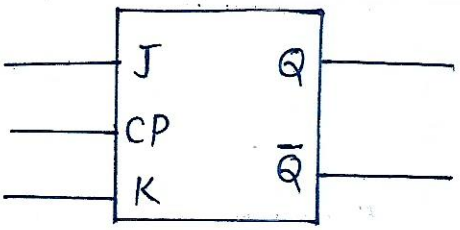
Q \ SR	$\bar{S}\bar{R}$	$\bar{S}R$	$S\bar{R}$	SR
Q	0	1	X 3	1 2
$\bar{Q}$	1 4	5	X 7	1 6

$$Q(t+1) = S + Q\bar{R}$$

(2) JK Flip Flop -

- A JK Flip Flop is a refinement of the SR Flip Flop to solve the problem of (independent) indetermined state when both inputs are 1.
- In JK Flip-Flop inputs J and K behaves like input S and R to set and Reset the flip-flop.





Q(t)	J	K	Q(t+1)
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

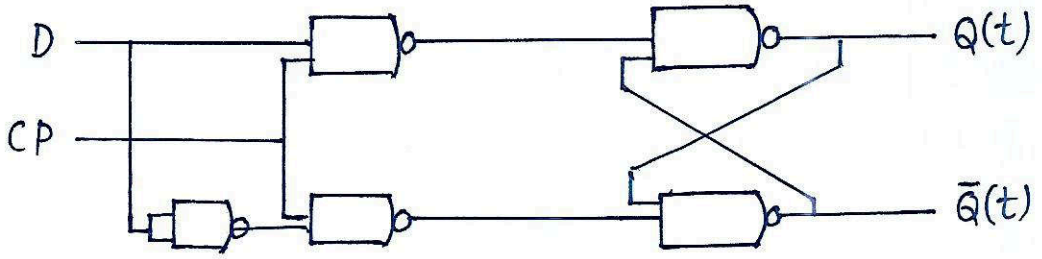
Q \	$\bar{J}\bar{K}$	$\bar{J}K$	JK	$J\bar{K}$
Q	0	1	1 (3)	1 (2)
$\bar{Q}$	1 (4)	5	7	1 (6)

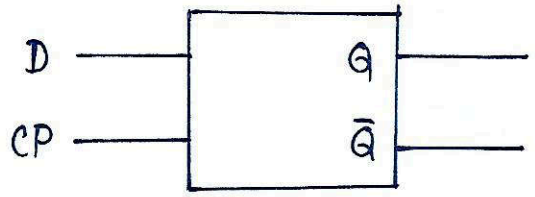
J	K	Q
0	0	Q(t)
0	1	0
1	0	1
1	1	$\bar{Q}(t)$

$$Q(t+1) = \bar{Q}J + Q\bar{K}$$

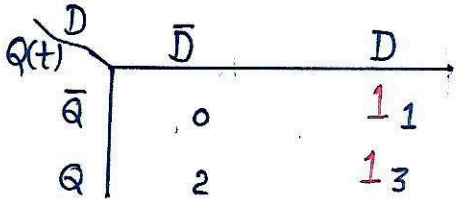
(3) D Flip Flop-

The D Flip Flop is a modification of the clocked SR Flip Flop as shown in the figure.





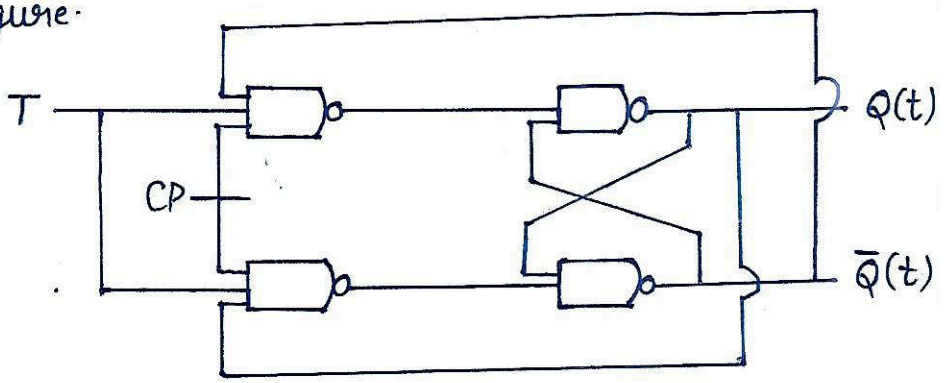
Q(t)	D	Q(t+1)
0	0	0
0	1	1
1	0	0
1	1	1

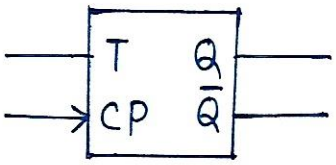


$$Q(t+1) = D$$

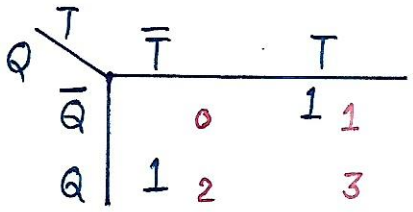
(4) T Flip Flop-

The T Flip Flop is a single version of JK Flip Flop as shown in figure.





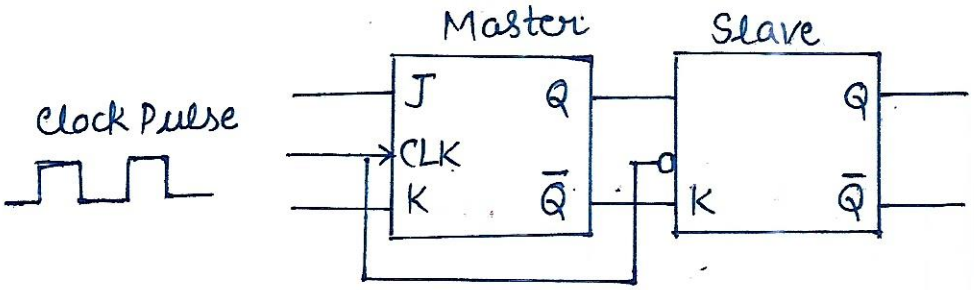
Q(t)	T	Q(t+1)
0	0	0
0	1	1
1	0	1
1	1	0

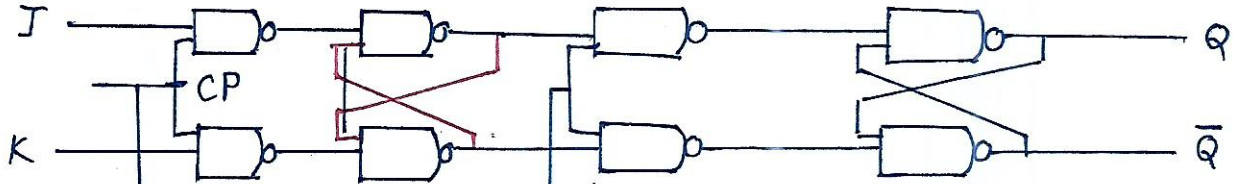


$$Q(t+1) = Q\bar{T} + \bar{Q}T$$

#Master-Slave Flip Flop-

- A master slave flip flop can be constructed using 2 JK Flip flops as shown in figure.
- First flip flop is called Master, is driven by the +ve clocked. The second flip flop is called Slave, is driven by the -ve clocked.





J	K	C	Output	Comment
0	0	⎍	Q_0	No change
0	1	⎍	0	Reset
1	0	⎍	1	Set
1	1	⎍	$\bar{Q}_0$	Toggle

IC Technology -

Full Form of IC - Integrated Circuit.

An Integrated circuit is just a packaged electronic circuit. An IC is a complete electronic circuit in which both the active and passive components are fabricated on an extremely tiny single chip of silicon.

Active components are those which have the ability to produce gain.
e.g. Transistor or FET.

Passive components or devices are those which do not have the ability to produce gain.
e.g. Resistors, Capacitors or Inductors

ICs are produced by the same procedures as are used for manufecturing individual transistors and diode etc.

J.S. Kilby was the first to develop an IC in 1958.

→ Advantages

1. Extreme small Physical Size.
2. Very small Weight.
3. Reduced Cost.
4. Extreme high Reliability.
5. Suitability for small-signal operation.
6. Low power consumption.
7. Easy Replacement.

→ Drawbacks

1. Coils or Inductors can't be febricated.
2. Can handle only limited amount of power.
3. Only Function at low voltage.
4. Quite delicate.
5. Can't withstand rough handling.

→ Scale of Integration

- SSI - Small Scale integration.
- MSI - Medium scale integration.
- LSI - Large scale integration.
- VLSI - Very Large scale integration.

#Thin Film Technology-

Such circuits are constructed by depositing films of conductive material through a mask on the surface of a substracte made of glass or ceramic. Resistors and conductors are formed by varying the width and thickness of the film and by using materials of different resistivity. Capacitors are produced by sandwiching an insulating oxide film between two conductive films. Small inductors can be made by depositing a spiral formation of film. The active components like transistors and diodes etc. are externally added and inter-connected by wire bonds.

Following two methods are used to produce thin-Films.

(i) Vaccum Evaporation-

In this method, the vaporised is deposited through a set of masks on a glass or ceramic substrate contained in vaccum.

(ii) Cathode Sputting-

In this method, atoms from a cathode made of the desired film material are deposited on a substrate which is located b/w the cathode and the anode.

Chemical
Plating Sol-Gel CVD

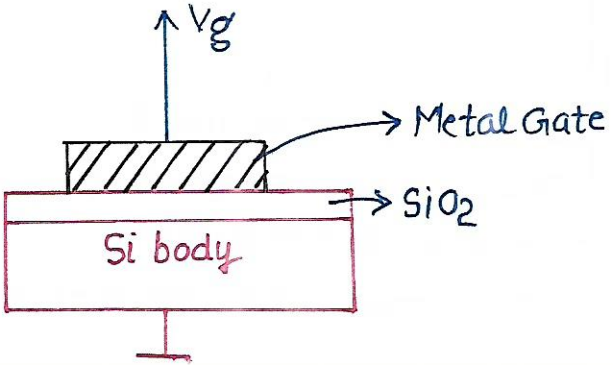
The Bipolar Transistor-

Basically, it consists of two back to back P-N junctions manufactured in a single piece of semiconductor crystal. These two junctions give rise to three regions called emitter, base and collector.

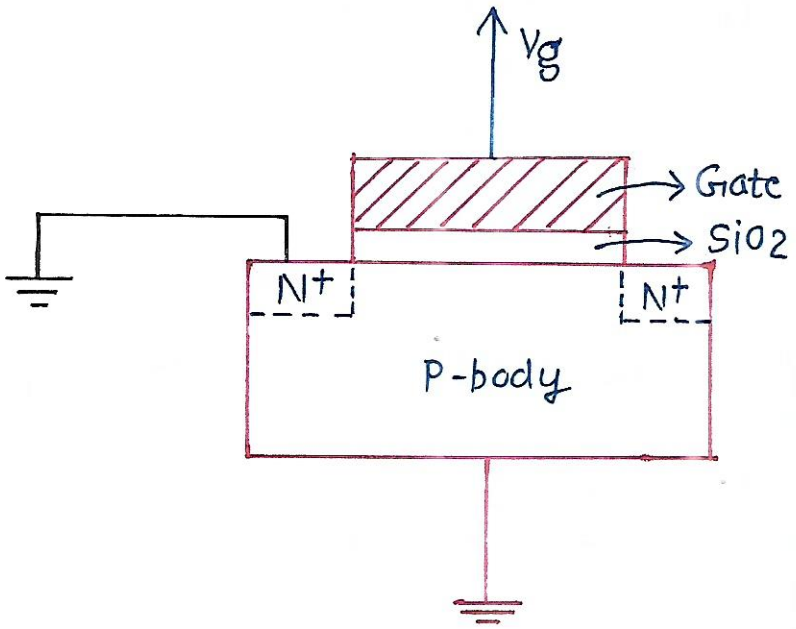
It is a contraction of two words ! transfer and Resistor. It is so because a transistor is basically a resistor that amplifies electrical impulses as they are transferred through it from its input to output terminals.

MOS Capacitor-

An MOS (Metal-Oxide-Semiconductor) Capacitor is made of a semiconductor body or substrate, an insulator film, such as SiO_2, and a metal electrode called a gate. The oxide film can be as thin as 1.5 nm. One nanometer is equal to 10 Å, or the size of a few oxide molecule.



Before 1970, the gate was typically made of metal such as Al (hence the M in MOS). After 1970, heavily doped polycrystalline silicon has been the standard gate material because of its ability to withstand high temperature without reacting with SiO_2.

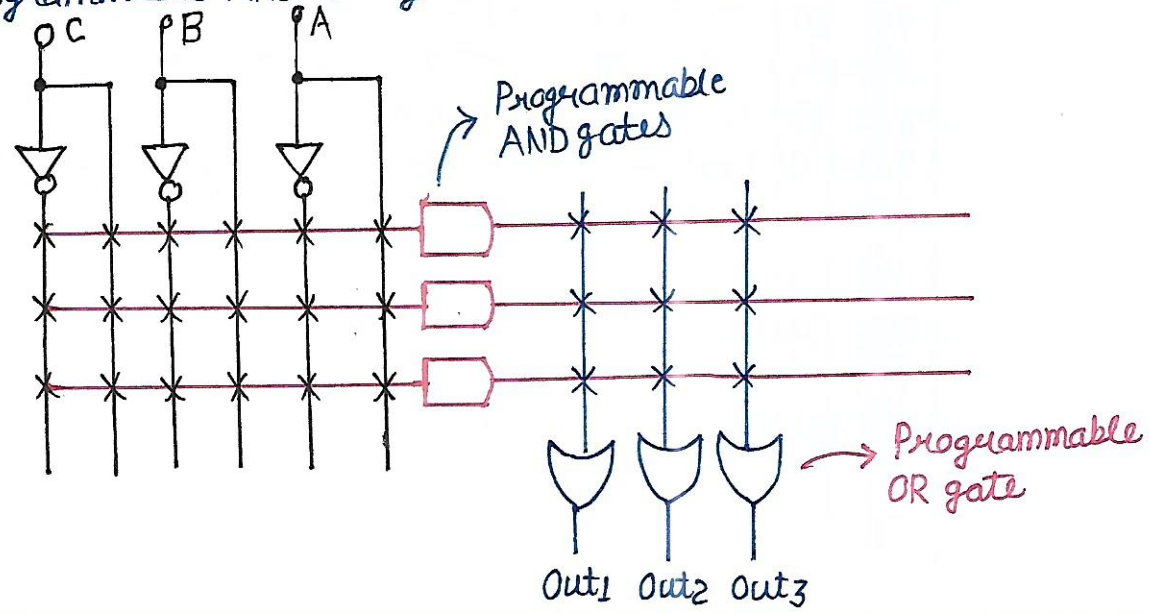


But the MOS name stuck. Unless specified otherwise, you may assume that the gate is made of heavily doped, highly conductive, polycrystalline silicon, or poly-Si for short. After 2008, the trend is to reintroduce metal gate and SiO_2 with more advanced dielectrics for the most advanced transistors.

The MOS capacitor is not a widely used device in itself. However, it is part of the mos-transistor.

#Programmable Logic Array-

The PLA represents another type of programmable logic but with a slightly different architecture. The PLA combines the characteristics of the PROM and the PAL by providing both a programmable OR Array or a programmable AND array.



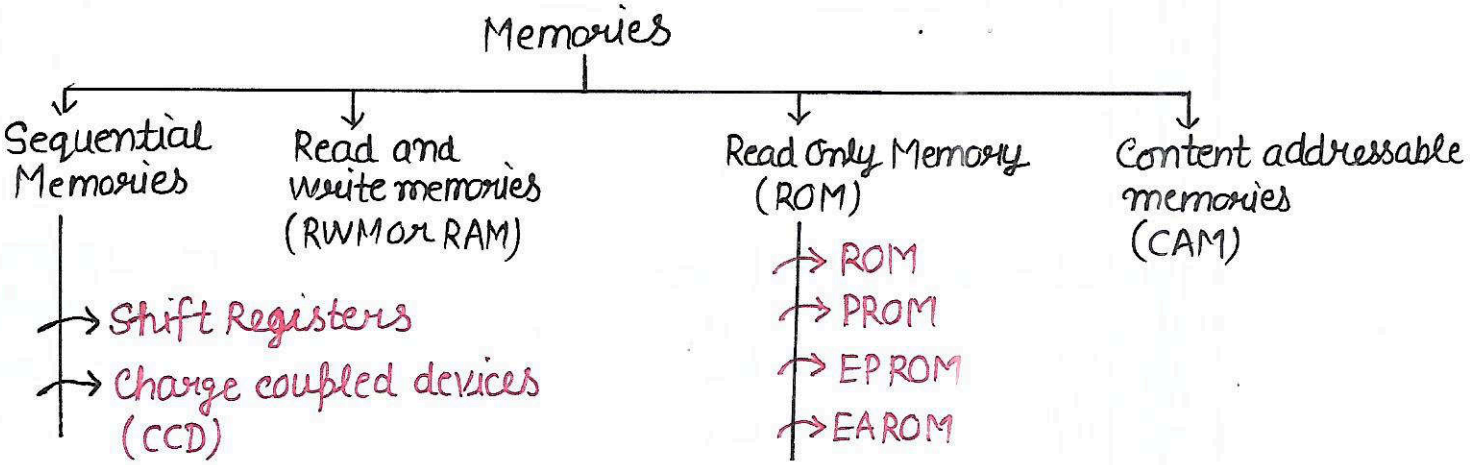
#CLASSIFICATION OF MEMORY-

The memory devices may be classified on the basis of various parameters. The Parameters used for basis of classification may be listed as under:

- (i) Operating Principle.
- (ii) Physical characteristics.
- (iii) Mode of access.
- (iv) Technology used for febrication process.

(i) Classification based on operating Principle-

The classification of memories based on the principle of operation is shown in figure.



(A) Sequential Memory- The example of the sequential memory are magnetic tape audio/video cassette. In the sequential memories, the memory locations are organized in a sequence (one after the other). The reading/writing from such memories is a sequential process. Hence, the time required to access a memory location is different for different locations.

(A_1) Shift Registers: The shift Registers may be of two types i.e. static or dynamic. In the static memory, the memory contents do not change with time as long as power is ON. In the dynamic memory, information is stored in MOS capacitors. The memory contents can change with time. Then, it is essential to refresh such memories at regular intervals.

Advantages:

→ Simpler than static memories.
→ Less expensive.
→ Require less power for operation.

(A2) CCD:

The CCD are manufactured using MOS technology. The salient features of CCD are high density and low cost.

(B) RAM-

Random Access memory is also known as Read write memory (RWM). The memory location is this type of memory are organised in such a manner that the access time required for accessing any location remains the same. Actually, this is an advantage of using RAM over the sequential memories. RAMs also may be further classified into two types namely the static RAM and dynamic RAM. RAM may be fabricated using either bipolar technology or unipolar technology.

(C) ROM-

These memories are (desired to) designed only for Reading the information which is already stored on them. The user can't write any new information on them. These are similar to the prerecoded cassette. A menufecturer or someone else can write ROM, but the writing process is much more complicated as compared to that of a RAM. Also, ROM is used to store fixed informations such as look up tables, instructions, fixed information etc. It is also possible to organise ROM in such a manner that reading time required for any location remains the same. The types are:

1. Read only memory (ROM)
2. Programmable Read only memory (PROM)
3. Erasable and Programmable ROM (EPROM)

1. ROM- These ROM are true sense memories, because they are programmed at the time of manufacturing. The programming is formulated as per the requirements of the user and it is the last process in manufacturing of ROM. Such ROMs are known as custom programmed or mask programmed. We can't change the programmed data after fabrication. These ROMs are quite costly and hence are suitable only for bulk use (no. of chips in millions).

2. PROM- A user can programme these ROMS using PROM programmer. A PROM can be programmed only once after its

fabrication. After programming, the content will become permanently fixed in the ROM. The PROMs are manufactured without any data stored into them.

3. EPROM-

As the name suggests, these ROMs can be erased and programmed again and again. The commonly used techniques for erasing may be described as under.

I. Using UV Radiation → EPROM
II. Using Electricity → EAROM

(D) CAM- It is a special type of RAM. It can perform association operation in addition to the read/write operation performed by the convientional memories.

(ii) CLASSIFICATION BASED ON Physical Characteristic-

Memories

Erasable or Non Erasable

An Erasable memory is that kind of memory in which the stored information can be erased and new information can stored.

The information stored in non-erasable memory can't be erased. The erasable memory can be categorised in two classes as under:

(i) Location by Location
(ii) All Location Simultaniously

All location can be deleted at same time.

Location by Location delation of data

Volatile or Non-volatile

If the information stored in the memory chip is lost when the electrical power is switched OFF, then the memory is called as Voltalile memory.

If the information once stored in memory chip does not change unless altered deliberately are known as non-volatile memory. Such memories can be hold the information even after switched OFF the power suffply. Infact, ROMs are non-volatile memories.

(iii) Classification based on mode of Access-

Mode of Access means the manner in which a memory location is accessed while performing the read or write operation. These are following two possible mode of access.

A. Sequential Access → The sequential memories have a sequential access to various memory locations. Hence, the time required to various locations shall be different.

B. Random Access → We can access any memory location without going sequential. Accessing any memory location for read or write require same amount of time.

(iv) Classification based on fabrication Technology-

There are two fabrication technologies.

A. Unipolar Technology → MOS Technology

B. Bipolar Technology → TTL, ECL

MOS Technology → RAM, ROM, EPROM

LASER PHYSICS-A

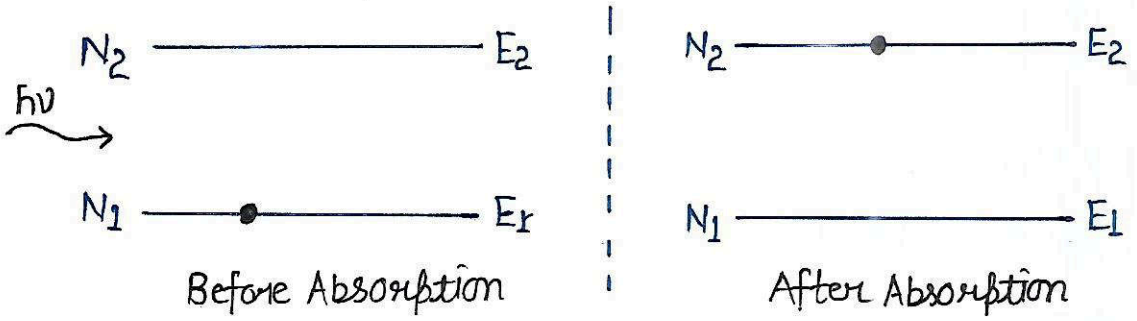
- UNIT-1 [BASIC PRINCIPLES]
- UNIT-2 [TYPES OF LASER]
- UNIT-3 [NON-LINEAR OPTICS]
- UNIT-4 [LASER SPECTROSCOPY]

• Theory of Absorption and Emission ; Einstein Cofficients-

- Einstein, 1916
- EMR can interact with matter through three processess.

1. Stimulated Absorption → B_{12}
2. Spontaneous Emission → A_{21}
3. Stimulated Amission → B_{21}

(1) Stimulated Absorption-



Before Absorption | After Absorption

$h\nu = E_2 - E_1$

Let $E(\nu)$ be energy density.

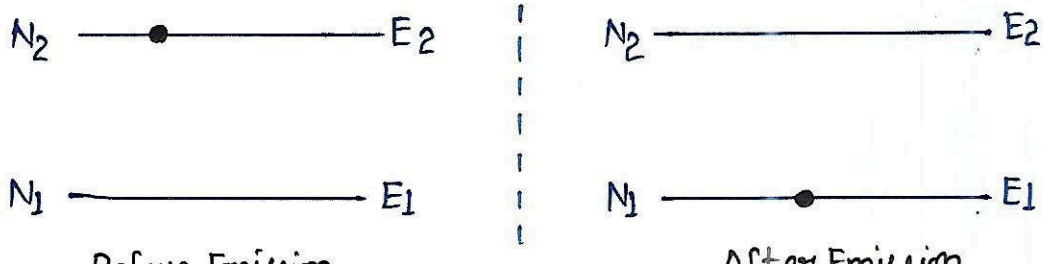
$R_{StA} \propto N_1$

$R_{StA} \propto E(\nu)$

$R_{StA} \propto E(\nu) N_1$

$R_{StA} = B_{12} N_1 E(\nu) \ldots\ldots\ldots (1)$

(2) Spontaneous Emission-

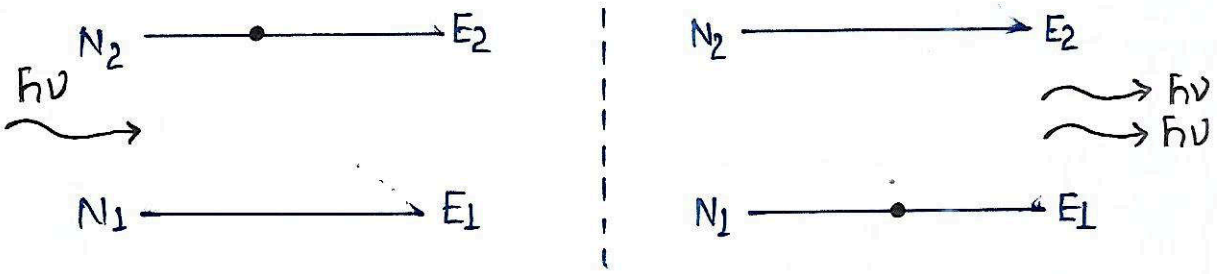


Before Emission | After Emission

Rate of Spontaneous Emission $R_{SpE} \propto N_2$

$$R_{SpE} = A_{21} N_2 \quad \ldots\ldots\ldots (2)$$

(3) Stimulated Emission -



$$R_{StE} \propto N_2$$

$$R_{StE} \propto E(\nu)$$

$$R_{StE} \propto B_{21} N_2 E(\nu) \quad \ldots\ldots (3)$$

In Equilibrium,

$$R_{StA} = R_{StE} + R_{SpE} \qquad [\because \text{From eq}^{n}(1),(2),(3)]$$

$$B_{12} N_1 E(\nu) = A_{21} N_2 + B_{21} N_2 E(\nu)$$

$$B_{12} N_1 (E(\nu)) = A_{21} N_2 + B_{21} N_2 E(\nu)$$

$$E(\nu) = \frac{A_{21} N_2}{[B_{12} N_1 - B_{21} N_2]}$$

$$E(\nu) = \frac{A_{21}}{B_{21}\left[\frac{B_{12}}{B_{21}} \cdot \frac{N_1}{N_2} - 1\right]} \quad \ldots\ldots\ldots (4)$$

$$\frac{N_1}{N_2} = e^{h\nu/KBT} \quad \ldots\ldots\ldots (5) \text{ MB distribution}$$

From eq^{n}(4),(5)

$$E(\nu) = \frac{A_{21}}{B_{21}} \cdot \frac{1}{\frac{B_{12}}{B_{21}}\left(e^{h\nu/KBT} - 1\right)}$$

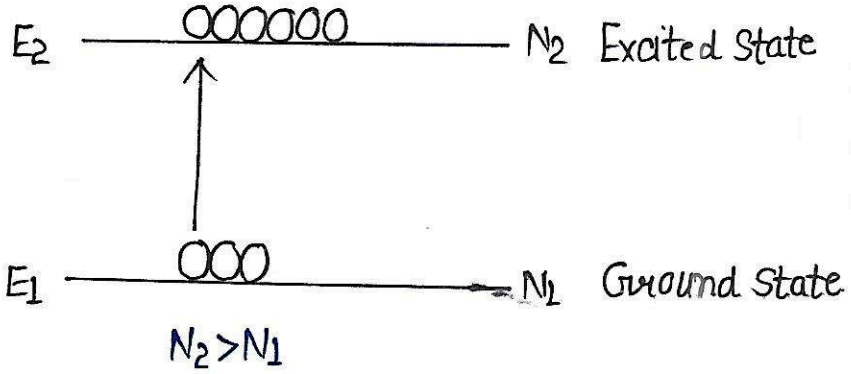
$$E(\nu) = \frac{8\pi h\nu^3}{c^3}\left[\frac{1}{e^{h\nu/KBT}-1}\right]$$

$$\boxed{\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}}$$

$$\frac{B_{12}}{B_{21}} = 1$$

$$\boxed{B_{12} = B_{21}}$$

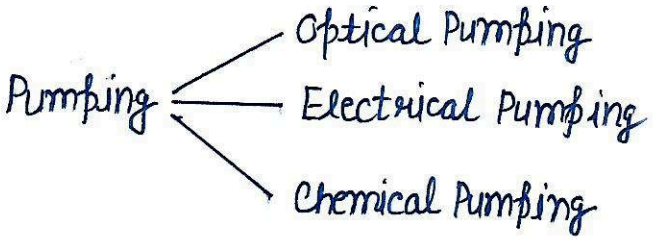
Population Inversion-



The no. of Atom are greater in excited state than ground state that situation is called Population Inversion. To achieve population Inversion, we use pumping methods.

In general, $E_1 > E_2$
But in pumping

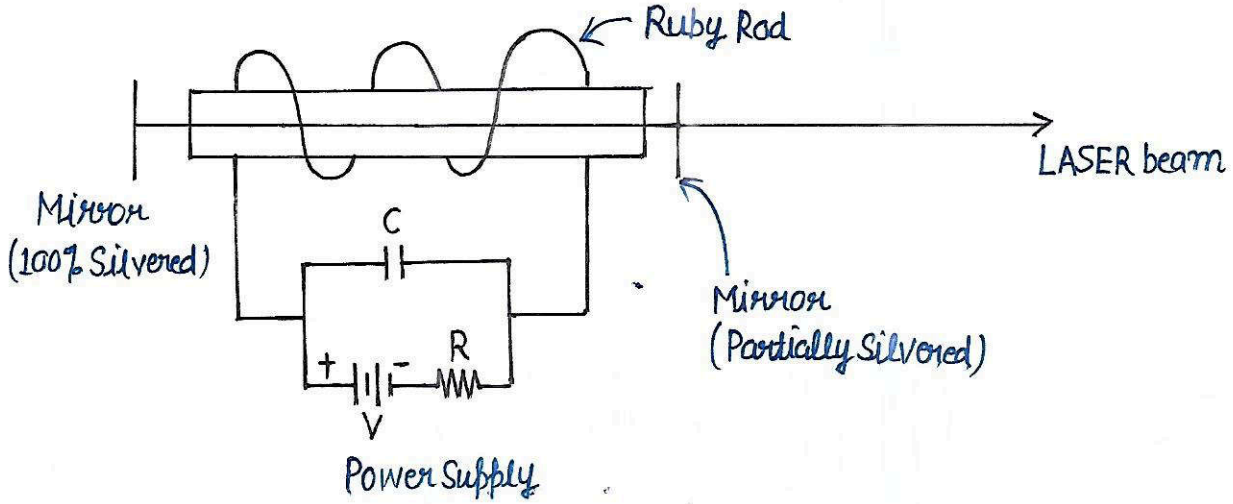
$$E_2 > E_1$$

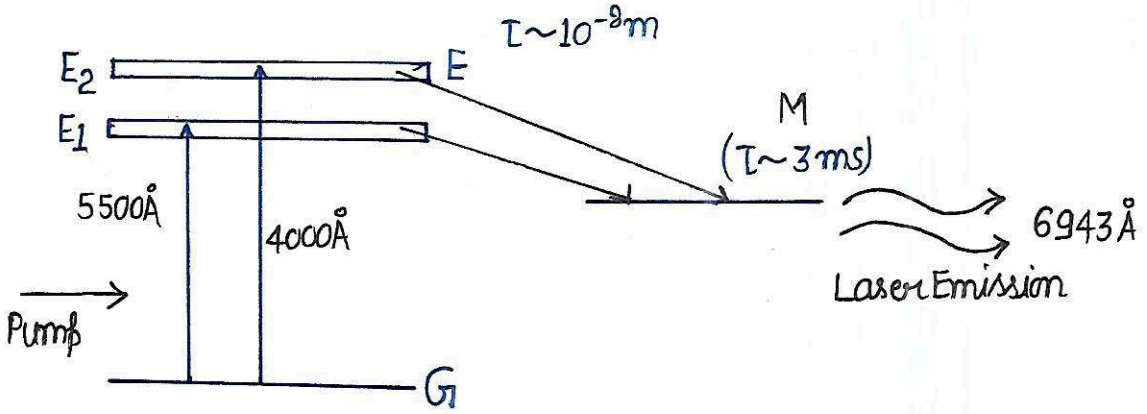


UNIT - 2
TYPES OF LASERS

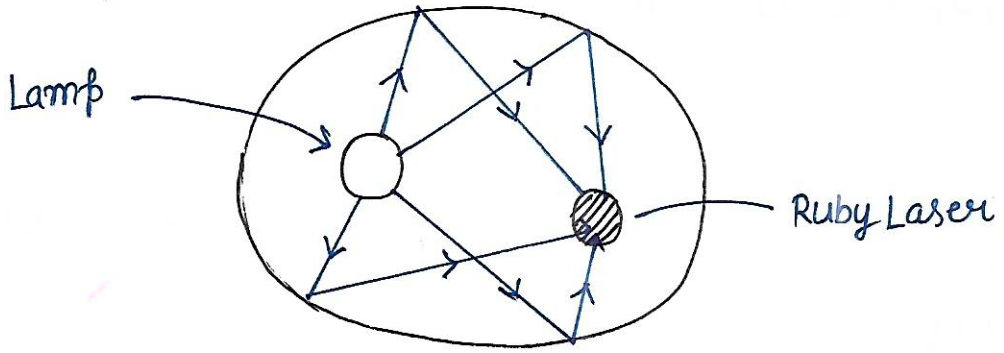
The Lasers considerd are
(a) Gas Lasers : He-Ne, argon ion and CO_2 Laser
(b) Solid state Lasers : Ruby Lasers, Nd:YAG, Nd:Glass Laser
Reference : Springer Publication.

#Ruby LASER : The First Laser to be operated successfully was the Ruby laser which was febricated by Maiman in 1960. Ruby, which is the lasing medium, consists of a matrix of Aluminium Oxide (Al_2O_3) in which some of the Aluminum ions are replaced by Cromium ions (Cr_3^+)-It is the energy level of the Cr_3^+ which take part in the lasing action. Typical concentration of the chromium ions are ~0.05% by weight. The energy level diagram of the chromium ions is shown in the figure. This is a three level Laser. The pumping of the Cr_3^+ ions is performed with the help flash Lamp (A xenon flashlamp) or (Kr Laser flashlamp) and the chromium ions in the groundstate absorb radiation around wavelength of 5500Å and 4000Å and are excited to the level maked E_1 and E_2. The Chromium ions excited to these levels relax rapidly through a non-radiative transition (in a time $\sim 10^{-8} - 10^{-9}$ s) to the level





marked M which is the upper laser level. The Level M is a metastable level with a lifetime of ~3ms. Laser emission occurs between level M and the ground level G at an output wavelength of 6943 Å.
The Flashlamp operation of the laser leads to a pulsed output of the Laser. As soon as the flashlamp stops operating, the population of the upper level is depleted very rapidly and lasing action stops till the arrival of the next flash. Even during the short period of a few tens of microseconds in which the laser is oscillating, the output is a highly irregular function of time with the intensity having random amplitude fluctuations of varying duration. This is called laser spiking, the formulation on which can be understood as follows; When the pump is turned on, the intensity of light at the laser transmission is small and hence the pump builds up the inversion rapidly. This cycle repeats itself to produce the characteristic spiking in lasers.
A typical setup of a flashlamp pumped pulsed ruby laser. The helical flashlamp is surrounded by a cylindrical reflector to direct the pump light onto the ruby rod efficiently. The ruby rod length is typically 2~20cm with diameter of 0.1~2cm. The typical input electrical energies required are in the range of 10-20 KJ.



One may use other pumping schemes such as that shown in figure, in which the pump lamp and the laser rod are placed along the foci of an elliptical cylinder reflector. It is well known that the elliptical reflector focuses the light emerging from one focus into the other focus of the ellipse, thus leading to an efficient focusing of pump light on the laser rod.

- Properties-

(1) Chemical Formula - $Al_2O_3:Cr$
(2) Crystal Structure - Cylindrical
(3) Melting point - 2040°C
(4) Hardness - 9

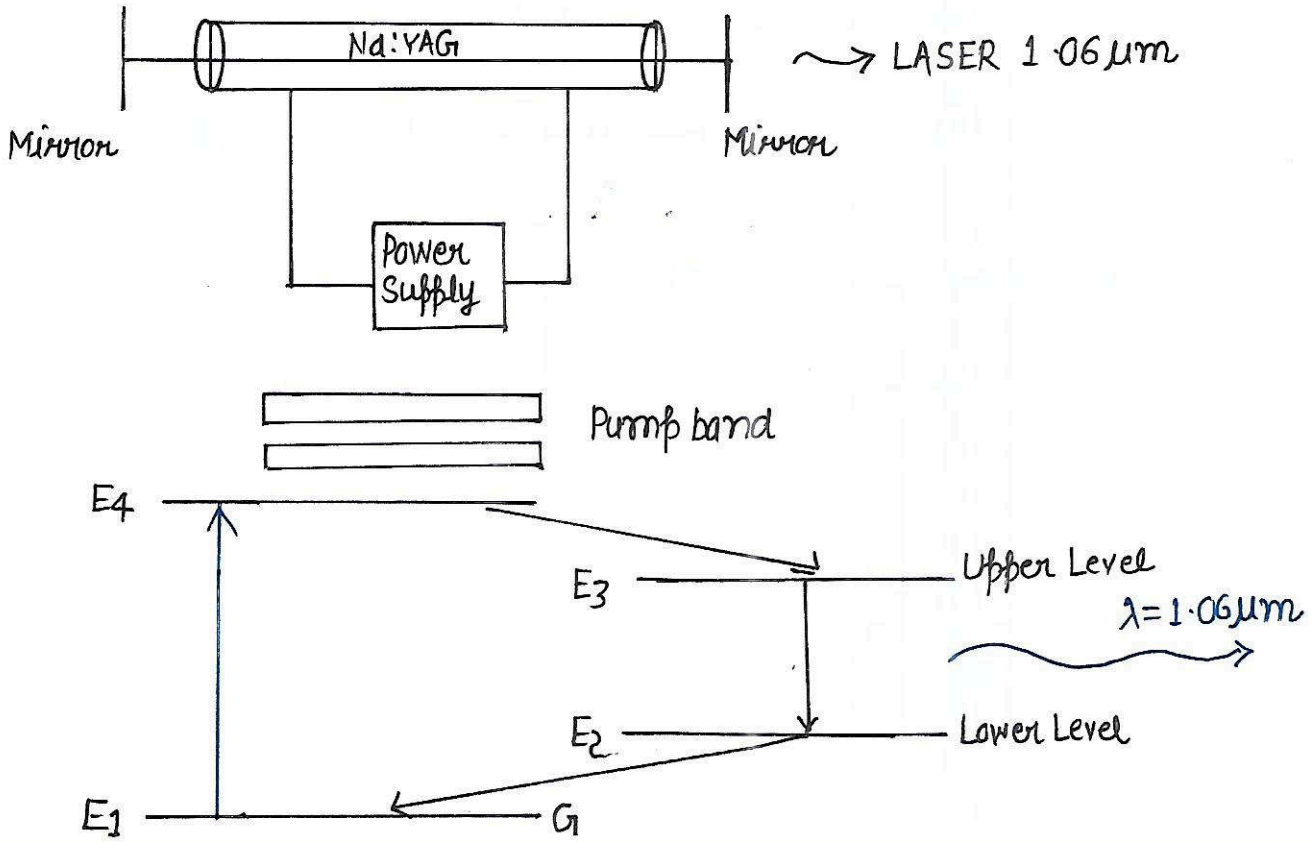
• Applications-

(1) High power Q-Switched system.
(2) Drilling holes in hard material.
(3) Frequency doubling into the UV spectrum.
(4) Tatto Removal and cosmetic dermatology.
(5) Drilling holes in diamonds.

#Nd:YAG LASER- The Nd:YAG Laser is a four-level laser and the energy level diagram of the neodymium is shown in figure. The laser emission occurs at 1.06 μm. Since the energy difference b/w the lower level and the ground level is ~0.26 eV, the radio of its population to that of the ground state at room temperature is,

$$\frac{e^{-\Delta E}}{K_B T} \approx e^{-9} << 1$$

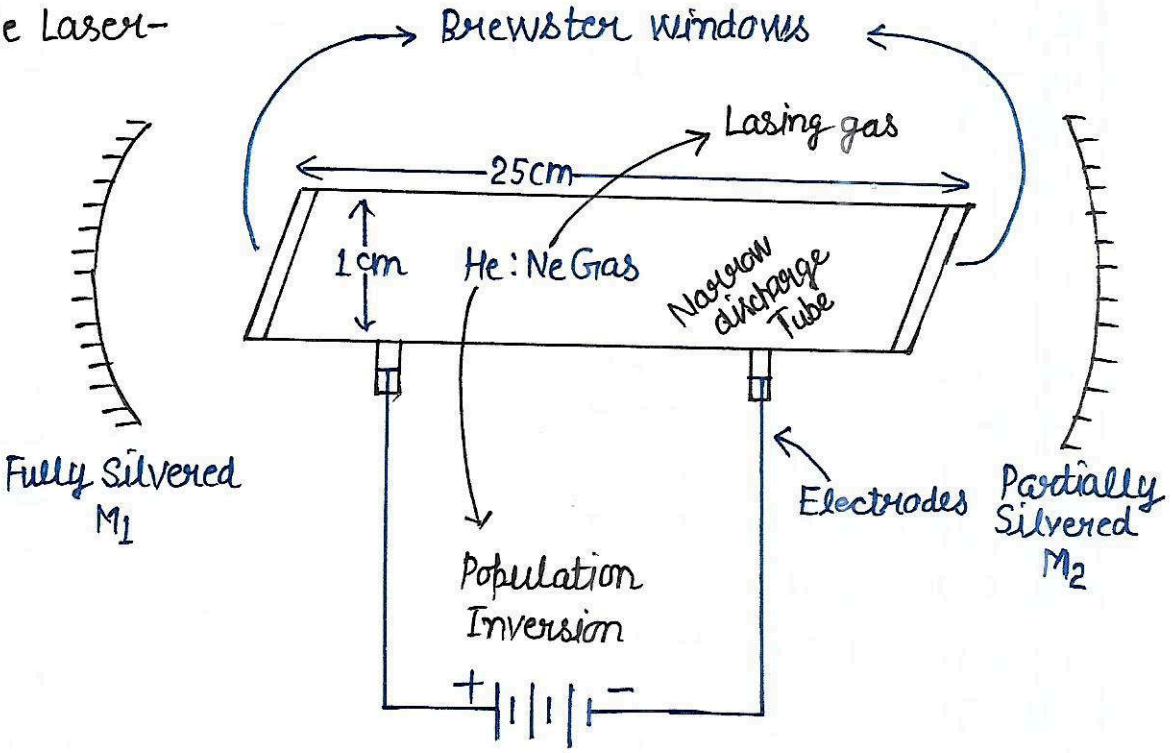
Thus the lower level is almost unpopulated and hence inversion is easy to achieve. The main pump for exeitation of the neodymium ions are the 0.81 and 0.71 μm wavelength regions and pumping is done using arc lamp (e.g. the Krypton arc lamp). Typical neodymium ion concentration used are $\sim 1.38 \times 10^{20}$ cm^{-3}. The spontaneous lifetime corresponding to the laser transition is 550 μs and the emission line corresponds to homogenous broadening and has a width $\Delta\nu \sim 1.2 \times 10^{11}$ Hz which corresponds to $\Delta\lambda \sim 4.5$ Å. Nd:YAG Laser has a much lower threshold of oseillation than a ruby laser.



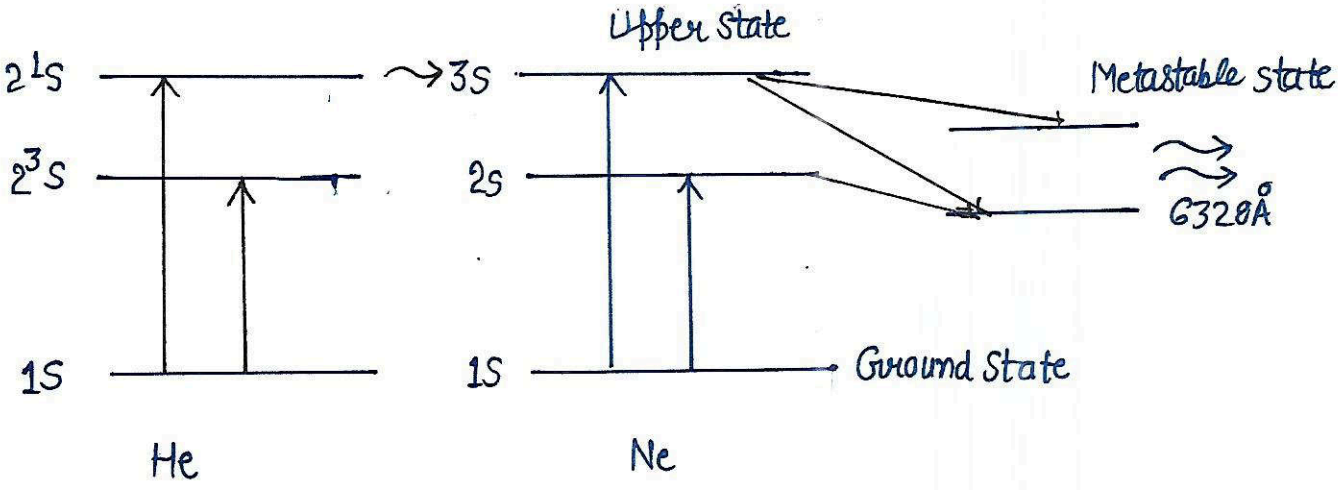
Nd:Glass Laser-

→ Four Level laser system.
→ Laser Emission around 1.06 μm.
→ Neodymium concentrations ~ 2.8×10^{20} cm^{-3}
→ Silicate and Phosphate glasses as the host material.
→ Leads to inhomogenous broadning.
→ Width is larger than Nd:YAG.
→ Suitable for homogenous high energy pulse operation.

He-Ne Laser-

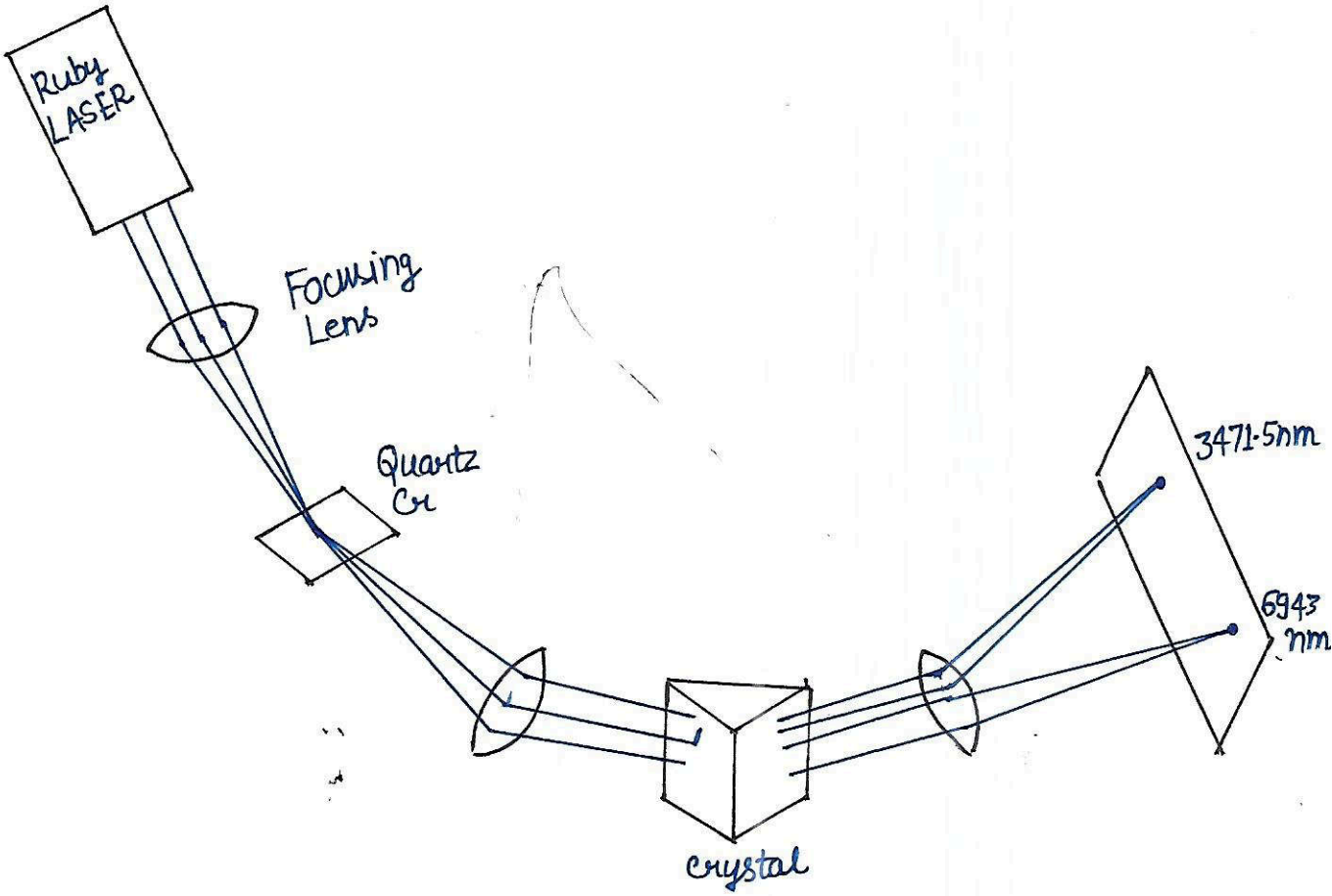


→ Coherence
→ High Intensity, directionality, Monochromaticity
→ First gas LASER
→ Ali Javan and his coworkers at Bell Laboratory in USA(1961).

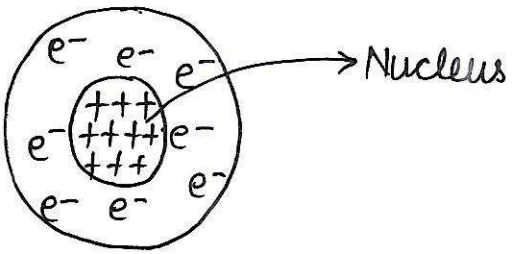


#Second Harmonic Generation-

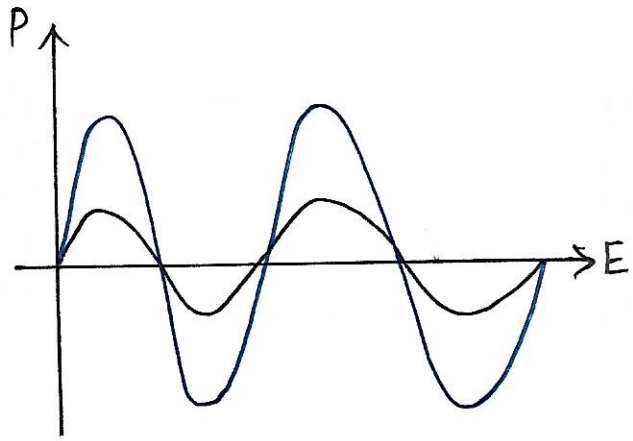
With the availability of high power laser beam, there has been a considerable amount of work on non linear interaction of optical beams with matter. One of the most striking non-linear effects of the phenomenon is second harmonic generation, in which one generates an optical beam of frequency 2ω from the interaction of a high power laser beam of frequency ω with a suitable crystal.

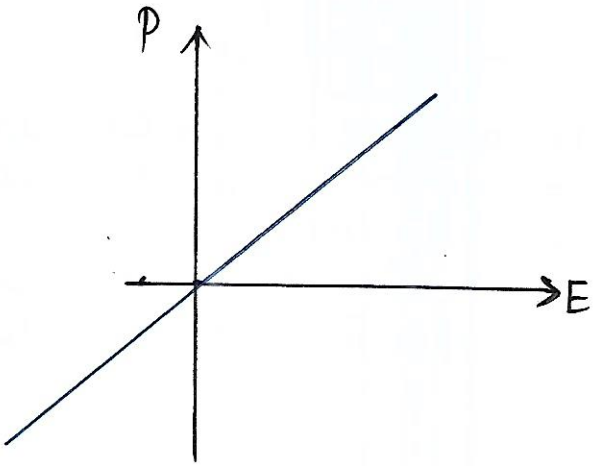


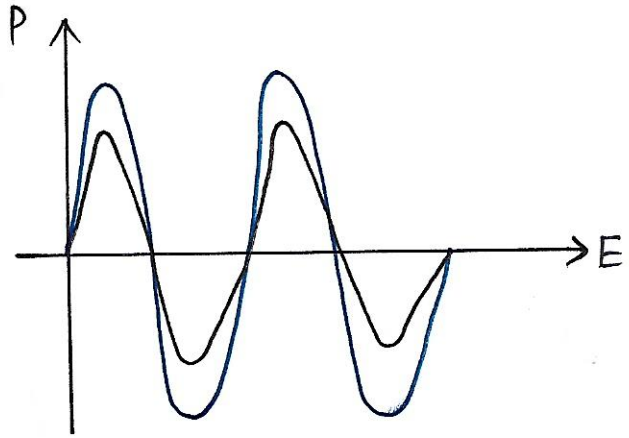
In order to understand second harmonic generation, we consider the non-linear defendance of the electric polarization on the electric field. It will be known that atom consisting of a positively charged nucleus surrounded by an e^-.

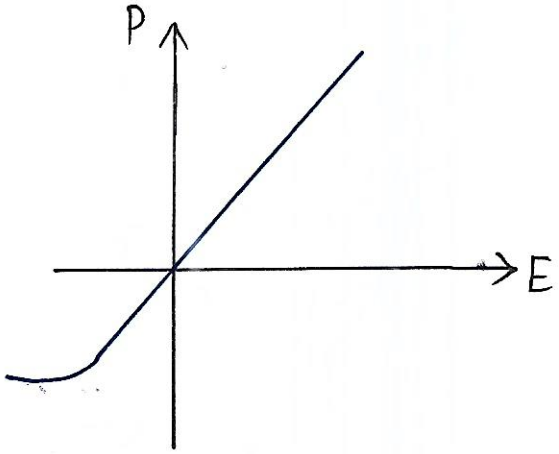


→ Electric Field is applied, e^- clouds get displaced and each atom behaves like electric dipole.



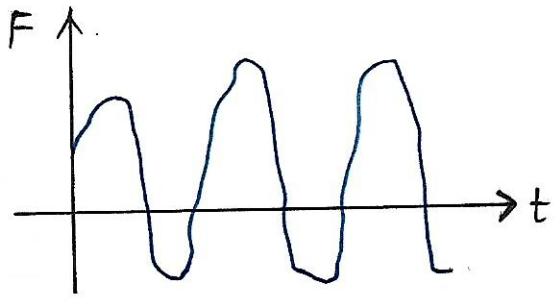


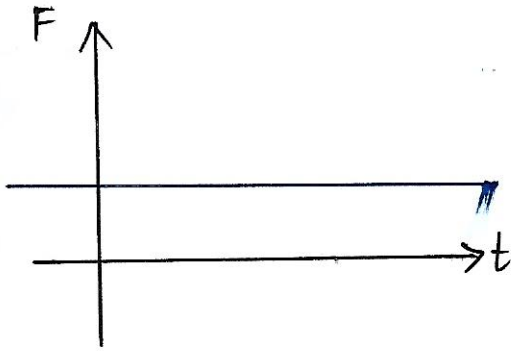


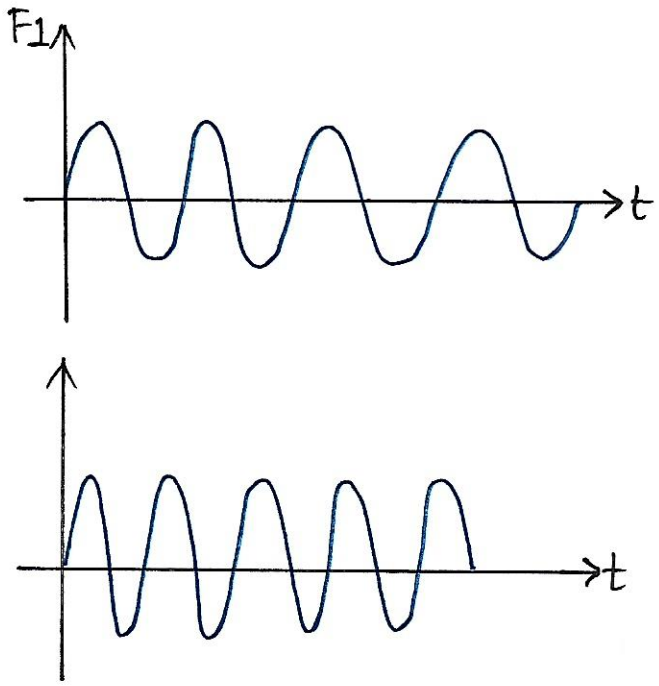


$P = \epsilon_0 \chi E$

$P = \epsilon_0 \chi E + 2\epsilon_0 d E^2$ → does not possess inversion Symmetry







→ It is forier Analysis.
→ d is tensor of Rank 3 of elements

$$E = E_0 \cos(\omega t - Kz) \ldots\ldots\ldots (1)A$$

Then

$$P = \epsilon_0 \chi E_0 \cos(\omega t - Kz) + 2\epsilon_0 d E_0^2 \cos^2(\omega t - Kz)$$

$$P = \underbrace{\epsilon_0 \chi E_0 \cos(\omega t - Kz)}_{\omega} + \underbrace{\epsilon_0 d E_0^2}_{\text{constant}} + \underbrace{\epsilon_0 d E^2 \cos 2(\omega t - Kz)}_{2\omega}$$

Then

$$\boxed{K(2\omega) = 2K(\omega)}$$

And

$$\boxed{n(2\omega) = n(\omega)}$$

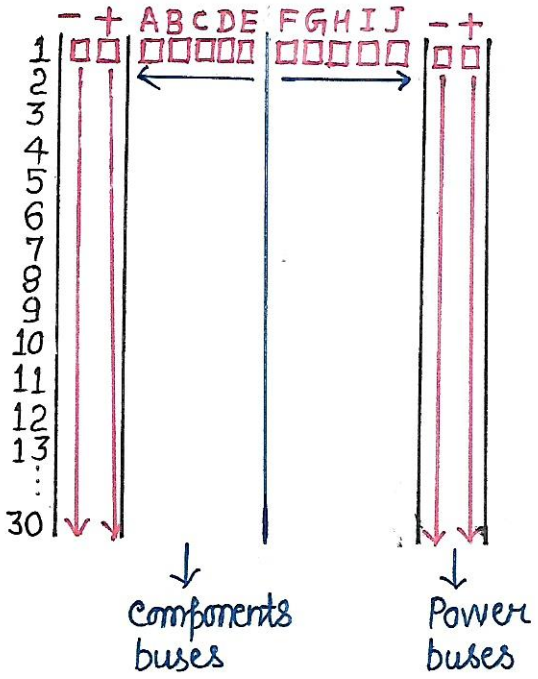
SOP/FOS/PHY/E006
Circuit Designing

Laboratory Course-2

Topic-Study of timer (555 IC)

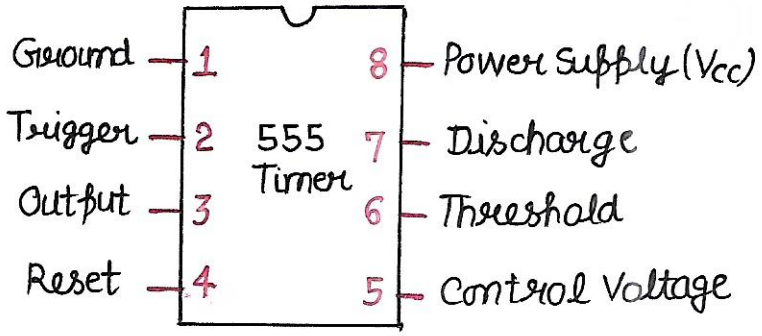
How to use Breadboard?

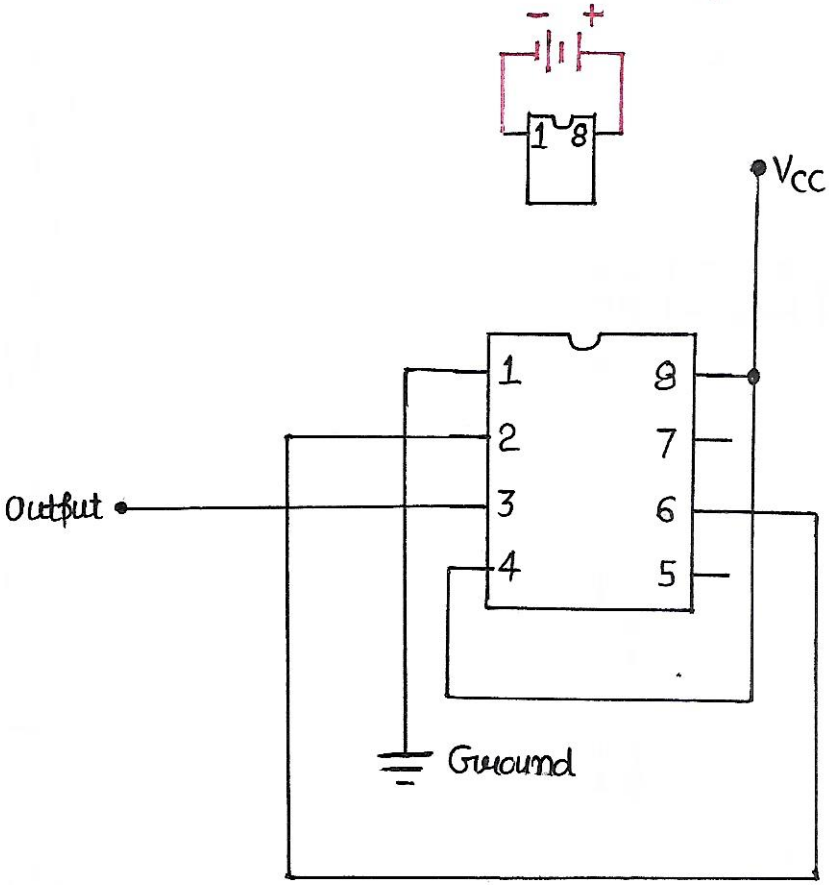
→ To much points
→ connecting points.
→ 840 holes.
→ can add 2 or more breadboards.
→ Ni Alloy clips.
→ Arranged symmetrically.
→ 0.1 inch gap b/w each point from neighbour point
→ Jumper wire for connections.
→ can substitute the single breadbord in 3 pieces
→ Can built circuits.
→ Solderless boards.



→ A1 to E1, all are connecting.
→ F1 to J1, all are connecting.
→ Check Path of flow?
→ Always check correct row of connection.
→ Check wires?
→ Loose connections.
→ Always check Polarity.
→ Can use hookup wires.
→ Can use solid core wires.

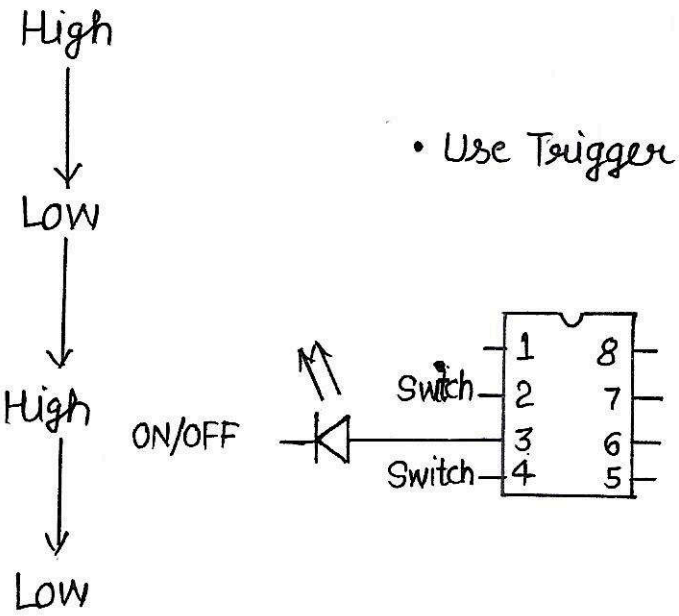
What is IC 555 timer?





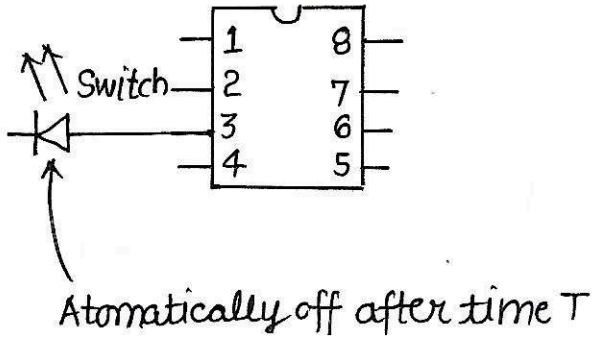
→ 8 Pin IC
→ Frequency generator
→ Astable Mode
→ Bistable Mode → Digital 0 or 1 (Use Trigger)
→ Multistable (Monostable) Mode

(1) Bistable Mode-



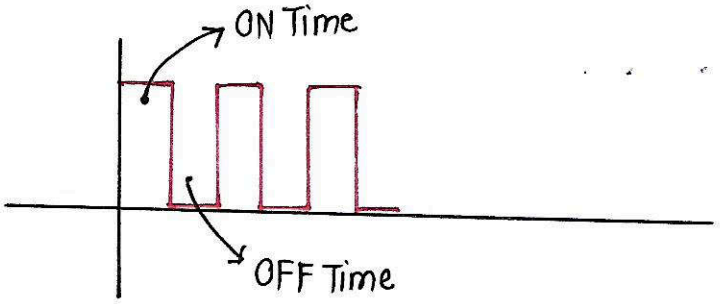
• Use Trigger

(2) Monostable state-



$T = 1.1\,RC$

(3) Astable Mode-

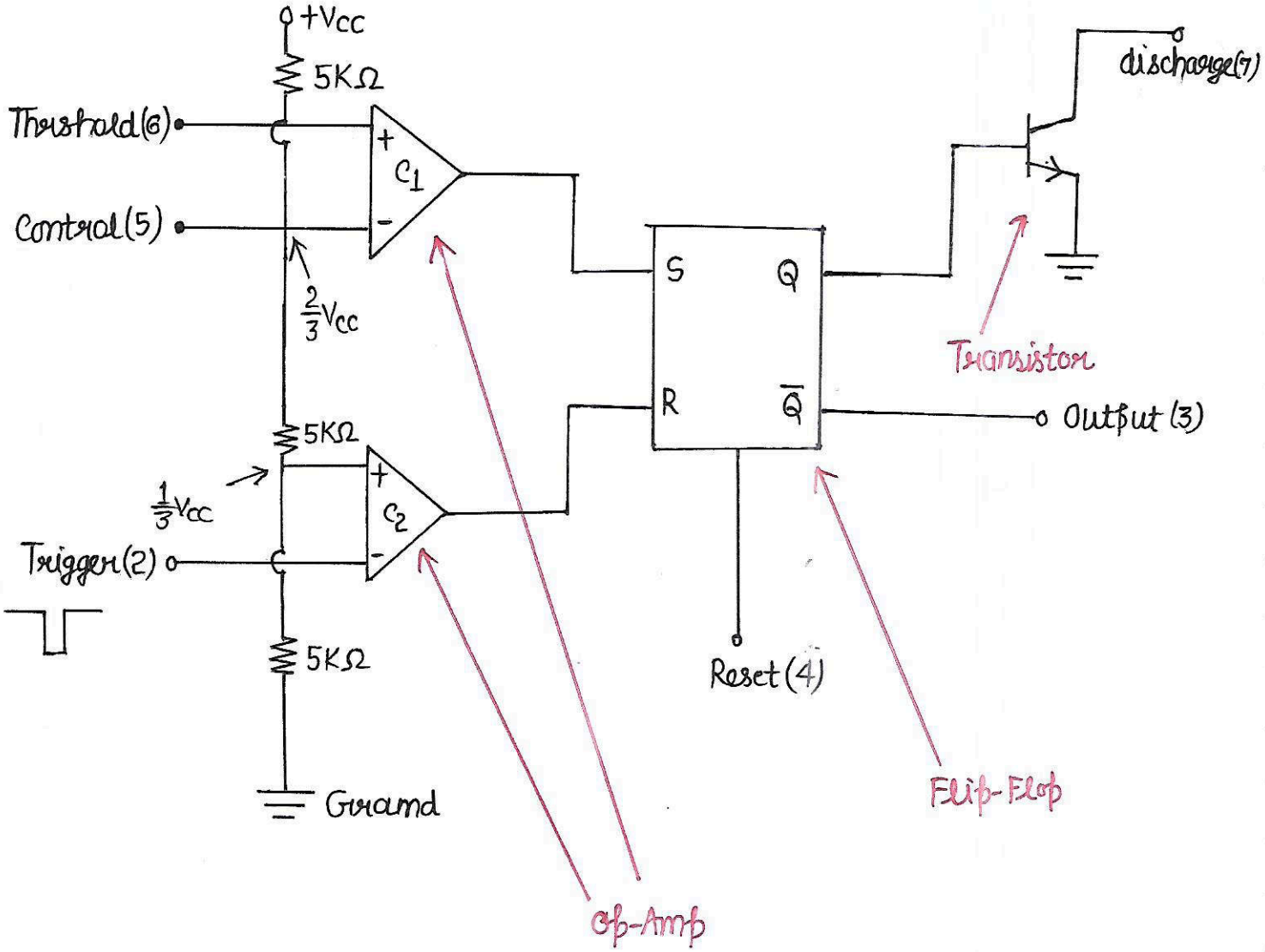


$T_{ON} = 0.693(R_1 + R_2)C$

$T_{OFF} = 0.693\,R_2C$

→ Introduced in 1970 by Signatics co.

→ IC 555 timer is used for generation of square wave (Asymmetrical and symmetrical), Saw-tooth wave, various other Application such as Asstable, Monostable and bistable multiviberator.



→ Due to these 5KΩ Resistor, this IC is known as IC 555 timer.

→ Newer Version IC 556.

Applications / Advantages and Disadvantages

- **Advantages-**

  (1) Supply voltage is low (4.5V to 15V)
  (2) Supply current (3 to 6 mA)
  (3) Operating temperature 0 to 75°C
  (4) Compact in size

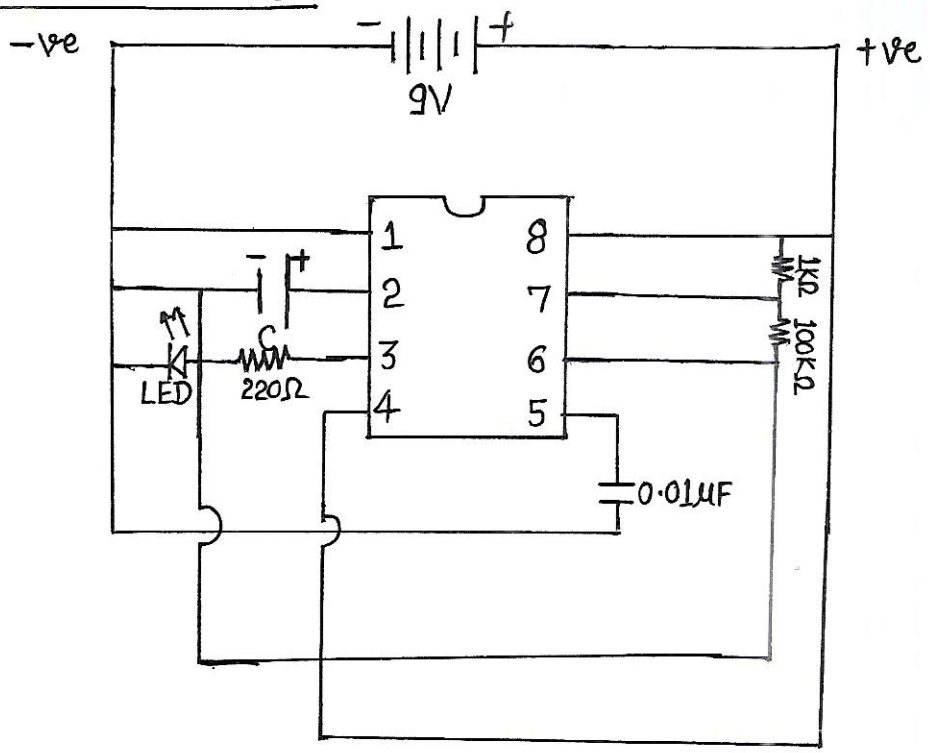
- **Disadvantages-**

  (1) Hard to solder with components on PCB

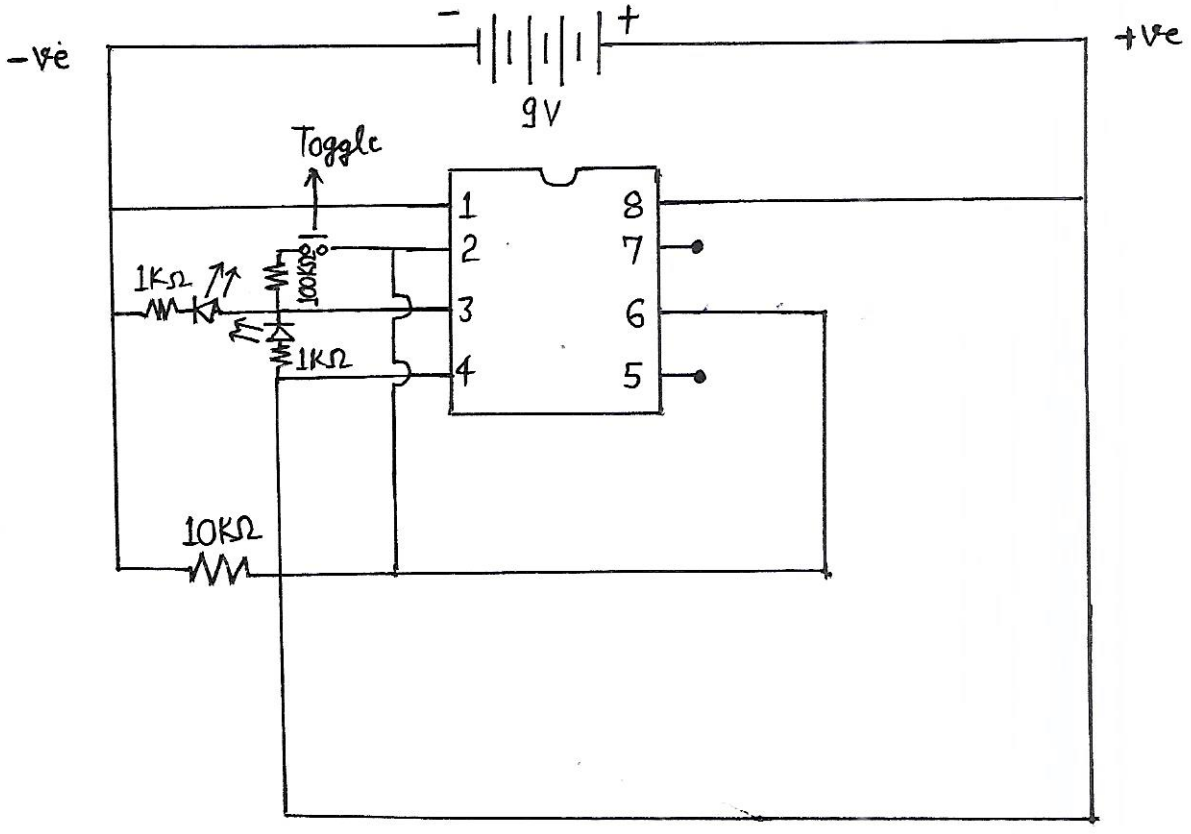
- **Applications-**

  (1) Blinking LED.
  (2) Police CAR siren.
  (3) Light detectors.
  (4) Joysticks and Game Paddels.
  (5) Lamp dimmer.
  (6) Digital Logic probes.
  (7) Analog Frequency meters.
  (8) Quard Timer Applications.

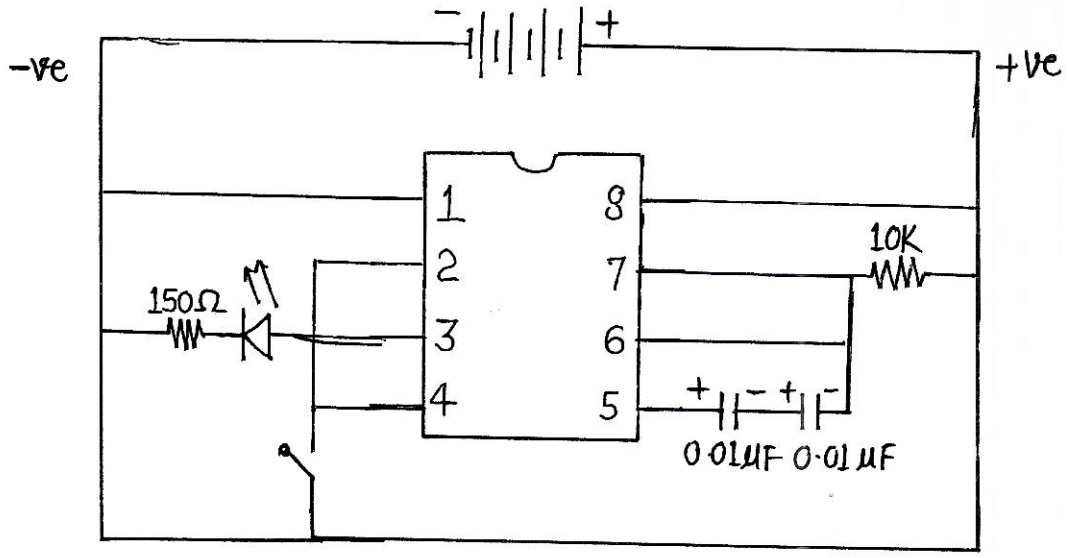
(1) Astable Circuit Design -



(2) Bistable Circuit Diagram -



(3) Monostable circuit Diagram-



PRESENTATIONS

SEMINAR

→ Development of Magnetically Impelled Arc Welding Setup

Hello guys,
This is Abhishek Bohare. Today, I'm going to present my work on magnetic pulse welding setup which was carried out at the Department of Mechanical Engineering under visiting student Programme of Indian Institute of Technology Powai, Mumbai.
So,
The outlines of my work are;

- → What is interdisciplinary Approach when we start research
- → Introduction about the project.
- → Block diagram of our setup
- → Experimental setup
- → RC circuit and why RC circuit is used
- → PPE and why PPE
- → Results
- → How I connect this Electromechanical Approach to Quantum Mechanics?
- → Some discussion about this work.
- → Application related to EM Pulse.
- → Publication and all
- → At last I will conclude this work with Thanks to all of you.

What is Interdisciplinary Approach when we start research?
Lets take some examples first,
Pacemaker is a medical device that generates electrical impulse delivered by electrodes to contract the heart muscles and regulate the electrical conduction system of the heart.

Medical Physics → Biomedical Engineering
↓
Nanotechnology
↓
Electrical Engineering

Heart Valve: A heart valve normally allows blood to flow in only one direction through the heart.

Geomathematics: Geomathematics or Mathematical Geophysics is the application of mathematical intution to solve problem in geophysics.

→ gravitational
→ geomagnetism
→ Seismology
→ Climate & Weather
→ Ocean Circulation
→ Imaging Science
→ Plate Techtonics
→ Geomechanics/Geodynamics
→ Remote Sensing

NOW I come on the Introduction part.

→ High Speed Welding process
→ Similar and dissimilar metals
→ Cold Welding process

This is the block diagram.

Capacitor Bank:

KVAR

$P = VI\cos\theta$

delta → Parallel 3C
Star → Series $\frac{C}{3}$

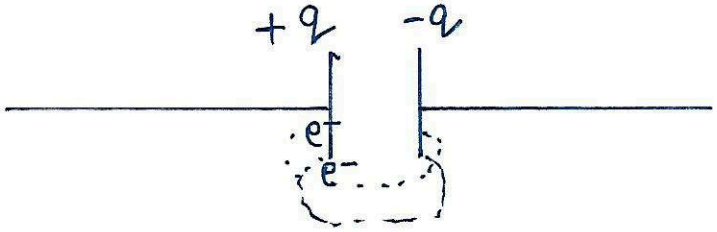
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \qquad C\uparrow, X_C\downarrow, Z\uparrow, I\downarrow, \cos\phi\uparrow$$

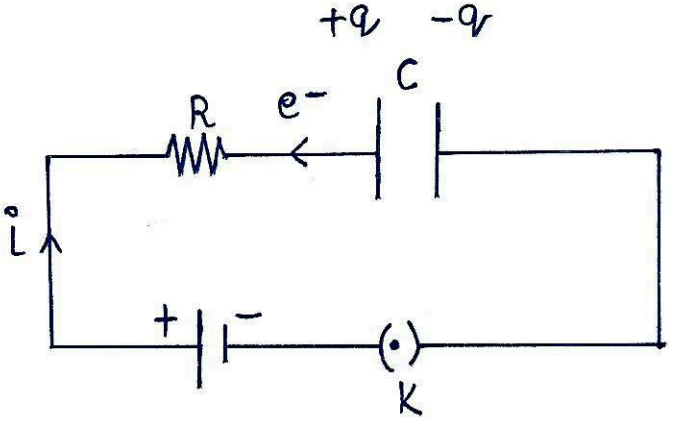
*delta collections always gives 3 time more reactive power than star connection of same rating of capacitor.

⇒ Charging and discharging of capacitor (RC circuit):

Charging → Growth of charge

Potential diff = $\frac{Q}{C}$





$$i = \frac{V}{R}$$

At $t = 0$

$q = 0$

$$\Delta V = \frac{q}{C} = 0$$

At some time t

q = q on capacitor

ΔV across capacitor $= \frac{q}{C}$

$$i = \frac{V - \frac{q}{C}}{R}$$

$q \uparrow \Leftrightarrow i \downarrow$ in charging

$$\frac{dq}{dt} = \frac{CV - q}{RC}$$

$$\int_0^q \frac{dq}{CV - q} = \int_0^t \frac{dt}{RC}$$

$$\left[\frac{\ln(CV - q)}{(-1)}\right]_0^q = \frac{1}{RC}(t)_0^t$$

$$\left[\ln(CV - q) - \ln(CV - 0)\right] = \frac{-t}{RC}$$

$$\ln\left(\frac{CV-q}{CV}\right) = \frac{-t}{RC}$$

$$\frac{CV-q}{CV} = e^{-t/RC}$$

$$1-\frac{q}{CV} = e^{-t/RC}$$

$$\frac{q}{CV} = 1-e^{-t/RC}$$

$$q = CV(1-e^{-t/RC}) \Rightarrow q = q_0(1-e^{-t/RC})$$

$$\boxed{V = V_0(1+e^{-t/RC})}$$

→ A capacitor can't be charged 100%.

When $t = T$

$$q = q_0(1-e^{-T/T})$$

$$q = q_0(1-e^{-1})$$

$$q = q_0(0.63)$$

$$q = q_0 \times 0.63$$

In time T, capacitor 63% charged.

Important Questions for Final Examinations

Quantum Mechanics

- Born Approximation and its Validity ✓✓
- Scattering Amplitude ✓✓
- Differential & Total cross section ✓✓
- Partial wave analysis
- Asymptotic behaviour of partial wave
- Phase Shift
- Optical Theorem ✓✓
- Schrodinger Equation for a system consisting of Identical particles ✓✓
- Symmetric and Anti-symmetric wave functions ✓✓
- Ortho and Para helium
- Schrodinger Relativistic Equation ✓✓
- Klein-Gorden Equation and drawbacks ✓✓
- Current and charge densities ✓✓
- Klein-Gordon Equation for Electromagnetic field ✓✓
- Dirac relativistic Equation ✓✓
- Negative Energy state ✓✓
- Spin-Orbit Coupling
- Quantization of non-relativistic schrodinger Equation ✓✓
- Second Quantization/First Quantization ✓✓
- N-Representation, Creation & Annihilation operator ✓
- Scattering of Identical Particles.
- Resonant and Non-Resonant Scattering ✓
- Numeric Problems ✓

Nuclear Physics

- Nuclear Radius
- Nuclear Angular momentum
- Nuclear Magnetic moments + Electric Quardpole Moment
- Binding Energy
- Liquid drop Model ✓
- Shell Model ✓
- Collective Model

} Applications

- Ground State of Deutron
- GM Counter ✓✓
- Exchange and Tensor Force
- Radioactive Equilibrium ✓✓
- Gamow Theory of α decay and Geiger Nuttal Law ✓✓
- Fermi Theory of β decay
- Electromagnetic Decay
- Low energy n-p, p-p scattering
- Q Value of nuclear reaction
- Bohr Theory of compound Nucleus.
- Breit Wigner Theorem

Electronics-A

- Binary Conversions ✓
- Binary Postulates ✓✓
- De-Morgan's Theorem ✓✓
- K-Map ✓✓
- Boolean Expression and Function ✓✓
- Logic gates ✓✓
- TTL Logic ✓✓
- CMOS Logic ✓✓
- Adder/Subtractors ½ ✓✓
- BCD Adder ✓
- Mux/DeMux ✓✓
- Encoder/Decoder ✓✓
- Parity Checker ✓✓
- Flip Flops : Latches, RS, D, JK, T ✓✓
- Master Slave Flip Flop ✓✓
- Shift Registers
- Classification of Memories ✓✓
- RAM ✓✓
- ROM ✓✓
- Registers and Counters
- Programmable Logic Array ✓✓
- IC Technology ✓✓
- MOS Capacitors ✓✓
- Bipolar Transistors ✓✓
- Thin Film Technology ✓✓

LASER Physics

- Theory of Absorption and Emission and Einstein Cofficients
- Population Inversion
- Theory of optical Resonator
- Laser Modes
- Spatial and Temporal Coherence → 1
- He-Ne Laser ✓
- CO_2 Laser ✓
- Semiconductor Laser ✓
- Ruby Laser ✓
- Nd: YAG Laser ✓
- Laser Spikes ✓
- Mode Locking, Q Switching ✓ → 2
- Phase Matching
- Second and Third Harmonic Generation
- Parametric Generation
- Self focussing → 3
- Stimulated Raman Spectroscopy
- Coherent Antistokes Raman Spectroscopy → 4

Quantum Field Theory

PARTICLE PHYSICS

- UNIT 1 : Classification and Elementry properties of Particle.
- UNIT 2 : Conservation Laws and Gauge Invariance.
- UNIT 3 : Fundamental Interaction
- UNIT 4 : Quark Model

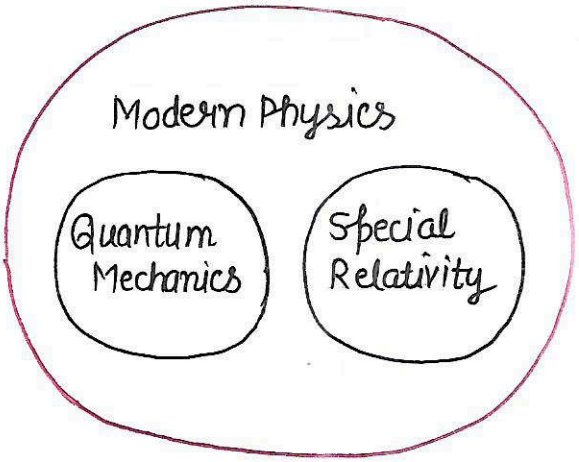
UNIT-1

Classification and Properties of Elementary Particl

Syllabus:
Elementary particles, their classification on the basis of mass and spin. (Laptons, Mesons, Baryons) and Field Quanta, their general properties (mass, spin, lifetime and their production and decay modes), Antiparticle.

- **Elementary Particle Physics's Introduction-**

Elementary Particle Physics is the study of the universe at its smallest and most fundamental level.



The synthesis of these two proved surprisingly difficult but when completed, resulted in an exceedingly successful theory. Modern particle physics is a completed theory explaining all about known phenomena except gravity and neutrino masses in terms of a handful of particles and parameters.

Einstein's Special theory of Relativity

↓

Maxwell Equations

↓

Electromagnetic theory

→ Time (4th dim.) = 3 Space dimensions

↓

Invariance under Lorentz transformation

↓

$v \not> 3 \times 10^8$ m/s

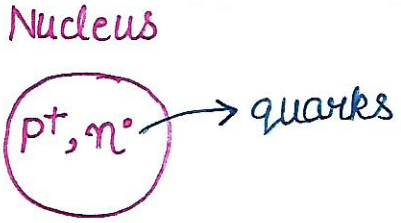
Quantum field theory, or particle physics, is the union of special relativity and quantum mechanics. If we start with classical field theory, and then quantize it, we end up with quantum field theory, in short QFT. The consequences are many; for example, whereas in conventional EMT wave of any strength are possible, in quantum field theory, only discrete value are allowed.

Questions :

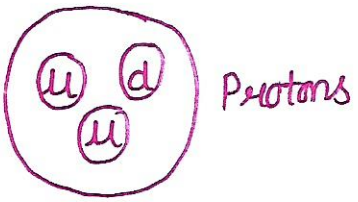
1. Fundamental particles or Elementary particles
2. Classification on the basis of mass, symbol, spin, decay scheme, lifetime.
3. Muon decay
4. Tau Theta Puzzle
5. Two component theory of neutrino or Helicity of Neutrino.
6. Strange particles (Production and decays).

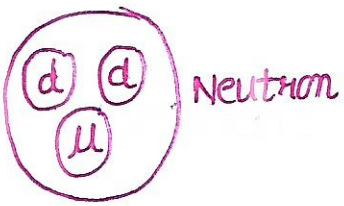
#Elementry Particles -

The neucleus of an atom is made of positively charged protons and electrically neutrons.
neutral

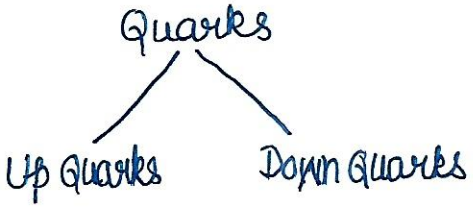


Protons and neutrons are made of more fundamental particles called quarks





This we know from scattering experimental.



(u) $\longrightarrow +\frac{2}{3}e$

(d) $\longrightarrow -\frac{1}{3}e$

Electrons, on the other hand, are structur less to the extent we know

- All matter around us is made up of up quarks and down quarks and the electrons.

- Wolfgang pauli $\longrightarrow$ β decay
- A neutral particle called neutrino was observed in β decay.

Many more particles found in cosmic rays (1930-1950)

$\Downarrow$

Radiation

$\downarrow$

muon μ, similar to the e^- but heavier
pions π, Kaon K, Lambda Λ etc.
(Identified as the composition made of quarks)

These were then rediscovered in various laboratory exp.

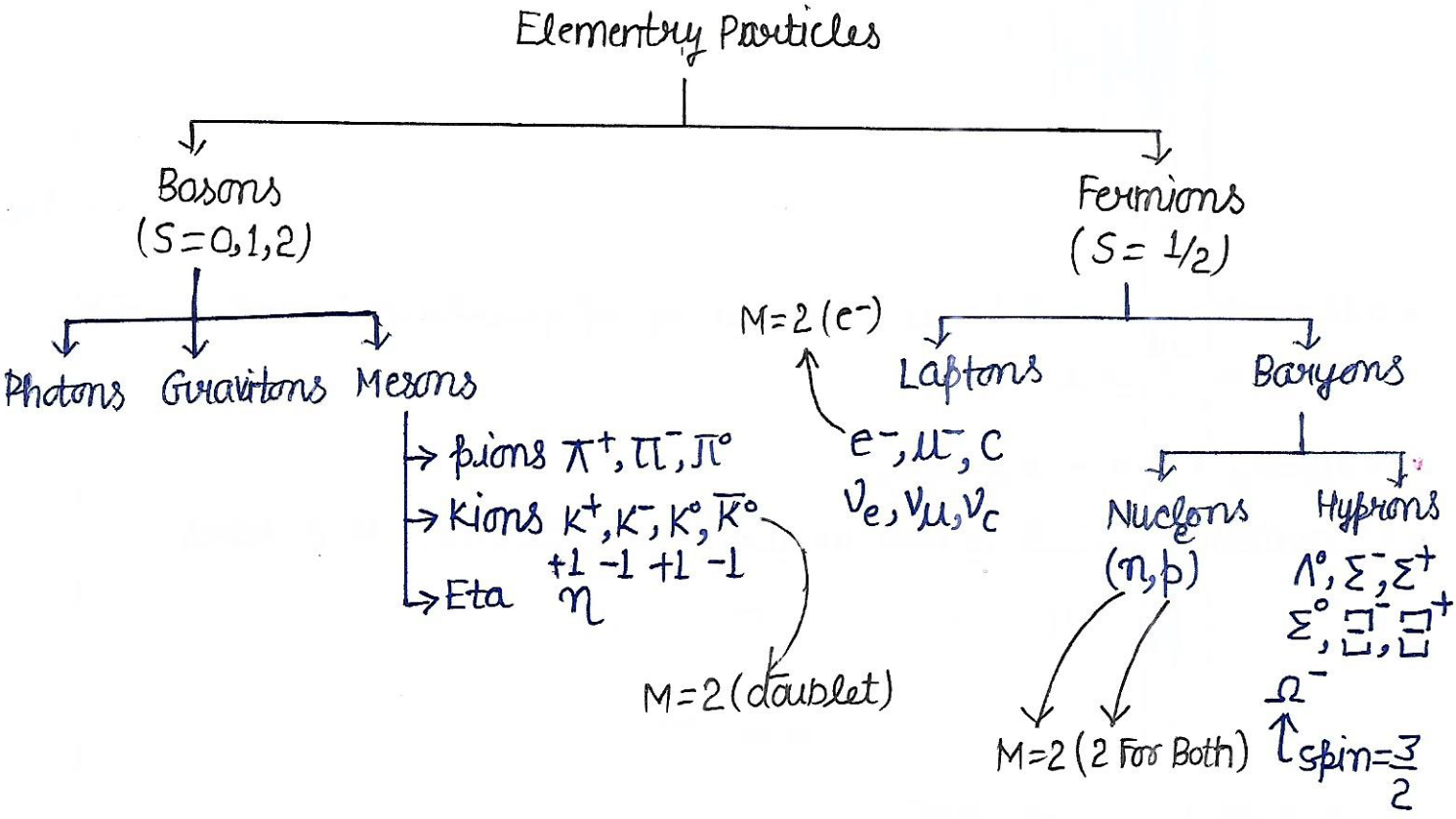
Particle Collider experiments also discovered newer particles, otherwise not found in cosmic rays elsewhere in nature.

Reference: The particle Hunters, Y. Ne'eman and Y. Kirsh, Cambridge.
Conceptual Foundations of Modern Physics (particle) R.E. Marshak

Elementary Particles

Leptons		Quarks	
electron e	charge = −1, Mass = 0.5 MeV/c^2	Up quark, u, c = +2/3, m = 2.3 MeV/c^2 ?	
e Neutrino ν_e	charge = 0, Mass = ?	down quark, d, c = −1/3, m = 4.8 MeV/c^2 ?	
Meon μ	charge = −1, Mass = 106 MeV/c^2	charm quark C, c = +2/3, m = 1275 MeV/c^2	
μ Neutrino ν_μ	charge = 0, Mass = ?	strange quark S, c = −1/3, m = 95 MeV/c^2	
Tau Laptons T	charge = −1, Mass = 1777 MeV/c^2	top quark t, c = +2/3, m = 173.5 GeV/c^2	
T Neutrino ν_t	charge = 0, Mass = 0 ?	bottom quark b, c = −1/3, m = 4.8 GeV/c^2 ?	

- Questions only in Part C (5 Marks)



Properties or parameters:

- Baryons no. will be +1 for baryons only & for their antiparticles it will be −1. For all other particles it will be 0.
- In Laptons

  +1 For Laptons
  −1 For their Antiparticles
  0 For All other particles

- Strangeness, Only Exists for strange particles, shows different behaviour (different decay)

- Kiaons, Hyprons

+1 K^+	−1 $\Lambda^\circ, \Sigma^+, \Sigma^-, \Sigma^\circ$
−1 K^-	−2 Ξ^-, Ξ°
+1 K°	−3 Ω
−1 $\bar{K}^\circ$	

Problem 1: Reaction Allowed If — L (Lepton), B (Baryons), Q (Charge conservation)

Check whether the following high energy process are allowed by conservation laws or not?

(a) $p+\bar{p} \longrightarrow \Lambda^0 + \Lambda^0$

(b) $\pi^- + p \longrightarrow \pi^0 + n$ (✓)

(c) $n \longrightarrow p + e^- + \nu_e$

(d) $\mu^+ \longrightarrow e^- + \gamma$

$\pi^- + p \longrightarrow \pi^0 + n$

$-1 + 1 \longrightarrow 0 + 0$

$0 + 0 \longrightarrow 0 + 0$

$0 + 1 \longrightarrow 0 + 1$

Problem 2: Check the reactions allowed or not?

(A) $\pi^- + p \longrightarrow \Lambda^0 + \pi^0$

Q $-1 + 1 \longrightarrow 0 + 0$

B $0 + 1 \longrightarrow 1 + 0$

L $0 + 0 \longrightarrow 0 + 0$

(B) $p + \bar{p} \longrightarrow 2\pi^+ + 2\pi^- + 2\pi^0$

Q $1 - 1 \longrightarrow 2 - 2 + 0$

B $0 \longrightarrow 0$

L $0 \longrightarrow 0$

- For general, check only Q, B, L

- Multiplicity:

M = different charge states of any particles

η

M = 1

$\pi^+, \pi^-, \pi^\circ \rightarrow M = 3$

$\Xi^-, \Xi^\circ \rightarrow M = 2$

Why we need multiplicity?

Isospin finding will be easy.

$M = 2I + 1$

- Strong reaction Interaction (All conservation Laws satisfied).
- Isospin will be conserved in strong interaction but in EM, Weak Interaction, not allowed.
- Isospin will have same value of all the particles having different charge states.
- I_3 or I_Z ⟶ 3rd component of Isospins.

↓

different for different charge states.

$I = 1$ for π^-, π°, π^+

↓ ↓ ↓

I_3 → −1 0 +1

↓

To know the exact value of I_3, we will use Gell-mann Nishijima formula.

$$Q = I_3 + \frac{B+S}{2}$$

I_3 → Strong ✓
EM ✓
Weak X

I → Strong ✓
EM X
Weak X

- Hypercharge $Y = 2\bar{Q}$ → Double of the average charge

$$Y = B + S$$

- Strangeness Number-

difference of the hypercharge Y and the Baryon number B

$$S = Y - B \Rightarrow Y = S + B$$

$$A + B \longrightarrow C + D$$

$$S_1 \quad S_2 \qquad S_3 \quad S_4$$

$$\Delta S = (S_3 + S_4) - (S_2 + S_1)$$

→ S is conserved in strong and EM reactions

→ For the weak interaction $\Delta S = 0$ or ± 1 → Not conserved

↓ ↳ Conserved

ΔS = Total Strangeness no. products - Total S of reacta

- Quarks-

$I_3 = +1/2$ # Up Quarks
$I_3 = -1/2$ # Down Quarks
$I = 1/2$

$I = 0$
- # Strange Quarks
- # Charm Quarks
- # Top Quarks
- # Bottom Quarks

$I = 0$

Baryon No. for all = $1/3$
Spin = $1/2$

	I	I_3	B	S	Q	Y = B+S	Spin
Up	$1/2$	$1/2$	$1/3$	0	$2/3$	$1/3$	$1/2$
down	$1/2$	$-1/2$	$1/3$	0	$-1/3$	$1/3$	$1/2$
Strange	0	0	$1/3$	-1	$-1/3$	$-2/3$	$1/2$
Top	0	0	$1/3$	0	$2/3$	$1/3$	$1/2$
Bottom	0	0	$1/3$	0	$-1/3$	$1/3$	$1/2$
Charm	0	0	$1/3$	0	$2/3$	$1/3$	$1/2$

- Mesons-

#Quarks structures

#Quarks-Antiquarks

$\bar{u}d$

Baryon uud

uud

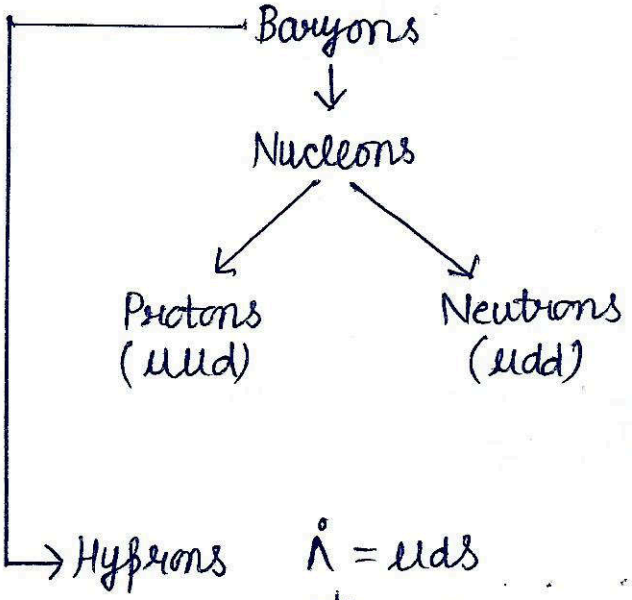
$\frac{2}{3}+\frac{2}{3}-\frac{1}{3}$

$\frac{4}{3}-\frac{1}{3}$

$\frac{3}{3}$

$Q=1$

→ Protons, Σ^+



$\Lambda^0 = uds$

$\Sigma^+ = uus$

$\Sigma^0 = uds$

$\bar{\Sigma} = dds$

$\Xi^0 = uss$

$\Xi^- = dss$

$\Omega^- = sss$

Mesons —

$\pi^+ (u\bar{d})$

$\pi^0 \ \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$

$\pi^- (d\bar{u})$

$K^+ (u\bar{s})$

$K^0 (d\bar{s})$

$K^- (s\bar{u})$

$\eta^0 \ \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$

- Baryon Resonance*

Particles	I	I_3	B	S	Quark Structure
Δ^{++}	3/2	3/2	1	0	uuu
Δ^+	3/2	1/2	1	0	uud
Δ^0	3/2	-1/2	1	0	udd
Δ^-	3/2	-3/2	1	0	ddd
Σ^{*+}	1	1	1	-1	uus
Σ^{*0}	1	0	1	-1	uds
Σ^{*-}	1	-1	1	-1	dds
Ξ^{*0}	1/2	1/2	1	-2	uss
Ξ^{*-}	1/2	-1/2	1	-2	dss
Ω^-	0	0	1	-3	sss

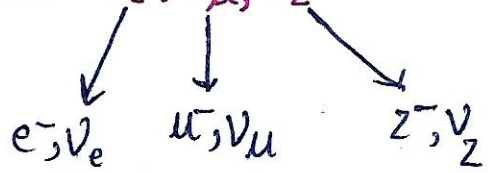
Types of Interactions-

	Range	Strength
1. Strong	10^{-15} m or 1 fm	1
2. Weak	10^{-18} m (Also known as contact Forces)	10^{-13}
3. Electromagnetic	Range ∞	10^{-2}
4. Gravitational	Range ∞	10^{-42}

Exchange Particles-

Strong $\longrightarrow$ π $\longrightarrow$ Spin 1
EM $\longrightarrow$ Photon $\longrightarrow$
Weak $\longrightarrow$ $W^{\pm}, Z^{\circ}$ $\longrightarrow$ 1
Gravitational $\longrightarrow$ Graviton

For various particles which exist in Lepton catagory, we can write their Lepton no. as L_e, L_μ, L_Z



L_e	+1	e^-, ν_e
	−1	$e^+, \bar{\nu}_e$
	0	others
L_μ	+1	μ^-, ν_μ
	−1	$\mu^+, \bar{\nu}_\mu$
	0	others
L_Z	+1	Z^-, ν_Z
	−1	$Z^+, \bar{\nu}_Z$
	0	others

If any reaction is possible then L no will be conserved for [Also B,Q].

In general, for all particles $L = 1$

for Lepton Antiparticle $L = -1$

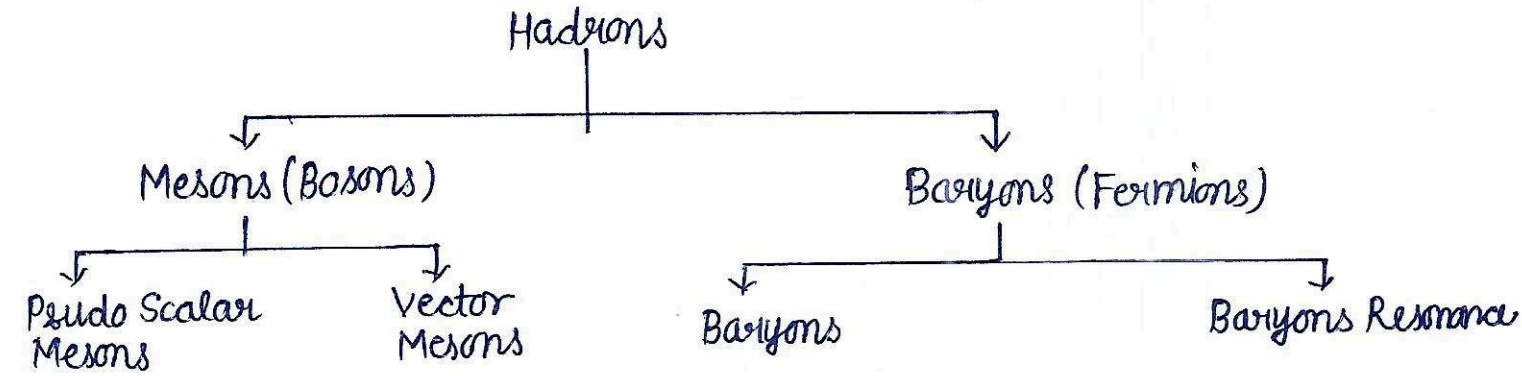
But L_e, L_u, L_Z as we have considered.

How we define particles?

↳ depending on the interactions in which they are involved.

Leptons → do not participate in strong interactions.

Hadrons → do not participate in strong interactions.



(Psudo means virtual here)

Conservation Laws -

Exact Conservation Laws

↳ If they voiletes, reaction will not be conserved or will not be allowed.

Lepton no., Baryon no., charge, Energy, Linear momentum, Angular momentum

Approximate Conservation Laws

↳ If they violets, these reactions can be allowed.

strangeness no., Isospin, CPT

CPT → charge configuration, Time reversal, Parity

If Anytime you get the information about isospin written as

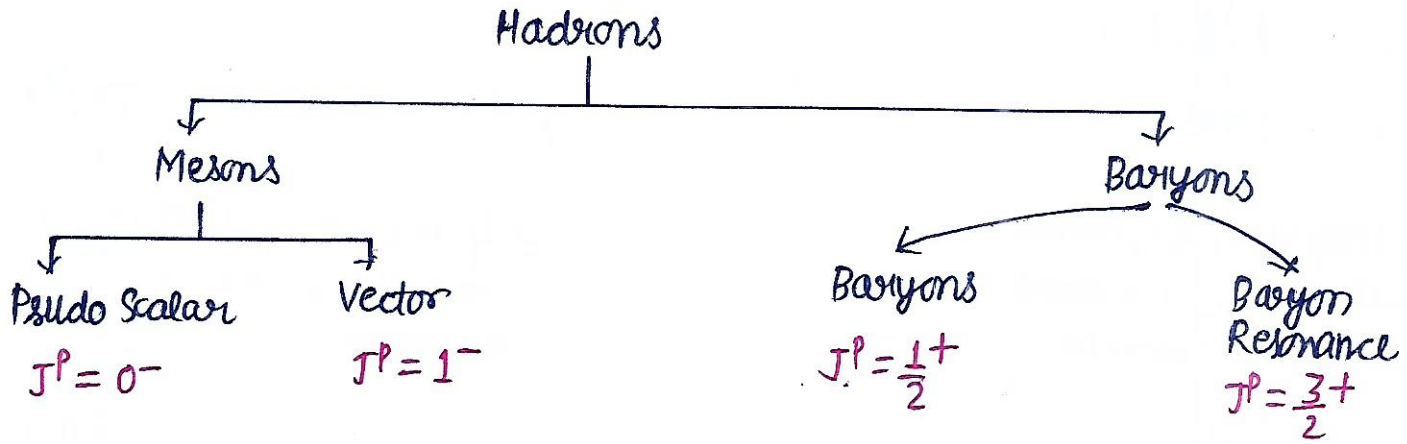
$|I, I_3\rangle$ Like for proton $|\frac{1}{2}, \frac{1}{2}\rangle$

- Multiplicity of $\Delta^{++} = 4$
- Why strange particles known as strange?
  Because strange Particles form with stronge Interaction and decay with weak Interaction.

Gell-Mann Nishijima Formula!

$$Q = I_3 + \frac{B+S}{2}$$

$[(Y = B+S)$ Hypercharge$]$



$J \rightarrow$ Angular momentum Quantum no.
$P \rightarrow$ Parity

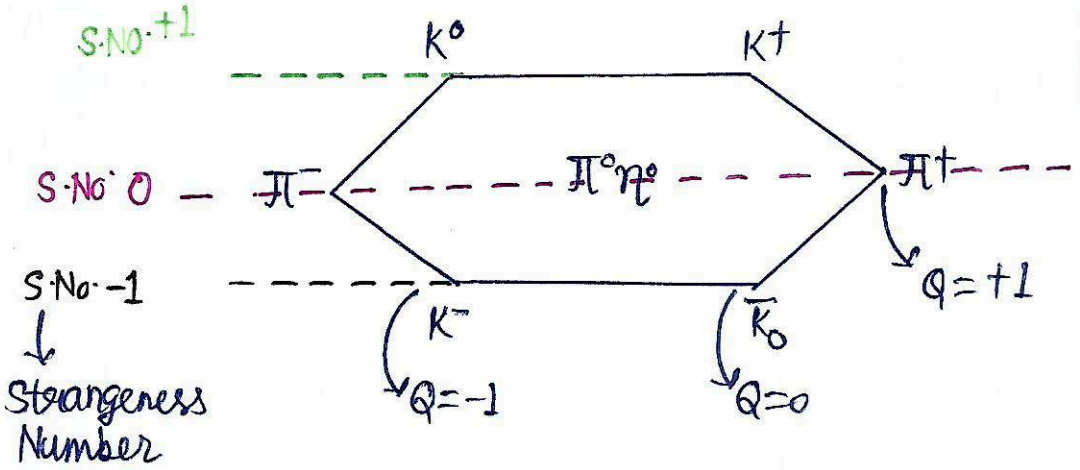
Vector Mesons $J^P = 1^-$

Particles	I	I_3	B	S	Y	Quark Structure
φ^+	1	+1	0	0	0	$u\bar{d}, u\bar{s}, d\bar{d}, s\bar{s}$
φ^0	1	0	0	0	0	$d\bar{u}\ u\bar{u}, d\bar{d}, s\bar{s}$
φ^-	1	-1	0	0	0	$u\bar{s}\ d\bar{u}$
K^{*+}	1/2	+1/2	0	+1	+1	$d\bar{s}\ u\bar{s}$
K^{*0}	1/2	-1/2	0	+1	+1	$s\bar{u}\ d\bar{s}$
$\bar{K}^{*0}$	1/2	+1/2	0	-1	-1	$s\bar{u}, s\bar{d}$
K^{-*}	1/2	-1/2	0	-1	-1	$s\bar{u}$
W^0	0	0	0	0	0	$u\bar{u}, d\bar{d}, s\bar{s}$

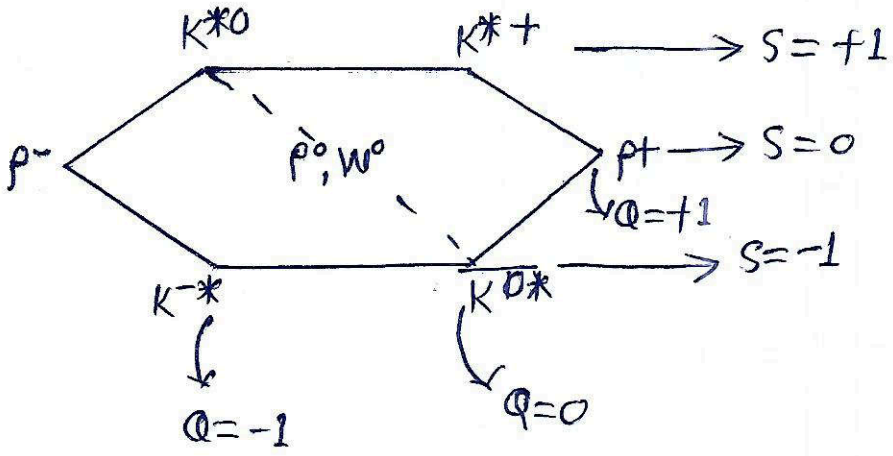
- Psude Mesons $J^P = 0^-$

Particles	I	I_3	B	S	Y	Quark structure
π^+	1	+1	0	0	0	$u\bar{d}\ \frac{2}{3}+\frac{1}{3}=1$
π^0	1	0	0	0	0	$u\bar{u}, d\bar{d}, s\bar{s}$
π^-	1	-1	0	0	0	$\bar{u}d$
K^+	1/2	1/2	0	+1	+1	$u\bar{s}$
K^0	1/2	-1/2	0	+1	+1	$d\bar{s}$
$\bar{K}^0$	1/2	1/2	0	-1	-1	$\bar{d}s$
K^-	1/2	-1/2	0	-1	-1	$\bar{u}s$
η^0	1/2	0	0	0	0	$u\bar{u}, d\bar{d}, s\bar{s}$

Baryon Octate



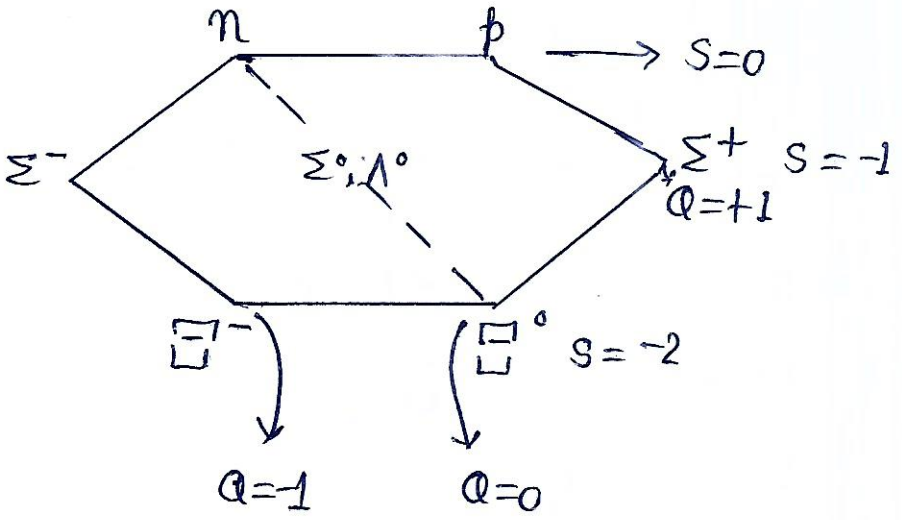
For Vector Mesons



Baryons $J^P = \frac{1}{2}^+$

Particles	I	I_3	B	S	Quark Structure
p	1/2	1/2	All +1	0	uud
n	1/2	-1/2		0	udd
Λ°	0	0		-1	uds
Σ^+	+1	+1		-1	uus
Σ°	+1	0		-1	uds
Σ^-	+1	-1		-1	dds
Ξ°	1/2	1/2		-2	uss
Ξ^-	1/2	-1/2		-2	dss

Baryon Octate



Baryons Resonance $J^P = \frac{3}{2}^+$

Particles	I	I_3	B	S	Quark Structure
Δ^{++}	3/2	+3/2	All 1	0	uuu
Δ^{+}	3/2	+1/2		0	uud
Δ^{0}	3/2	-1/2		0	udd
Δ^{-}	3/2	-3/2		0	ddd
Σ^{+*}	1	+1		-1	uus
Σ^{0*}	1	0		-1	uds
Σ^{-*}	1	-1		-1	dds
Ξ^{0*}	1/2	+1/2		-2	uss
Ξ^{-*}	1/2	-1/2		-2	dss
Ω^{-}	0	0		-3	sss

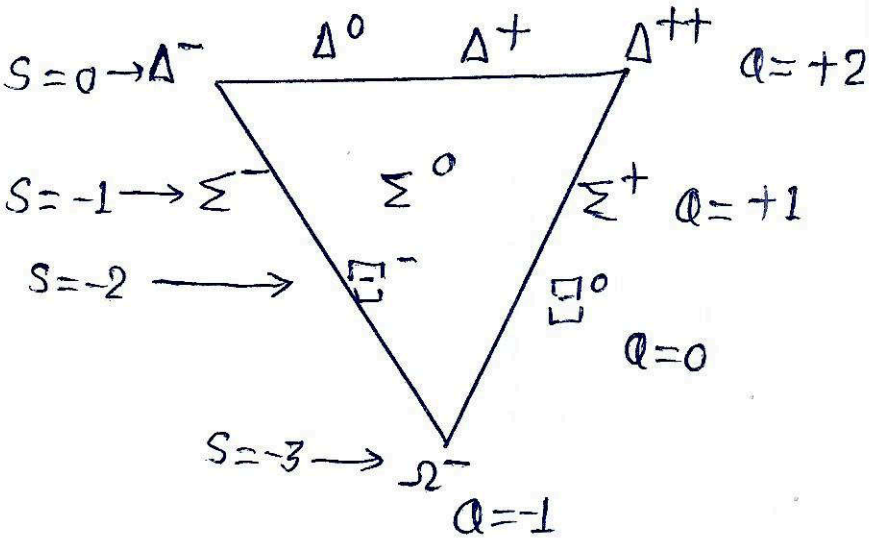
For Δ^{++}

$M = 4$

$M = 2I + 1$

$I = 3/2$

Decuplet



Characteristic Time

Strong	$< 10^{-22}$ Sec.	P, n, K, π
EM	10^{-14} to 10^{-20} sec.	e; μ, π, K, n, p
Weak	10^{-8} to 10^{-13} sec.	All ν

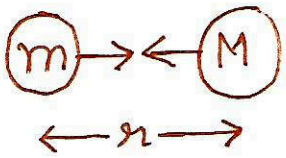
* If neutrino & Antineutrino present then weak possibly.

Fundamental Forces-

Fundamental Forces

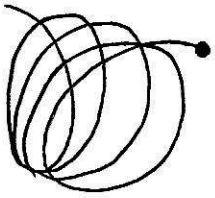
- Gravitational Force
- Electromagnetic Force
- Strong Force
- Weak Force

- Always Attaract
- Infinite Range
- Acts on anything that has mass and Energy

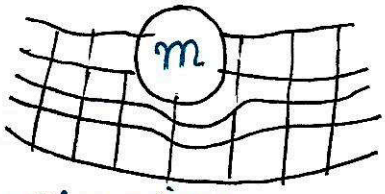


$F \propto Mm$

$F \propto \frac{1}{r^2}$



→ Perturbations in mercury Orbit



Space-time

→ According to Einstein, Gravity is concequence of space time contennum.

Modern Explaination

Lensing, Redshift, Black Hole

• EM Force –



→ Molecules → Crystal Structure

Atom

Macroscopic Properties

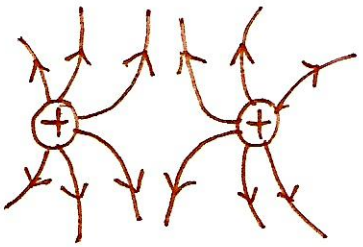
↳ optical properties

↳ Electric/Magnetic

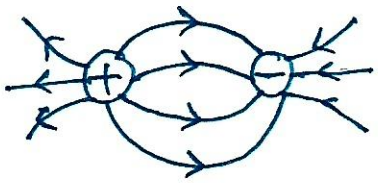
↳ Hardness/Softness

↳ Solids, Liquids, Gasss

↳ Light

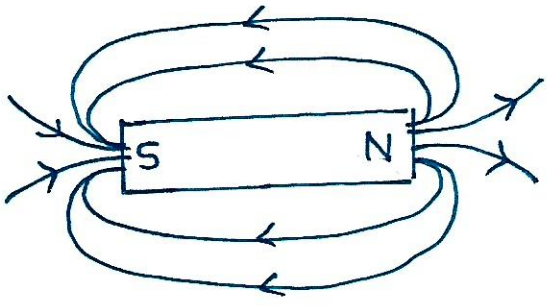


Repel

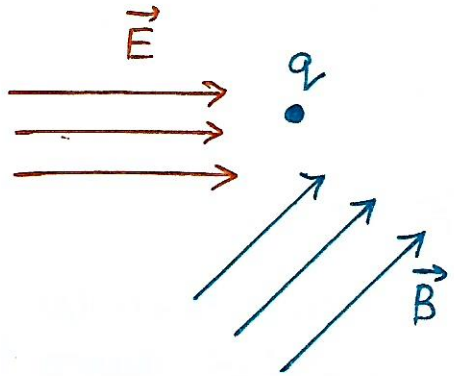


Attaract

$$F = K\frac{q_1 q_2}{r^2}$$



$$B = B_1 + B_2 + B_3 + B_4$$



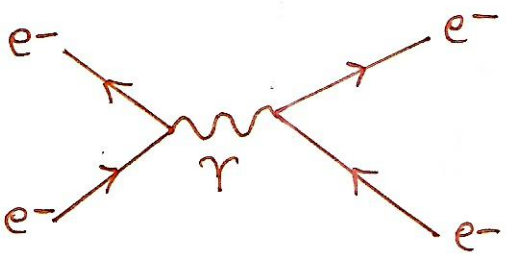
$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Maxwell Equations :

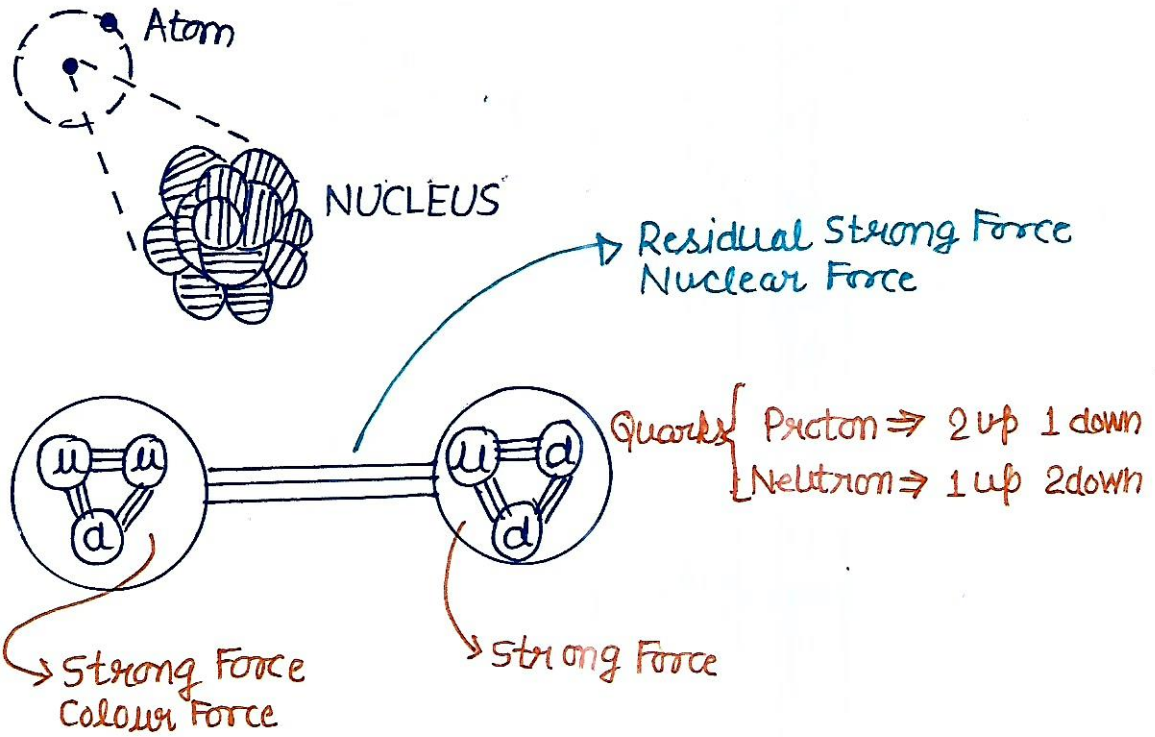
$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

→ Quantum Electrodynamics (QED)-



• Strong Force-



Strong Interaction → Acts between quarks via gluons (<0.8 fm)
Residual Strong Force → Acts between hadrons via pions (1-3 fm)

gluons → Boson particles

pions → Meson particles (Kind of Hadron particle)

QUANTUM CHROMODYNAMICS → colour is a property.

QUARK COLOR

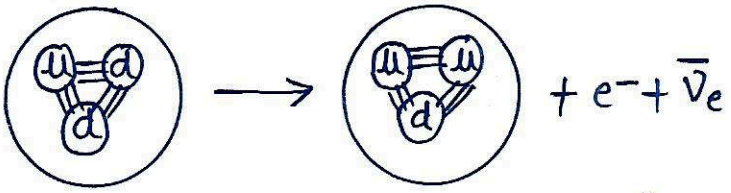
R
G
B

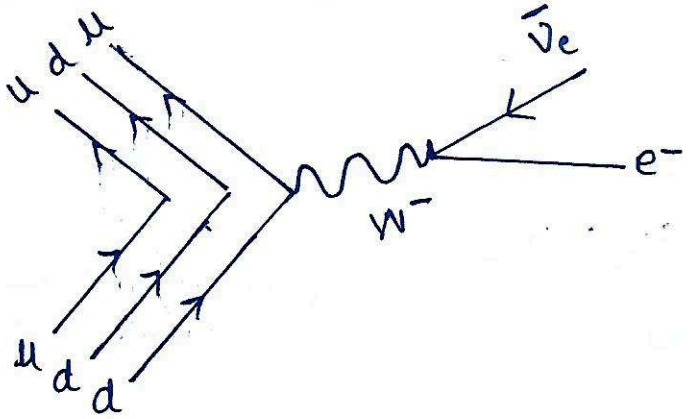
- Weak Force-

β^- decay

$$^{14}_{6}C \longrightarrow {}^{14}_{7}N + e^- + \bar{\nu}_e$$

$$^{1}_{0}n \longrightarrow {}^{1}_{1}p + e^- + \bar{\nu}_e$$





$$W^- \longrightarrow e^- + \bar{\nu}_e$$

- Electroweak Theory

$$(P) + (P) \longrightarrow (P)(n) + e^{+} + \nu_e$$

→ Necessary for radioactive process.
→ Necessary for building up heavier elements.
→ Vailets parity conservation.

Interaction	Current Theory	Range	Relative Strength	Force carrier
Gravitational Force	GTR	∞	1	Graviton (QFT)
EM Force	QED	∞	10^{36}	Photon
S Force	QCD	$10^{-15}m$	10^{38}	quarks → gluons hadrons → π meson
W Force	EWT	$10^{-18}m$	10^{25}	W^{+}, W^{-}, Z boson

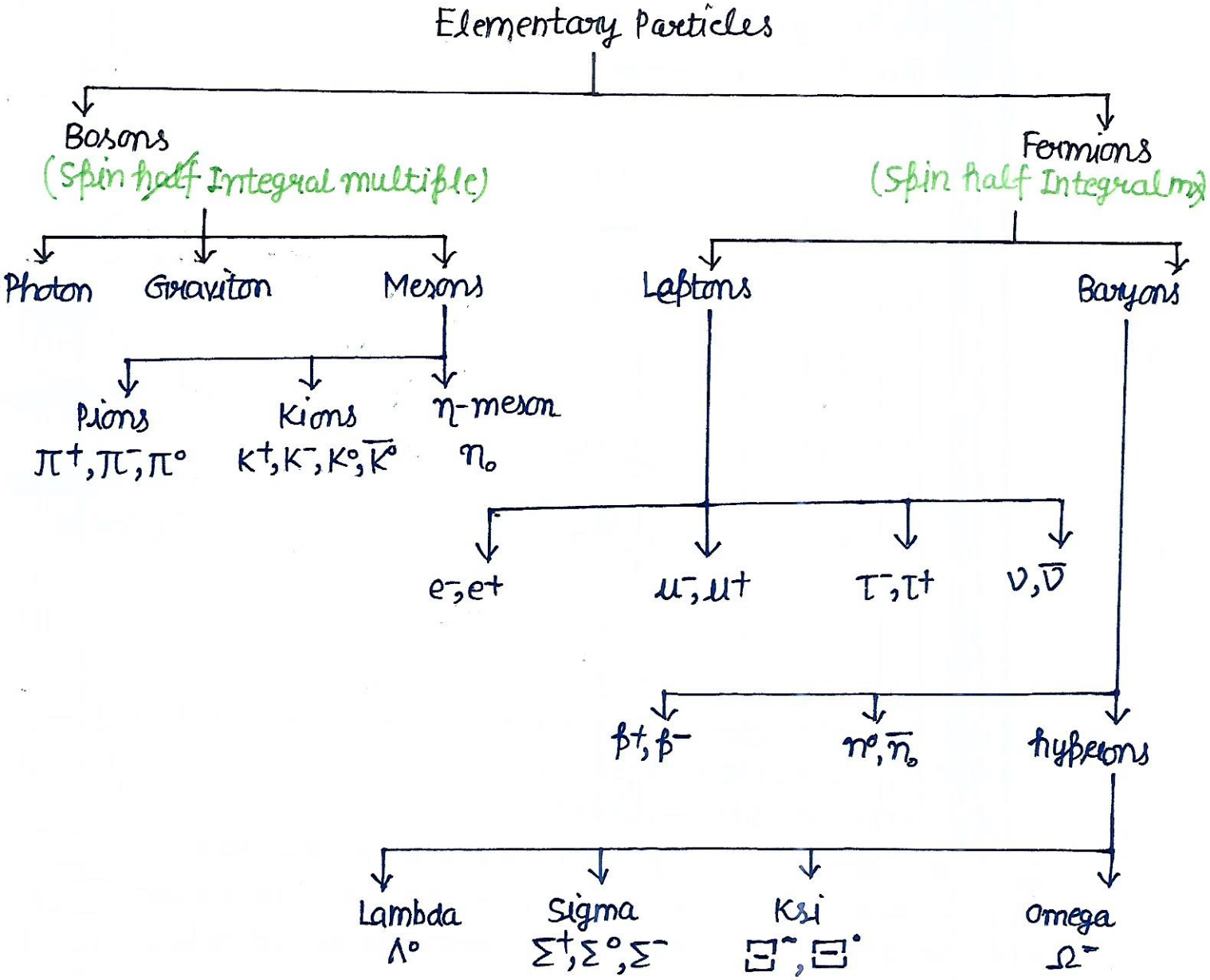
Elementary particles -

- Particle which have no further internal structure.
- According to QFT, elementary particles are nothing but the quanta of corresponding field.
- Study on the basis of radiation.
- If Antimatter exists (b/w nucleus and e^{-} in the atom) it would then be made up of antielectron, i.e. positron, antiproton and antineutron thus we see that 7 particles would explain both matter and antimatter.
- The Fundamental constituents of matter are three generations of leptons and three generation of quarks.
- The force b/w the fundamental constituents of matter arise due to the exchange of guage bosons.

Elementary particles

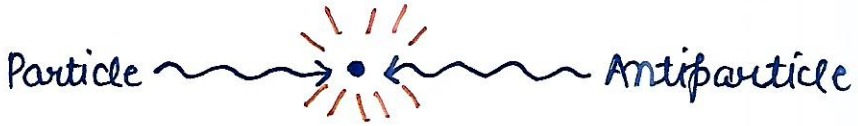
- Bosons
  - Angular momenta equal to integral multiple of $\hbar$
- Fermions
  - Spin is half integral.

- There is no conservation law controlling the total number of bosons in the universe whereas the total number of fermions is strictly conserved.



Anti particles-

- Universe is made up of different type of atoms (particle).



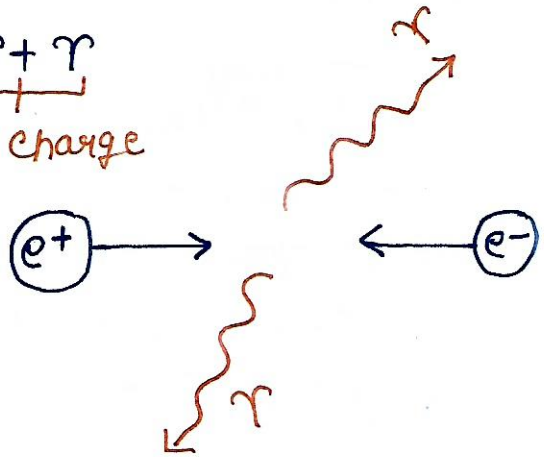
1. e^- and e^+ –

Electron — Antielectron or positron

e^- e^+

$e^- + e^+ \Rightarrow \gamma + \gamma$

0 charge



Rest mass of $e^- = m_0c^2$

$= 0.511$ MeV

e^- e^+

→ Same mass (9.1×10^{-31} kg)

→ Opposite charge ($q_e = +1.6 \times 10^{-19}$ C)

→ Same spin ($1/2$)

→ Opposite magnetic moment → External $\vec{B}$ Field

→ Opposite Lepton number

→ conservation of

- Charge
- Linear momentum
- Energy
- Angular momentum
- Lepton Quantum number

$e^- + e^+ \longrightarrow \gamma + \gamma$

$+1 \quad -1 = 0 \quad 0$

$0 = 0$

(2) p & $\bar{p}$ –

Proton (p)

Antiproton ($\bar{p}$) ← Composite particles

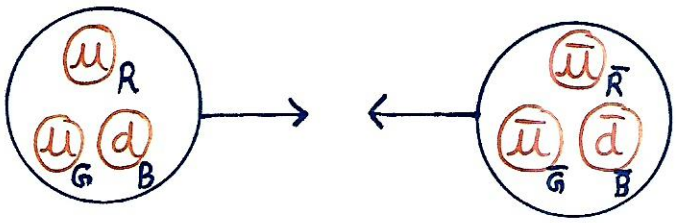
→ Same mass (1.6×10^{-27} kg)

→ Opposite charges

→ Spin spin ← Spin half (Fermions)

→ Opposite magnetic moment

→ Opposite Baryons numbers



→ Quarks

- Up, u
- down, d
- Strange, s
- charm, c
- top, t
- bottom, b

Antiquarks: $\bar{u}$, $\bar{d}$, $\bar{s}$, $\bar{c}$, $\bar{t}$, $\bar{b}$

→ different charge

→ different baryon numbers

(3) n & $\bar{n}$ –

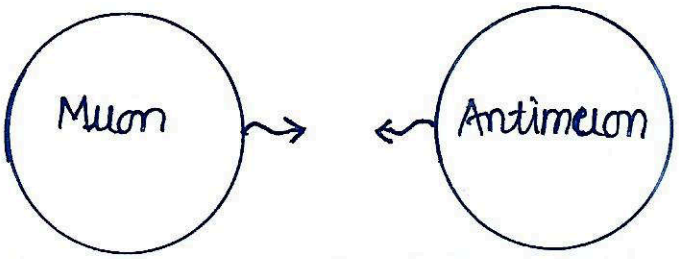
Neutrons (n)

Antineutron ($\bar{n}$)

- Opposite magnetic moment
- Opposite Baryon number



(4) μ^- & μ^+



(5) π^+ & π^-

Pimeson — Anti-pi-meson



(6) Photon



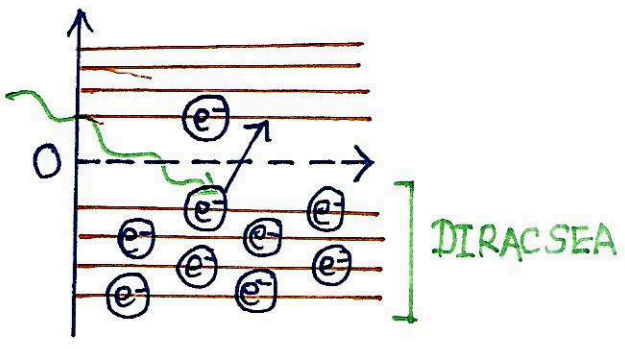
(7) $\pi^0, \bar{\pi}^0$



- Antimatter is also possible.
- Universe favour matter when begin.
- postulated by dirac.
- $\hat{H}\psi = E\psi$
- $i\hbar\gamma^\mu \partial_\mu \psi = mc\psi$

* DIRAC HOLE THEORY-

$$E = \pm\sqrt{m_0^2c^4 + p^2c^2}$$



- possible least energy of particle $+m_0c^2$.
- Creation of O in dirac sea.

$$\gamma \longrightarrow e^+ + e^-$$ ← Pair creation

- positron is nothing but the hole in the negative energy electrons.

$$e^- + e^+ \longrightarrow \gamma + \gamma$$ ← Pair annihilation

* CROSSING SYMMETRY-

$$A + B \longrightarrow C + D$$

$$A \longrightarrow \bar{B} + C + D$$

$$A + \bar{C} \longrightarrow \bar{B} + D$$

$$\bar{C} + \bar{D} \longrightarrow \bar{A} + \bar{B}$$

* COMPTON SCATTERING-

$$e^- + \gamma \longrightarrow e^- + \gamma$$

$$e^- + e^+ \longrightarrow \gamma + \gamma$$

$$\gamma + \gamma \longrightarrow e^+ + e^-$$

* POSITRON EMISSION-

$$P \longrightarrow n + e^+ + \nu_e$$ ← Nuetrino of e-

$$e^- + P \longrightarrow n + \nu_e$$

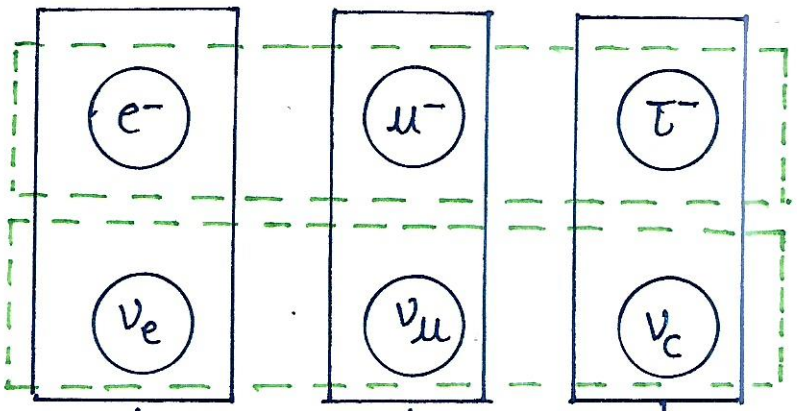
Electron capture

$$P + \bar{\nu}_e \longrightarrow n + e^+$$ ← Inverse β decay.

#Leptons-

Subatomic particles

- **Leptons** (Do not respond to Strong Interaction)
- **Hadrons** (Responds to Strong Interactions) e.g. ${}_0n^1$, p^+



⇒ Charged Leptons (Participate in Gravity, weak and EM Interaction)

⇒ Neutral Leptons (Participate in Gravity and weak Interaction)

$$m_{e^-} < m_{\mu^-} < m_{\tau^-}$$

$$n \longrightarrow p + e^- + \nu_e$$
$$p \longrightarrow n + e^+ + \bar{\nu}_e$$

$$\pi^+ \longrightarrow \mu^+ + \nu_\mu$$
$$\pi^- \longrightarrow \mu^- + \bar{\nu}_\mu$$

$$\tau^- \longrightarrow \pi^- + \bar{\nu}_\tau$$

- **Inverse Decay-**

$$\bar{\nu}_e + p \longrightarrow n + e^+$$
$$\nu_e + {}^{37}_{17}Cl \longrightarrow {}^{38}_{18}Ar + e^-$$

$\nu_\mu + e^- \longrightarrow \mu^- + \nu_e$

$m_e \longrightarrow 0.511\ MeV/c^2$ $(m_p/1800)$

$m_{\mu^-} \longrightarrow 106\ MeV/c^2$ $(m_p/10)$

$m_{\tau^-} \longrightarrow 1777\ MeV/c^2$ $(2\,m_p)$

$T_{e^-} = Stable$

$T_{\mu^-} = 2.1 \times 10^{-6} s$

$T_{\tau^-} = 2.9 \times 10^{-13} s$

$\tau^- \longrightarrow \pi^- + \nu_\tau$

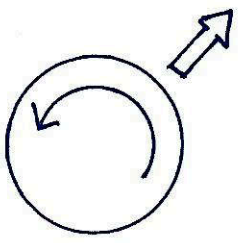
$\pi^- \longrightarrow \mu^- + \bar{\nu}_\mu$

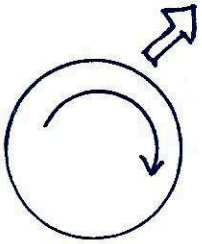
$\mu^- \longrightarrow e^- + \nu_\mu + \bar{\nu}_e$

↓

Stable

Charge $-1e$		Charge $+1e$
e^-	$\longleftrightarrow$	e^+
μ^-	$\longleftrightarrow$	μ^+
τ^-	$\longleftrightarrow$	τ^+
Neutrinos	$\longleftrightarrow$	Antineutrinos





⇒ Spin: These Leptons have half spin that means they follow Pauli Exclusion principle. They are Fermions.

$$spin = \pm \frac{1}{2}$$

LEPTON NUMBER-

* Electron Lepton No.

L_e

$L_e = 1 \quad (e^-)$

$= 1 \quad (\nu_e)$

$= -1 \quad (e^+)$

$= -1 \quad (\bar{\nu}_e)$

$= 0 \quad$ (For other Particles)

* Muon Lepton No.

$L_\mu = 1 \quad (\mu^-)$

$= 1 \quad (\nu_\mu)$

$= -1 \quad (\mu^+)$

$= -1 \quad (\bar{\nu}_\mu)$

$= 0 \quad$ (For other Particles)

* Tau Lepton No.

$L_\tau = 1 \quad (\tau^-)$

$= 1 \quad (\nu_\tau)$

$= -1 \quad (\tau^+)$

$= -1 \quad (\bar{\nu}_\tau)$

$= 0 \quad$ (For other particles)

#BARYON NUMBER:

$B = 1$ (For all Baryons) like n, p, Λ^0, Σ etc.

quarks $\longrightarrow \frac{1}{3} = B$

$= -1$ (For Anti-Baryons)

Antiquarks $\longrightarrow -\frac{1}{3} = B$

$= 0$ (For all others)

$\Lambda^0 = 1$

$\square^0 = 1$

Baryons = quark + quark + quark

$= \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

$= 1$

Anti-Baryon = Antiquark + Antiquark + Antiquark

$= \frac{-1}{3} - \frac{1}{3} - \frac{1}{3}$

$= -\left(\frac{1}{3} + \frac{1}{3} + \frac{1}{3}\right)$

$= -1$

Meson = quark + antiquark

$= 1 - 1$

$= 0$

#STRANGENESS AND STRANGENESS NUMBER:

- It is associated with a different kind of behaviour associated with particles.
- Produced either by Strong or EM Interaction.
- Decayed via weak interaction.
- Associated quantum number is called Strangeness quantum number.
- Cosmic rays enter the earth atmosphere leads to the creation of subatomic particles.

HADRON PARTICLES

Particle		S
n	⟶	0
p	⟶	0
π	⟶	0
Λ^0	⟶	−1 (uds)
$\Sigma^+, \Sigma^0, \Sigma^-$	⟶	−1 (uus, uds, dds)
Ξ^0, Ξ^-	⟶	−2 (uss, dss)
Ω^-	⟶	−3 (sss)
K^0, K^+	⟶	+1 ($d\bar{s}, u\bar{s}$)
K^-	⟶	−1

→ mesons are made up of two quarks.
→ Baryons are made up of three quarks.

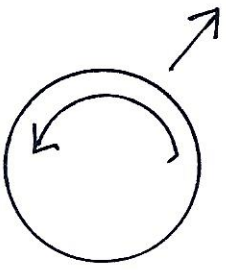
→ Strange quark −1
→ Other quark 0

→ Antistrange quarks +1

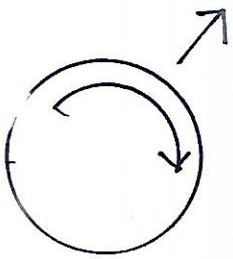
- Strangeness no. is conserved in only strong or EM Interaction
- Strangeness no. is not conserved in weak interactions

- $\Xi \longrightarrow \Lambda^0 + \pi^-$
  $-2 \xrightarrow{\Delta S = +1} -1 + 0$
- $\Lambda^0 \xrightarrow{\Delta S = +1} n + \pi^0$
  $-1 \longrightarrow 0 + 0$
- $\Sigma^0 \xrightarrow{\Delta S = 0} \Lambda^0 + \gamma$
  $-1 \longrightarrow -1 + 0$

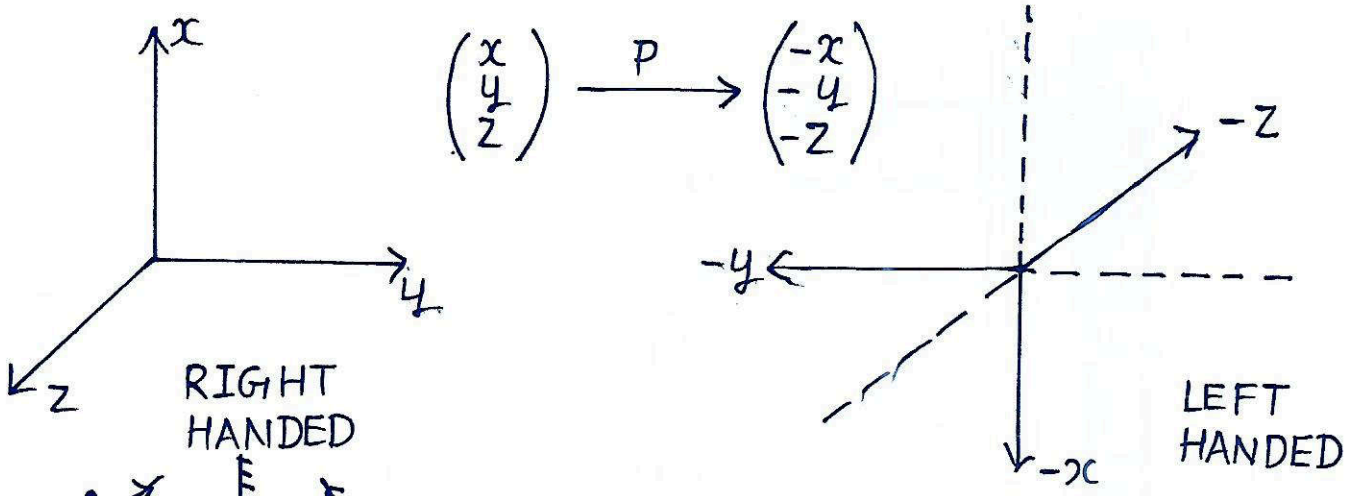
#PARITY VOILATION;



Neutrino



Anti-Neutrino



$\vec{r} \longrightarrow -\vec{r}$

$\vec{p} \longrightarrow -\vec{p}$

$\vec{a} \longrightarrow -\vec{a}$

$\vec{F} \longrightarrow -\vec{F}$

$\vec{F} = m\vec{a}$ $\qquad -\vec{F} = -m\vec{a}$

$\vec{L} \longrightarrow -\vec{r} \times -\vec{p}$ ← Angular momentum

$r \times p$

$= \vec{L}$

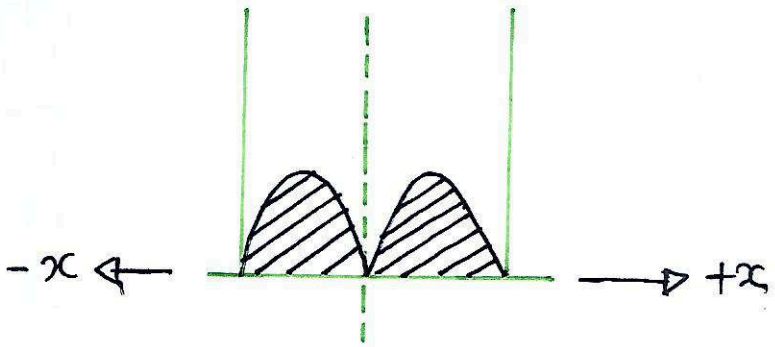
$\vec{S} \longrightarrow \vec{S}$ ← Spin Angular momentum

- Newton's Law, Law of Gravitation, Maxwell's Equations etc. are all invariant under a parity transformation.

Quantum Mechanics

$V(x,t) \longrightarrow V(-x,t)$

$|\psi(x,t)|^2 \longrightarrow |\psi(-x,t)|^2$



- Universe is symmatric from right and left, this is called Parity symmetry.
- Chien-Shiung Wu ⟶ β decay of $_{60}Co$

↓

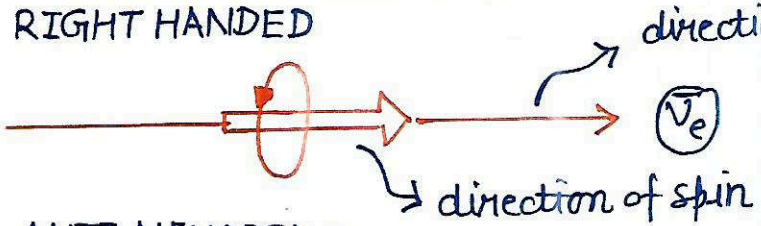
Wu Experiment

↓

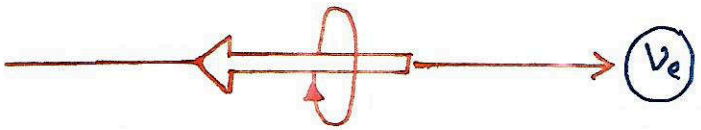
conservation of parity voilated atleast for the case of weak Interaction.

- **HELICITY OF NEUTRINOS-**

RIGHT HANDED



ANTI-NEUTRINO



LEFT HANDED NEUTRINOS

→ Angular momenta doesn't change in mirror image.
→ All neutrinos (ν_e) are left handed.
→ All antineutrinos ($\bar{\nu}_e$) are right handed.
→ If Universe is symmetric

$\nu_e, \bar{\nu}_e$: Right handed
$\nu_e, \bar{\nu}_e$: Left handed
} Exist

→ But practically, experimentally

ν_e : Left Handed
$\bar{\nu}_e$: Right-handed
} only exist

Universe is n't symmetric.

• Wu Experiment -

$$^{60}_{27}Co \longrightarrow {}^{60}_{28}Ni^{*} + e^- + \bar{\nu}_e$$

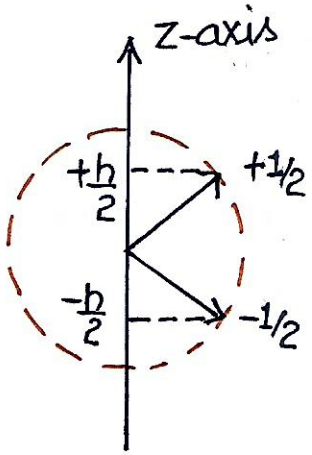
Spin 5 → 4 + $\frac{1}{2}$ + $\frac{1}{2}$

$$n \longrightarrow p + e^- + \bar{\nu}_e$$

$$udd \longrightarrow uud + e^- + \bar{\nu}_e$$

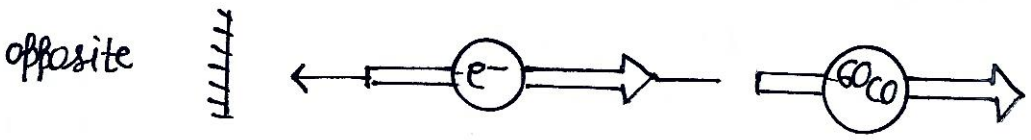
$$d \longrightarrow u + e^- + \bar{\nu}_e$$

* Weak interaction is only responsible for transformation of quarks.



* ^{60}Co placed in strong $\vec{B}$ Field.
* e^- and $\bar{\nu}_e$ should have opposite linear momenta.
* e^- have linear and spin angular momenta in different direction hence it should be left handed.
* $\bar{\nu}_e$ have linear and spin angular momenta in same direction hence it should be right handed.
* And may be vice-versa.

→ Voilation of Parity conservation happens.



→ Mirror image is voilating parity.

→ Universal Conservation quantities —▷ Energy, momentum, B, L

→ Approximate conservation quantities —▷ Strangeness, Isospin

#ENERGY CONSERVATION-

$A + B \longrightarrow C + D$

$\downarrow$

m_0c^2

$\pi^- + p \longrightarrow \Sigma^- + K^+$

140 938·3 1197 493

1028·3 1690

• Decay reaction :

$P \longrightarrow Q + R + S$

$m_Pc^2 \longrightarrow m_Qc^2 + m_Rc^2 + m_Sc^2 + K$

1) $K = 0$, $m_Pc^2 = m_Qc^2 + m_Rc^2 + m_Sc^2$

2) $K \neq 0$, $m_Pc^2 > (m_Q + m_R + m_S)c^2$

$P \longrightarrow n + e^+ + \nu_e$

938·3 939·6 0·511

#MOMENTUM CONSERVATION-

$p_i = p_f$

• $\pi^+ \longrightarrow \mu^+ + \nu_\mu$ (✓)

$p_i = 0$ $p_f = 0$

$p_\mu + p_\nu = 0$

$p_\mu = -p_\nu$

• $e^- + e^+ \longrightarrow \gamma$ (X)

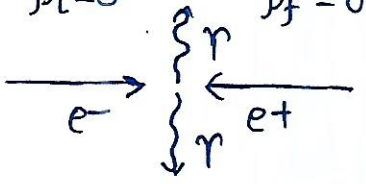
$p_i = 0$

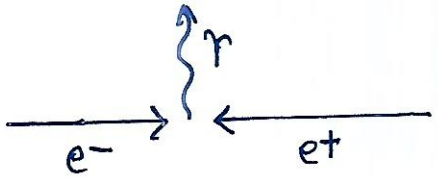
$p_{e^-} + p_{e^+} = 0$

$p_{e^-} = -p_{e^+}$

• $e^- + e^+ \longrightarrow \gamma + \gamma$ (✓)

$p_i = 0$ $p_f = 0$





$p_\gamma = \frac{h\nu}{c} \neq 0$

ANGULAR MOMENTUM CONSERVATION-

$$\vec{J} = \vec{L} + \vec{S}$$

$$J_i = J_f$$

- $\Sigma^0 \longrightarrow \pi^0 + \gamma$

Ground State

$l = 0$

J spin $\frac{1}{2}$ 0 1 $|J_1 + J_2| \ldots . |J_1 - J_2|$

$J_i = \frac{1}{2}$ $J_f = 1$

$J_i \neq J_f$

- $n \longrightarrow p + e^- + \bar{\nu}_e$

Spin J $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

0, 1 $\frac{1}{2}, \frac{3}{2}$

$J_i = \frac{1}{2}$ $J_f =$

Allowed ✓

$J_i = J_f$

CHARGE CONSERVATION-

$$A + B \longrightarrow C + D$$

$q_A \; q_B \qquad q_C \; q_D$

$$q_A + q_B \longrightarrow q_C + q_D$$

- $e^- \longrightarrow \gamma + \gamma$

$q = -1$ 0 0

$\Delta Q \neq 0$

$Q_i \neq Q_f$

- $e^- + e^+ \longrightarrow \gamma + \gamma$

q -1 +1 0 0

$\Delta Q = 0$

$Q_i = Q_f$

CHARGE CONJUGATION-

→ C

C : $q \longrightarrow -q$

→ Replace particle into Antiparticle

• $\pi^+ \longrightarrow \mu^+ + \nu_\mu$

⇩

$\pi^- \longrightarrow \mu^- + \bar{\nu}_\mu$

TIME REVERSAL-

$e^- + e^+ \longrightarrow \gamma + \gamma$ ← Pair Annihilation

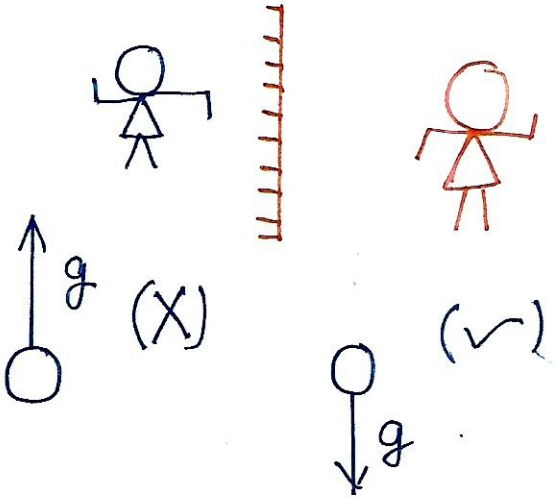
⇩

$\gamma + \gamma \longrightarrow e^- + e^+$ ← Pair production

Int.	T
Strong	✓
EM	✓
Weak	X

CP VOILATION-

- Mirroring of universe.
- But Physics is same.
- Things just flipping



- We can define left and right on earth but if there is an alien, he wouldn't understand at all.
- To find left and right is just an invension of human and doesn't mean much to Aliens.

→ Life ←

Alanine → Nature → Left handed

↓

Synthetic

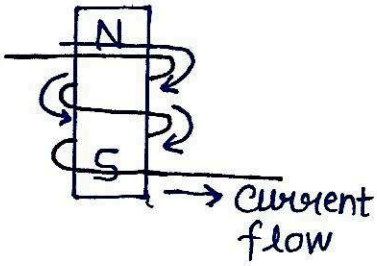
↓

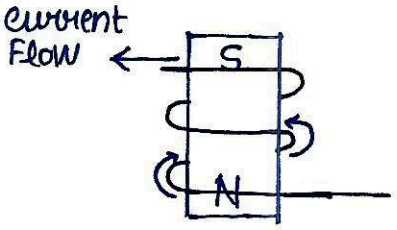
Right handed
+
Left handed

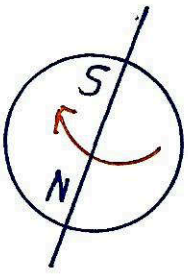
- Life on earth is Left handed.

Left handed molecules.

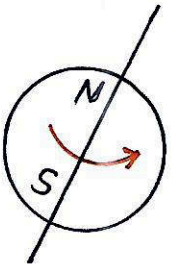
- Physics has left and right.





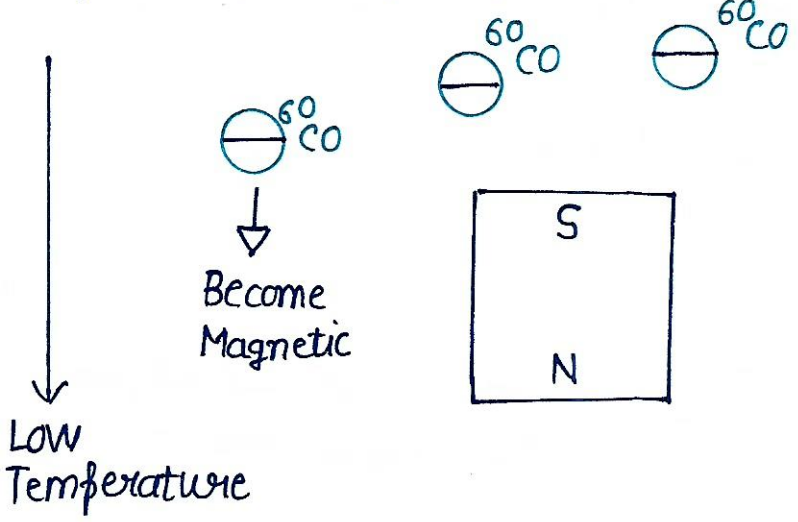


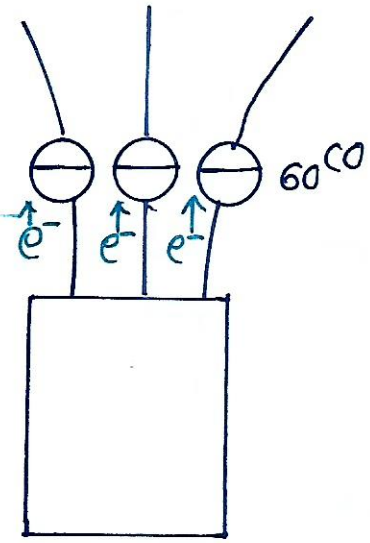
Earth



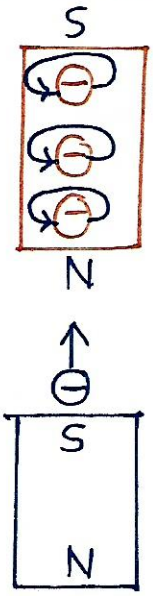
Earth(Might Be)

→ An experiment was performed at University of ~~colorado~~ columbia; using a very strong magnet and at a very low temperature.





OUR UNIVERSE



MIRROR UNIVERE



Fundamental Interactions

- Gravitational Interaction
- Electromagnetic Interaction
- Strong Interactions
- Weak Interaction

} discussed in UNIT-1

* Fermi Theory of β decay → Elementary particle physics Yorikiyo Nagashima et. al 2008 PP. 553

#NEUTRINOS (GHOST PARTICLE)-

→ β^- Decay-

$$^{12}_{5}B \longrightarrow {}^{12}_{6}C + e^-$$

$$^{1}_{0}n \longrightarrow {}^{1}_{1}p + e^-$$

→ β^+ Decay-

$$^{15}_{8}O \longrightarrow {}^{15}_{7}N + e^+$$

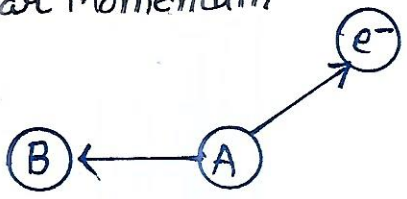
$$^{1}_{1}p \longrightarrow {}^{1}_{0}n + e^+$$

$$\boxed{{}^{A}_{Z}A \longrightarrow {}^{A}_{Z+1}B + e^-}$$ ⇒ Neutrino Hypothesis.

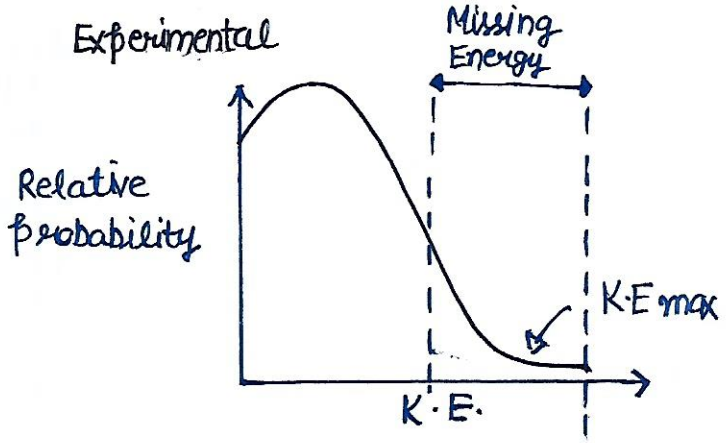
1. Energy conservation

Theoretical $E_e = \left(\frac{m_A^2 - m_B^2 + m_e^2}{2m_A}\right)c^2$

2. Linear Momentum



Experimental



3. Angular momentum

$$^{1}_{0}n \longrightarrow {}^{1}_{1}p + e^-$$

spin : $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

P	+	Q	⟶	R
1		1		0, 1, 2
$\frac{1}{2}$		$\frac{1}{2}$		0, 1
1		$\frac{1}{2}$		$\frac{1}{2}, \frac{3}{2}$

→ Vailation of Energy, Linear momentum, Angular momentum.
→ Wolfgang Pauli gave theory.

$$^{12}_{5}B \longrightarrow {}^{12}_{6}C + e^- + \bullet$$

Particle

→ Fermi gave theory ⇒

Neutrino

$$^{1}_{0}n = {}^{1}_{1}p + e^- + \bullet$$

⇩ Fermion

⇩ Called Fermion as Neutrino

⇩ $\bar{\nu}$

$n \longrightarrow p + e + \bar{\nu}_e$
$\beta \longrightarrow n + e^+ + \nu_e$
$\pi^- \longrightarrow \mu^- + \bar{\nu}_\mu$
$\pi^+ \longrightarrow \mu^+ + \nu_\mu$
$\tau^- \longrightarrow \pi^- + \nu_\tau$

Electron-Neutrinos
Muon-Neutrinos
Tau-Neutrinos

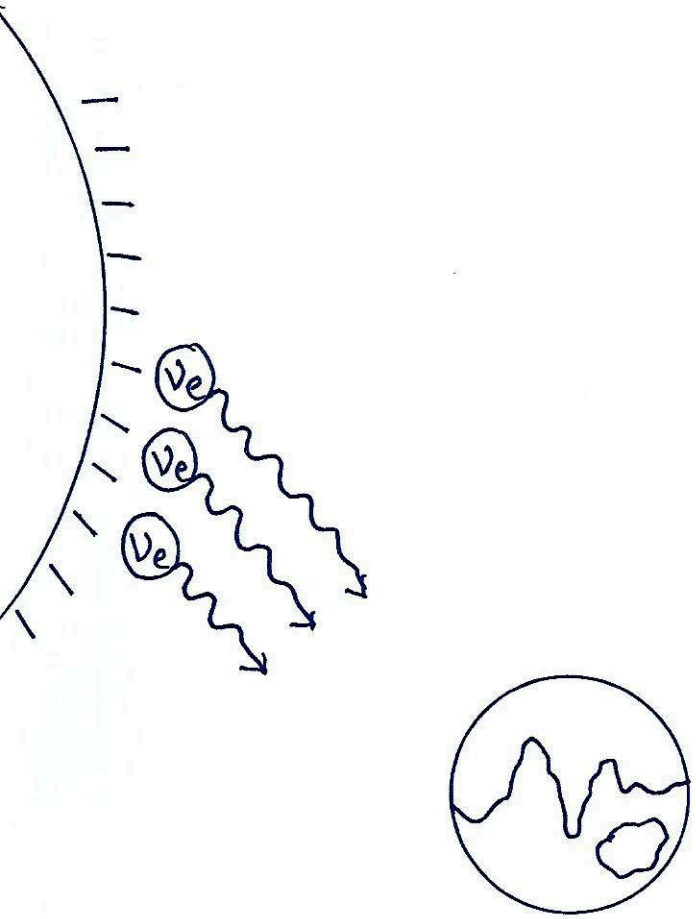
- NEUTRINOS PROPERTIES!
  - Extremely tiny
  - Do not intract with strong intraction.
  - Do not intract with EM interaction.
  - Only Interact with weak or gravitation interaction
  - Fermions
  - Spin $\frac{1}{2}$
  - Zero charge

- INVERSE β decay!

$$\bar{\nu}_e + p \longrightarrow n + e^+$$

#NEUTRINO OSCILLATOR-

- Neutrinos have multiple personality disorder.



- $p + p \longrightarrow {}^{2}_{1}H + e^+ + \nu_e$
- proton-proton cycle
- Solar neutrino deficit

- NEUTRINOS FLAVOR-

ν_e ν_μ ν_τ

- Interactions :

$$\beta \longrightarrow n + e^+ + \nu_e$$
$$\pi^+ \longrightarrow \mu^+ + \nu_\mu$$
$$\tau^- \longrightarrow \pi^- + \nu_\tau$$

- Inverse Decay :

$$\nu_e + {}^{37}_{17}Cl \longrightarrow {}^{37}_{18}Ar + e^-$$

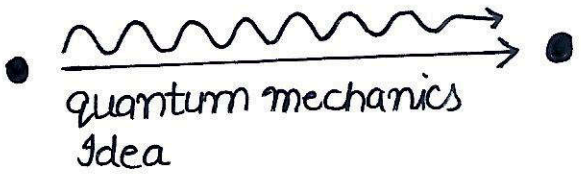
$$\nu_\mu + e^- \longrightarrow \mu^- + \nu_e$$

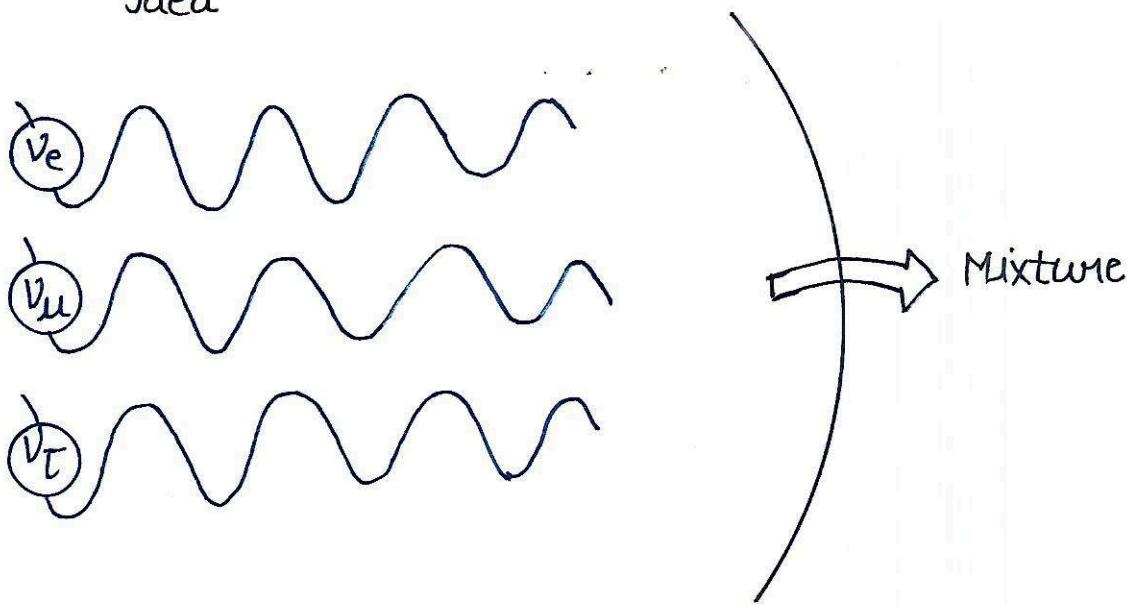
⇩

distinct identities

$$\nu_e + e^- \longrightarrow \mu^- + \nu_\mu \text{ [Not possible]}$$

- Bruno Pontecorvo ⟹ Neutrinos are in fact, mixure of some quantum state. As they travel, their identities change





$$\nu_e = U_{e1}\nu_1 + U_{e2}\nu_2 + U_{e3}\nu_3$$
$$\nu_\mu = U_{\mu 1}\nu_1 + U_{\mu 2}\nu_2 + U_{\mu 3}\nu_3$$
$$\nu_\tau = U_{\tau 1}\nu_1 + U_{\tau 2}\nu_2 + U_{\tau 2}\nu_3$$

⇓

Out of phase ⇒ Quantum Phasing.

⇓

Neutrinos oscillation

⇓

Probability of finding flavour

⇓

Frequencies are quite same. ⟹ Neutrinos have mass.

⇒ Neutrinos are massless

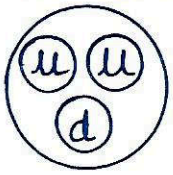
UNIT-4

QUARK MODEL

Subatomic particles

- **LEPTONS** (Do not experiance strong Interaction)
- **HADRONS** (Experiance strong Interaction)
  - Greek word means strong.
  - Hadrons are composite particles.
  - e.g. ${}^{1}_{0}n$, p^{+}
    - **Mesons**
      - $q\bar{q}$
      - • Properties–
        Electronic charge of $u = +\frac{2}{3}c$
        $d = -\frac{1}{3}c$
    - **Baryons**
      - $q\,q\,q$

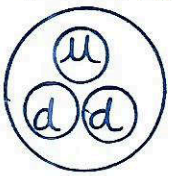
PROTON



Total Charge $= +\frac{2}{3} + \frac{2}{3} - \frac{1}{3}$

$= +1e$

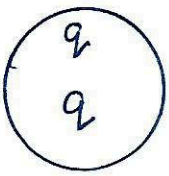
NEUTRON



Total Charge $= +\frac{2}{3} - \frac{1}{3} - \frac{1}{3}$

$= 0e$

Spin:- $q \rightarrow \frac{1}{2}$

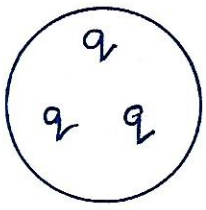


$\uparrow\uparrow$ $+1$

$\uparrow\downarrow$ 0

$\downarrow\downarrow$ -1

→ Bosons
→ B-E Statistics



$\uparrow\uparrow\uparrow$ $+\frac{3}{2}$

$\uparrow\uparrow\downarrow$ $+1/2$

$\uparrow\downarrow\downarrow$ $-1/2$

$\downarrow\downarrow\downarrow$ $-3/2$

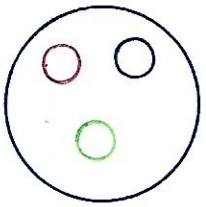
spin = $+\frac{3}{2}, +\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$

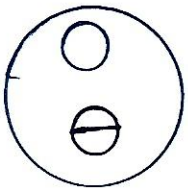
→ Fermions

→ Fermi-Dirac Statistics

→ Pauli Exclusion Principle

• Colour :-

BARYON  R + B + G

MESON  Colour + Anticolor

- π-mesons are force carrier.
- Mediators

• MESON ;

PIONS

	π^-	π^0	π^+
→ Zero spin	$u\bar{d}$	$u\bar{u}$ or $d\bar{d}$	$d\bar{u}$
→ charge	-1e	0e	+1e

→ Decay

$\pi^+ \longrightarrow \mu^+ + \nu_\mu$

$\pi^- \longrightarrow \mu^- + \bar{\nu}_\mu$

$\pi^0 \longrightarrow \gamma + \gamma$

ETA

η η'

→ Zero charge

→ Integer spin

$\eta \longrightarrow \gamma + \gamma$

$\eta' \longrightarrow \pi^+ + \pi^- + \eta$

KAON

	K^+	K^-	K^0
→ zero spin	$u\bar{s}$	$s\bar{u}$	$d\bar{s}$
→ charge	$+1e$	$-1e$	$0e$

• **BARYONS-**

PROTON

p

→ uud

→ Most stable hadron

NEUTRON

n

→ udd

→ $n \longrightarrow p + e^- + \bar{\nu}_e$

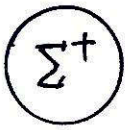
LAMBDA

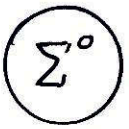
Λ^0

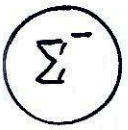
→ $\Lambda^0 \longrightarrow p + \pi^-$

→ $\Lambda^0 \longrightarrow n + \pi^0$

SIGMA







→ uus uds dds

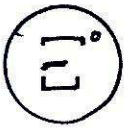
Charge +1e 0e -1e

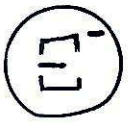
→ $\Sigma^+ \longrightarrow p + \pi^0$

→ $\Sigma^0 \longrightarrow \Lambda^0 + \gamma$

→ $\Sigma^- \longrightarrow n + \pi^-$

Xi





→ uss dss

Charge −1e 0e

Omega



→ sss

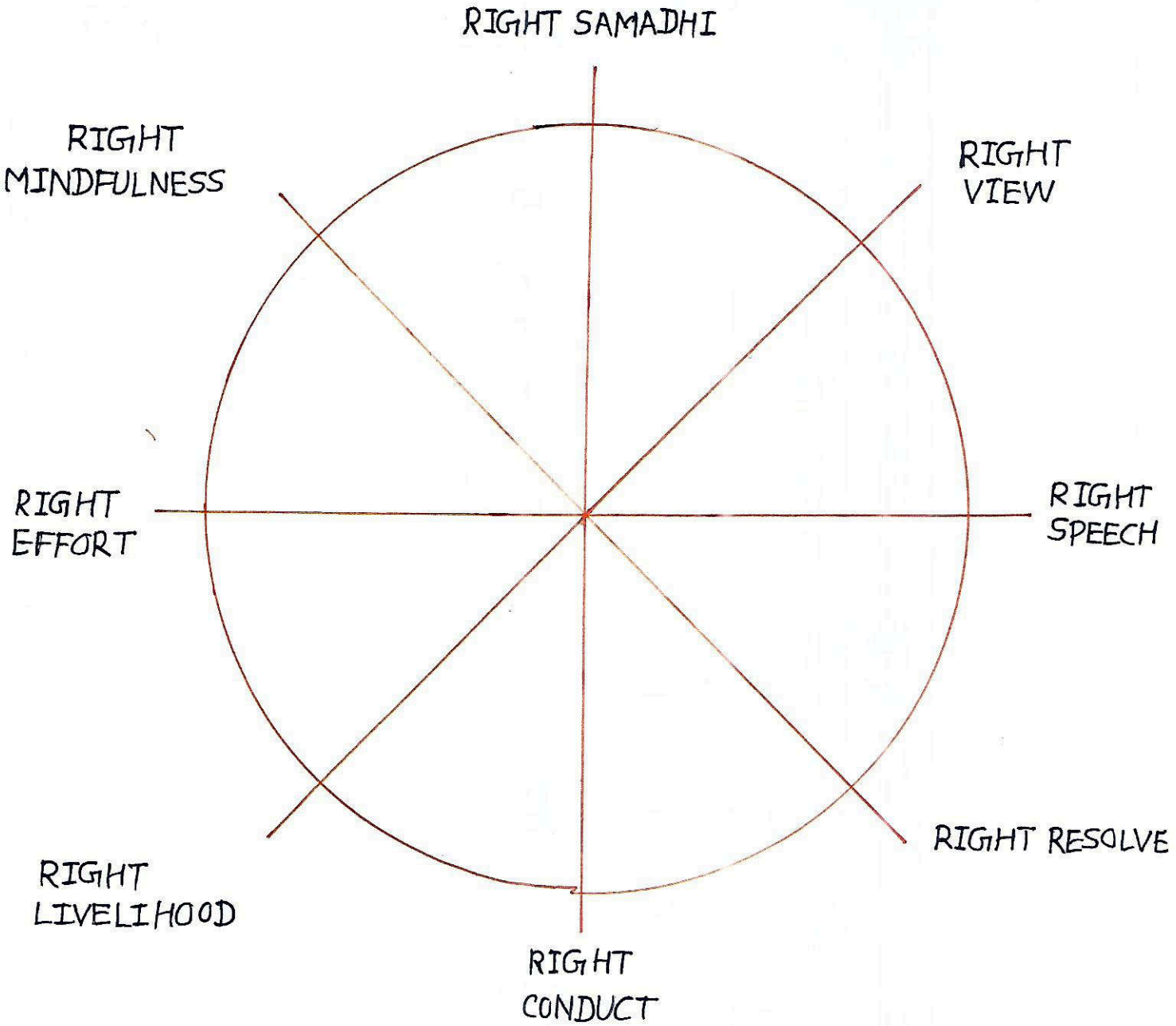
→ $\Omega^- \longrightarrow \Xi^0 + \pi^-$

$\Xi^0 \longrightarrow \Lambda^0 + \pi^0$

$\Lambda^0 \longrightarrow p + \pi^-$

#EIGHTFOLD WAY-

- In Ancient philosophy of Buddishm, there is a noble Eight fold path way. It simply consists of eight practices that lead to enlightenment and freedom from sansaram.



EIGHTFOLD WAY IN PARTICLE PHYSICS

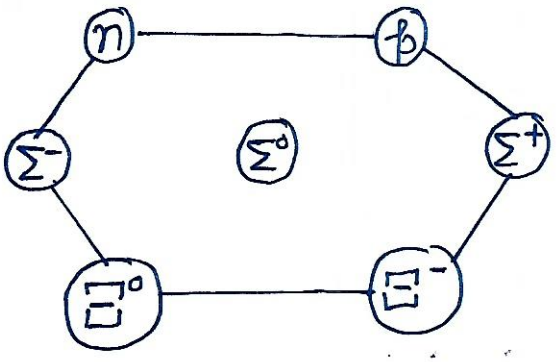
- HADRONS: Hadrons are a class of subatomic particles that experiance the strong force / Interaction.

  $n, p, \Lambda^0, \Sigma^+, \Sigma^-, \Sigma^0, \Xi^0, \Xi^-, \Omega^-$

  - Combining those particles in fundamental entities consisting of organising particles in multiplets.

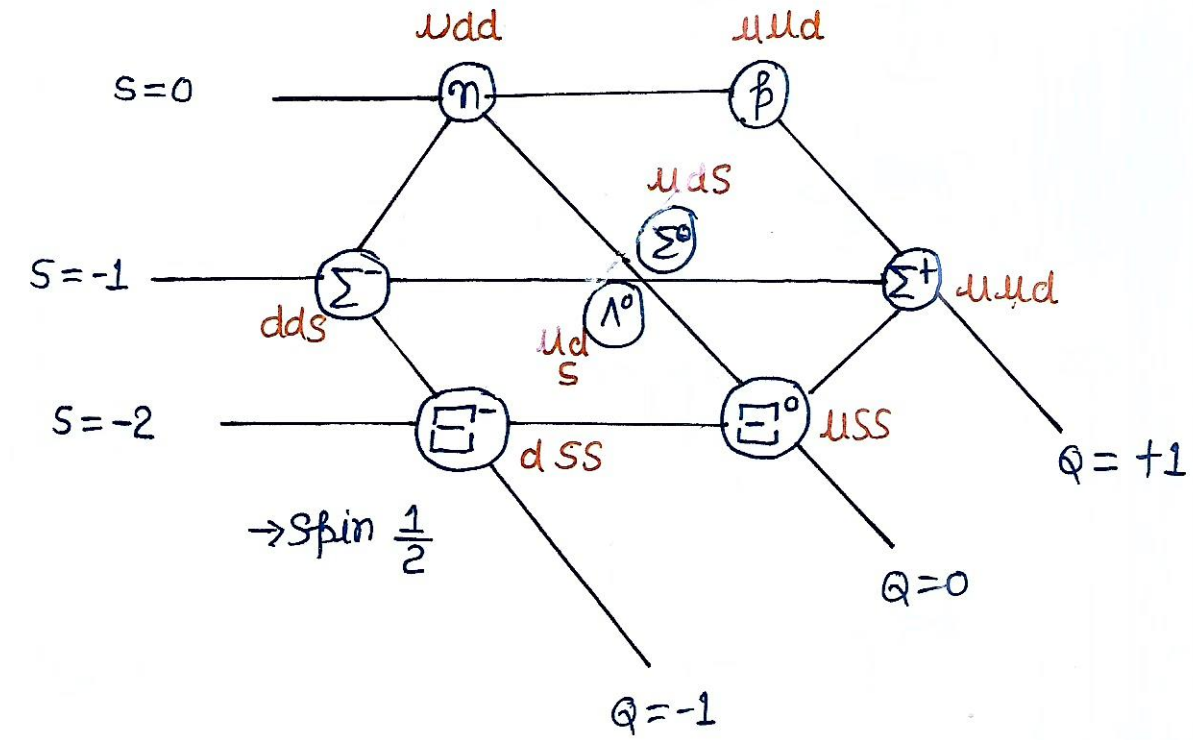
	n	p	
** mass	939.6	938.3	
	Σ^-	Σ^0	Σ^+
** mass	1197	1193	1189
	Ξ^0	Ξ^-	
** mass	1315	1321	

  - Supermultiplet.
  - Idea like Periodic Table.

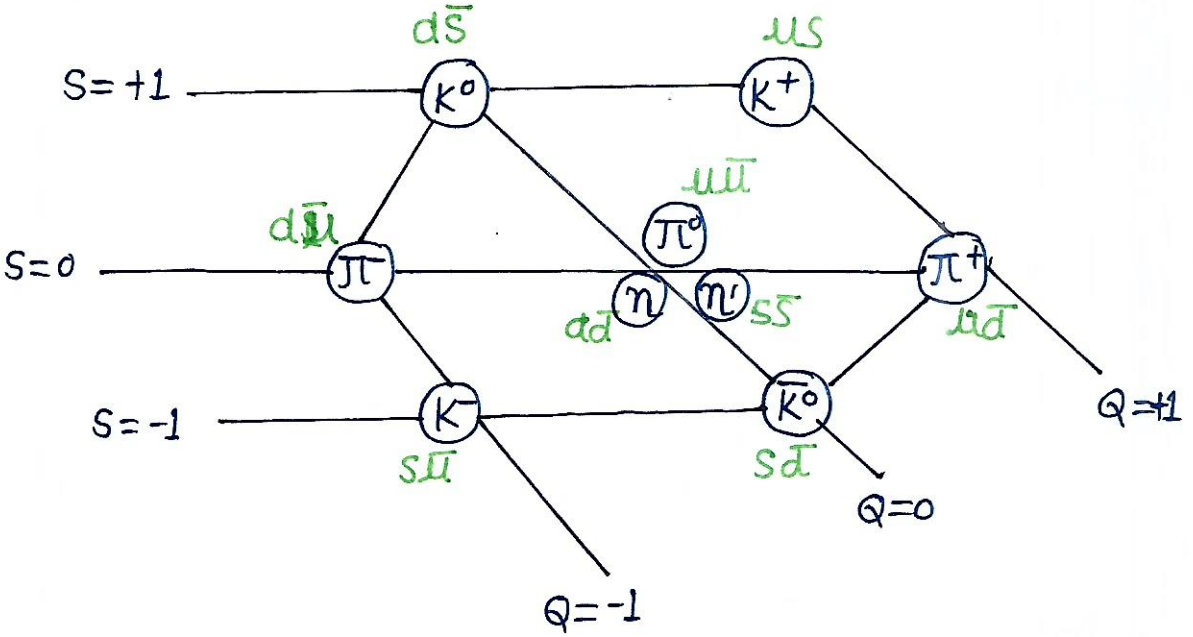


  - The Eightfold way is an organisational scheme, or classification of the Hadron particles in groups/charts or geomatric pattern based on same spin but different charge and strangeness.

  - Murray Gell-Mann
  - Yuval Na'eman

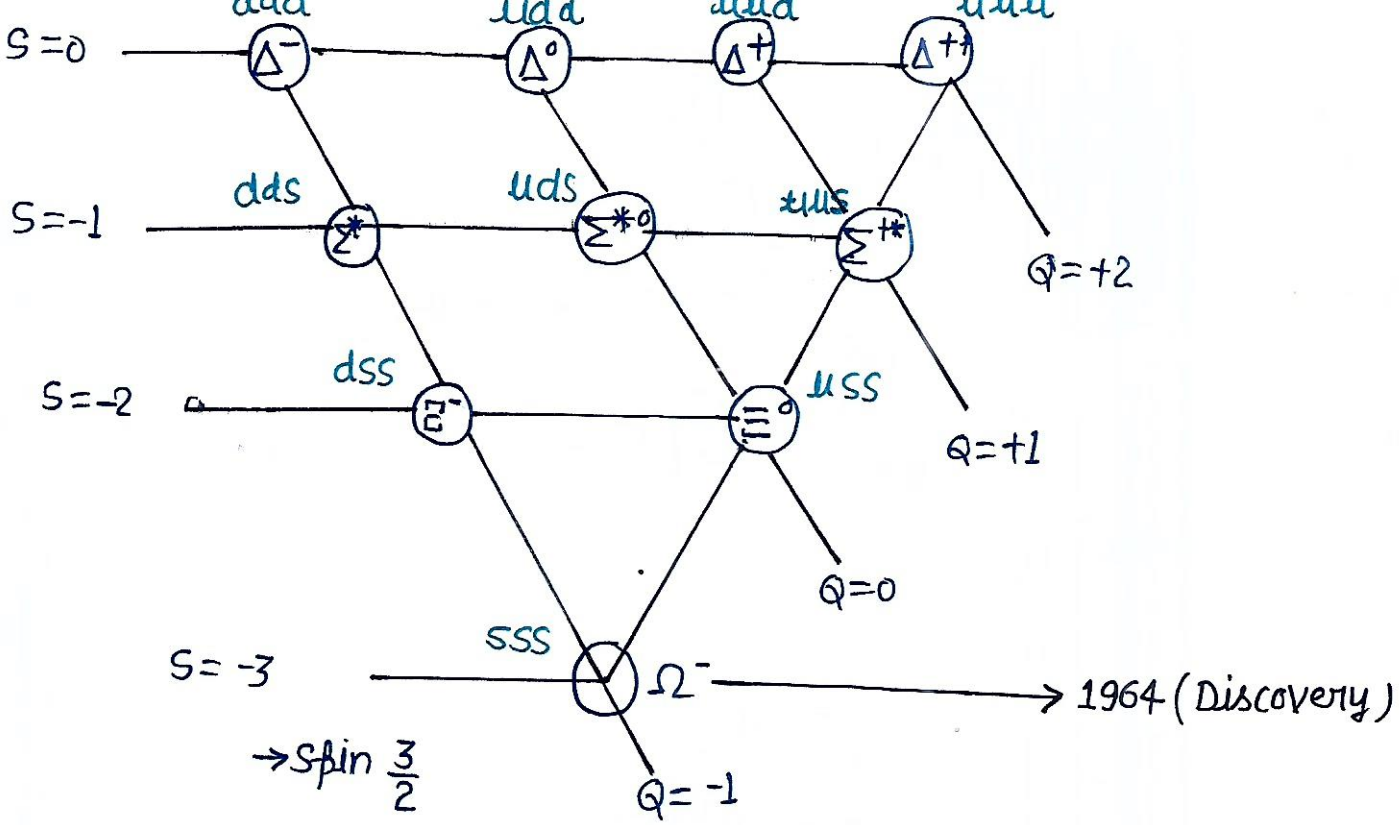


BARYON OCTET



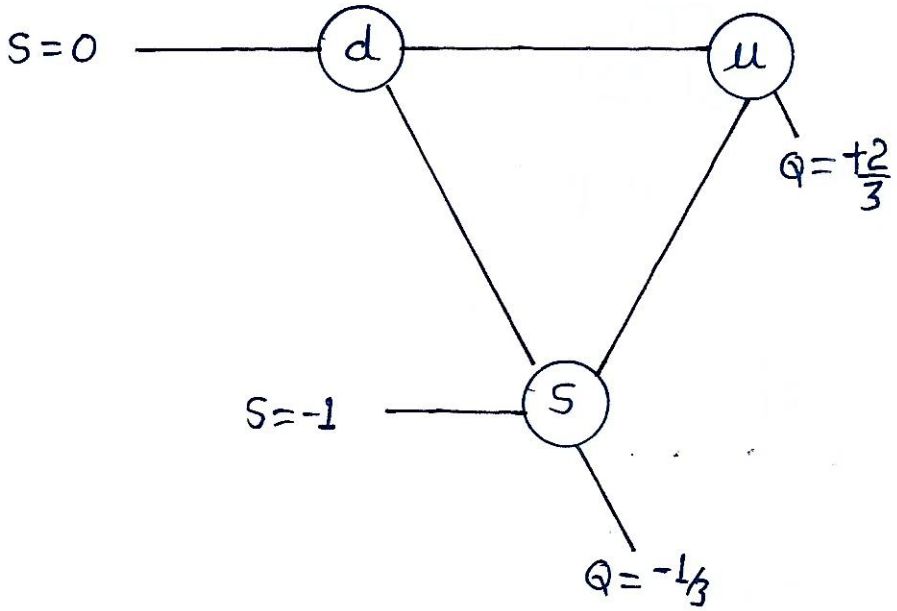
→ Spin 0

MESON OCTET
MESON NONET



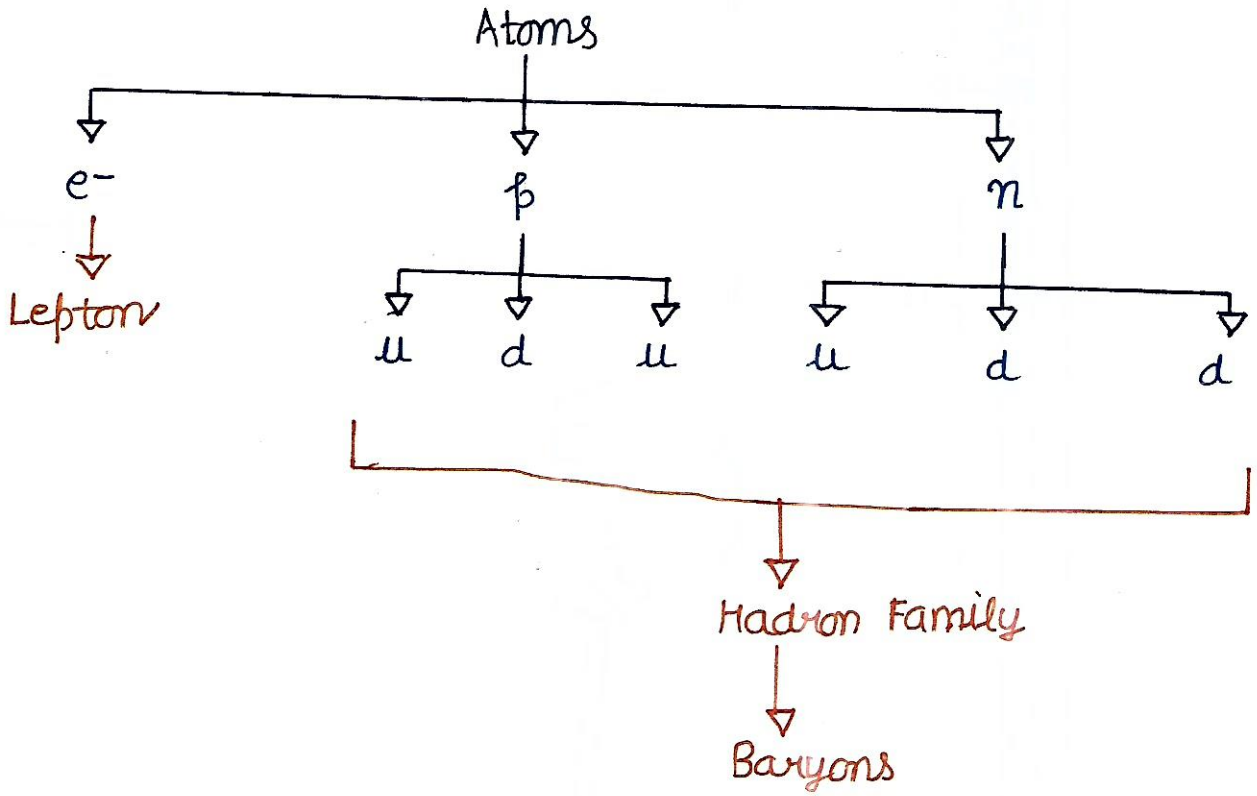
BARYON DECUPLET

- Quark model

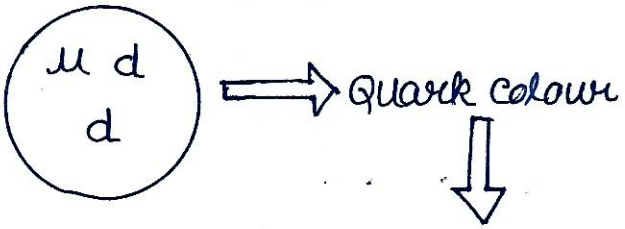


→ QUARK MODEL :

1. All Baryons are made up of three quarks. All anti-baryons are made up of three anti-quarks.
2. All mesons particles are made up of quark-antiquark pair.



EXCLUSION PRINCIPLE :



Quark colour Hypothesis

R	G	B
$\bar{R}$	$\bar{G}$	$\bar{B}$

→ Hadrons are colourless**

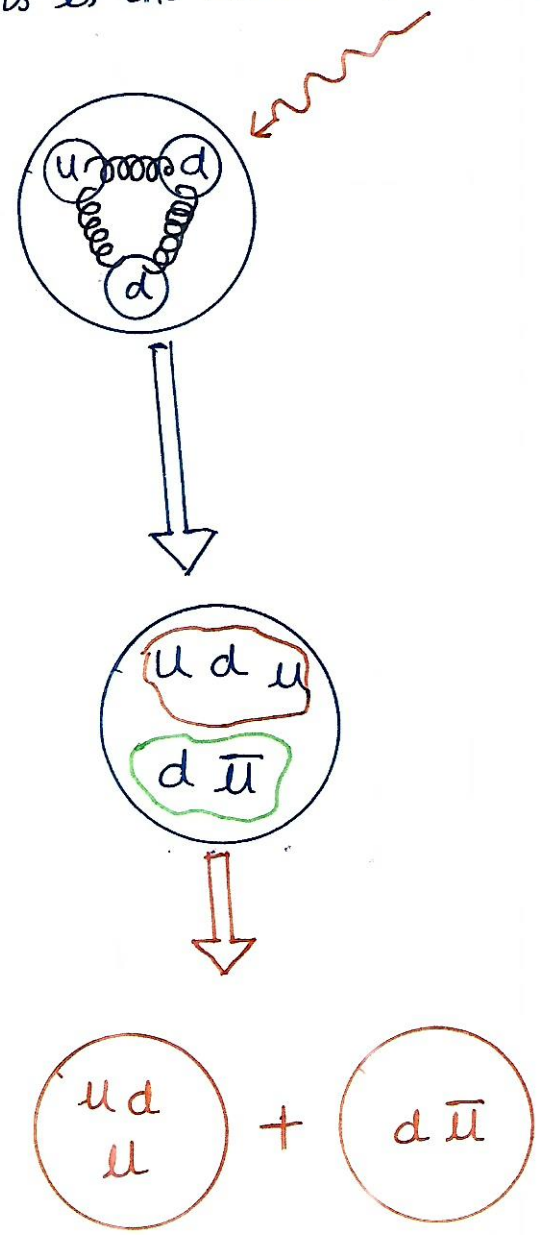
** colourless here means absent of colour property.

• QUARK FLAVOR–

u UP	d DOWN	s STRANGE
c CHARM	t TOP	b BOTTOM

• QUARK CONFINMENT–

– Strong interactions is the result of exchange of gluons.



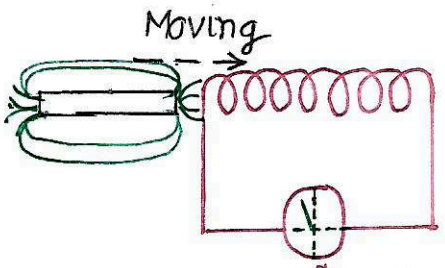
SPECIAL THORY OF RELATIVITY

- Einstein paper proposing Relativity
- Michelson Morley Experiment 1,2
- Galilean Transformation
- Lorentz Transformation
- Length contraction and Time dilation
- Relativity of Simultaneity.
- Syncronization of time.
- Transformation of Velocities
- Problems
- Position 4 Vectors
- $P=mv$?
- Momentum Refined
- 4 Vectors for Velocity and momentum
- $E=mc^2$
- Mass Energy Conservation
- Energy Momentum Relation
- The Massless Particle
- Magnetism in a Relativistic Phenomen
- Magnetic Field is consequence of Relativity
- Quiz 3
- Electric and Magnetic Field in R...
- Transformation of EB Field
- Electromagnetic Field Tensors
- Minkowski Diagrams

#Einstein Relativity Paper-

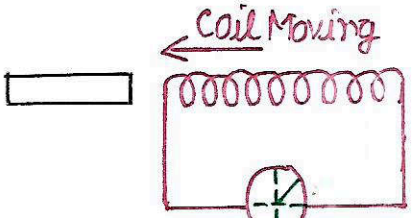
→ In 1905, Einstein proposed a paper.
→ On the Electrodynamics of moving bodies.

Space
Time
Time dilation
Length Contraction



→ Moving Magnet, changing Electric $\vec{B}$ field create $\vec{E}$.
→ Id $\vec{F} = q\vec{E}$

diffflection



diffflection

→ Motion of Inductor around magnet
$\vec{F} = q\vec{v} \times \vec{B}$

1. Principle of Relativity:
   - The Laws of optics and electrodynamics is same for Inertial frames.
   - Light always propogate in the empty space with definite velocity whether the body is moving or rest.
2. Constancy of Speed of Light:
   - Speed of Light in vaccum is same in All inertial frames.

#Michelson Morley's Experiment-

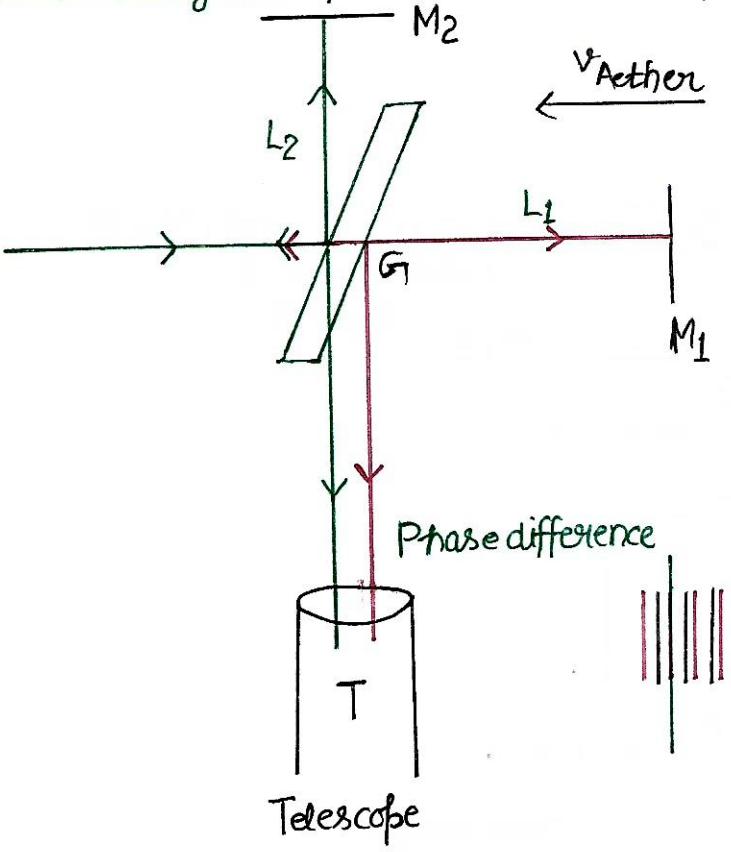
- Newton done Lots of Experiment.
- Opticks by Newton.

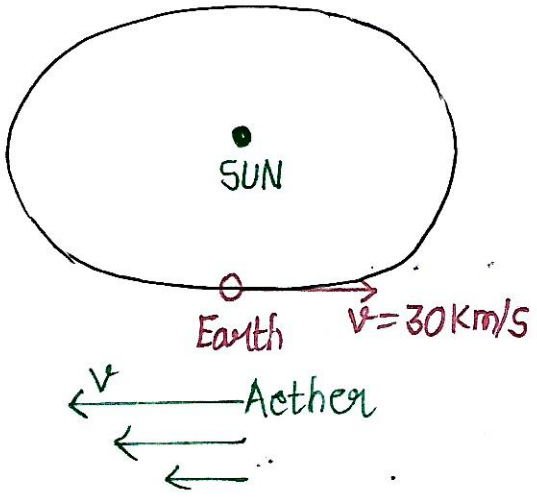
$\frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3 \times 10^8$ m/s Maxwell done this using electromagnetism.

Maxwell's Equation

- → EM Wave with speed $1/\sqrt{\epsilon_0 \mu_0}$.
- → Light is EM wave.
- → Aether medium

Michelson Morley did experiment to check speed of Earth in Aether medium.





Calculation ?

$$\vec{v}_{light,lab} = \vec{v}_{light,Aether} + \vec{v}_{aether,lab}$$

$$\vec{v}_{light,lab} = c - v$$

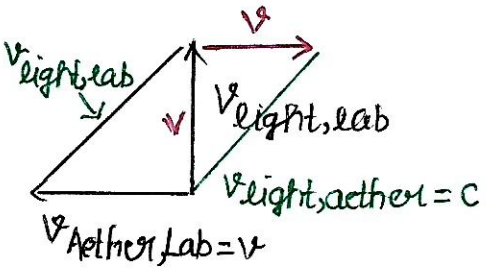
$$t_{1A} = \frac{L}{c-v}$$

$$\vec{v}_{light,lab} = c + v$$

$$t_1 = \frac{L}{c+v} + \frac{L}{c-v}$$

$$t_1 = \frac{2Lc}{c^2 - v^2}$$

$$\boxed{t_1 = \frac{2L}{c}\left(\frac{1}{1-\frac{v^2}{c^2}}\right)}$$



$$V = \sqrt{c^2 - v^2}$$

$$t_2 = \frac{L_2}{\sqrt{c^2-v^2}} + \frac{L_2}{\sqrt{c^2-v^2}}$$

$$\boxed{t_2 = \frac{2L_2}{\sqrt{c^2-v^2}}}$$

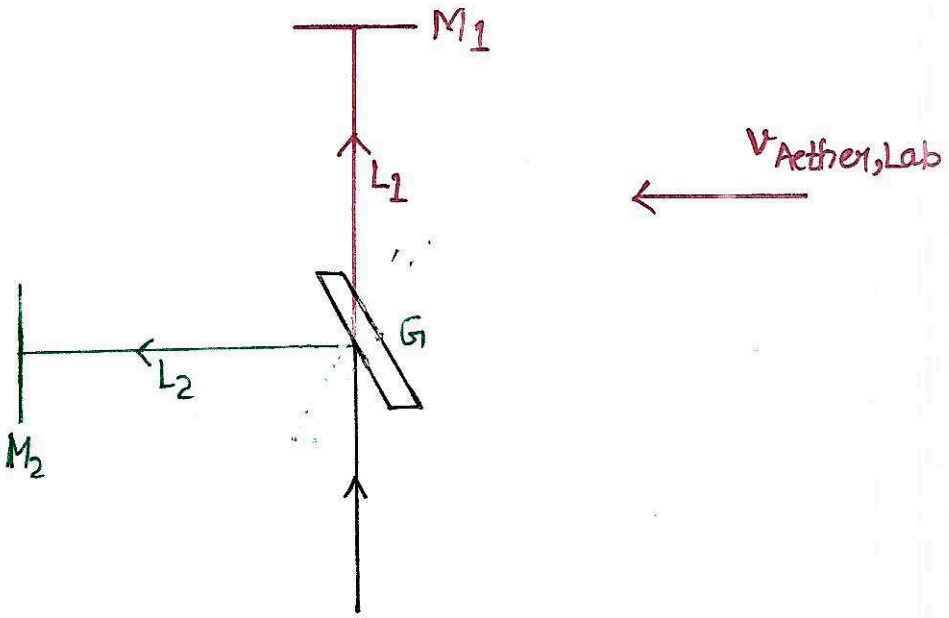
or

$$t_2 = \frac{2L_2}{c}\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$\boxed{\begin{aligned} t_2 - t_1 &= \frac{2L_2}{c}\left(\frac{1}{\sqrt{1-v^2/c^2}}\right) - \frac{2L_1}{c}\left(\frac{1}{1-v^2/c^2}\right) \\ \Delta t &= \end{aligned}}$$

$$\Delta\phi = \frac{\Delta t}{T} \times 2\pi \qquad \left[T = \frac{1}{\nu}\right] = \frac{\lambda}{c}$$

$$\boxed{\Delta\phi = \frac{c\Delta t}{\lambda} 2\pi}$$



$$t_1 = \frac{2L_1}{c} \cdot \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t_2 = \frac{2L_2}{c} \cdot \frac{1}{1 - \frac{v^2}{c^2}}$$

$$t_2 - t_1 = \frac{2L_2}{c} \cdot \frac{1}{1 - \frac{v^2}{c^2}} - \frac{2L_1}{c} \cdot \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\boxed{\Delta t' = \frac{2L_2}{c} \cdot \frac{1}{1 - v^2/c^2} - \frac{2L_1}{c} \cdot \frac{1}{\sqrt{1 - v^2/c^2}}}$$

$$\Delta\phi' = \frac{\Delta t'}{T} \times 2\pi$$

$$\Delta\phi = \frac{c\Delta t'}{\lambda} 2\pi$$

$\Delta\phi' = ?$

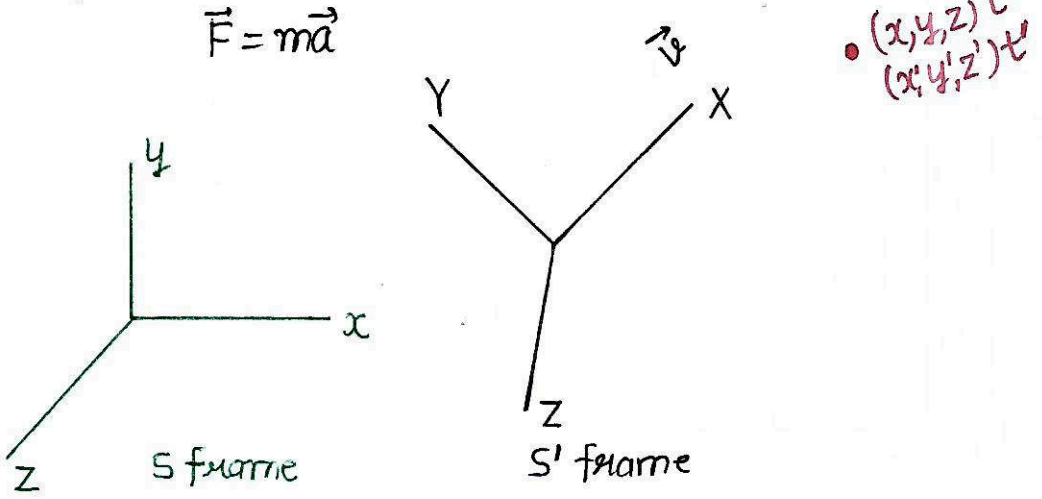
$$\eta = \frac{\Delta\phi' - \Delta\phi}{2\pi}$$

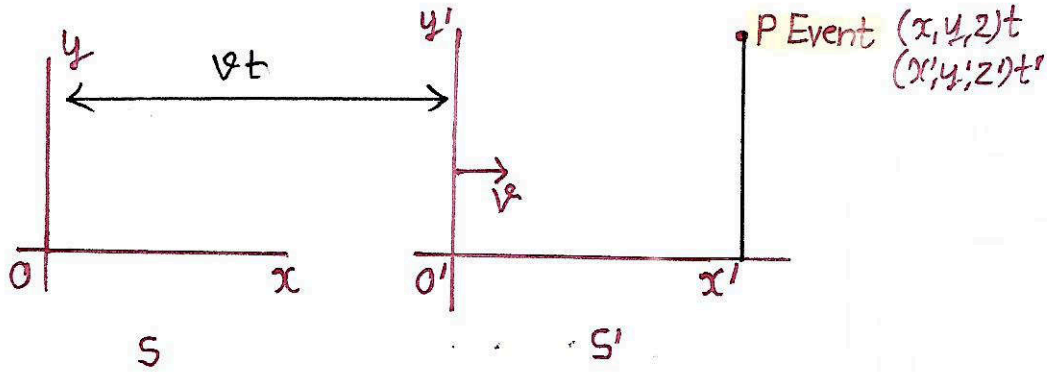
$\eta = 0.44$

#Galilean Transformation–

Laws of Physics have same form in all inertial frame.

$\vec{F} = m\vec{a}$





* Axes of S' are parallel to the corresponding axes of S.
* x and x' axes are coincidence
* At the crossing of O' and O
  $t = 0, t' = 0$
* S' moves in +ve x direction with a velocity of v.

Even? = Something happens	
Where? (x,y,z)	(x',y',z')
When? t	t'

Here

$$t' = t$$
$$x' = x - vt$$
$$y' = y$$
$$z' = z$$

} Galilean Transformation Equation

$$u_x = \frac{dx}{dt}, \quad u_y = \frac{dy}{dt}, \quad u_z = \frac{dz}{dt}$$

$$u_{x'} = \frac{dx'}{dt'}, \quad u_{y'} = \frac{dy'}{dt'}, \quad u_{z'} = \frac{dz'}{dt'}$$

$$u_{x'} = \frac{d(x-vt)}{dt} = \frac{dx}{dt} - v = u_x - v$$

$$u_{y'} = u_y$$

$$u_{z'} = u_z$$

} Galilean Transformation eqⁿ of velocity

$$a_x = \frac{du_x}{dt}, \quad a_y = \frac{du_y}{dt}, \quad a_z = \frac{du_z}{dt}$$

$$a_{x'} = \frac{du_{x'}}{dt'}, \quad a_{y'} = \frac{du_{y'}}{dt'}, \quad a_{z'} = \frac{du_{z'}}{dt'}$$

$$a_{x'} = \frac{du_x}{dt} - a_x$$

$$\boxed{a_{x'} = a_x}$$

$$a_{y'} = a_y$$

$$a_{z'} = a_z$$

$$a_{x'} = a_x$$
$$a_{y'} = a_y$$
$$a_{z'} = a_z$$

} Galilean Transformation Eqⁿ of acceleration

Lorentz Transformation –

$\vec{F} = m\vec{a}$

If it is valid in S

$\vec{F} = m\vec{a}$

If this result is same in S'.

→ Galilean Transformation.

→ Galilean Transformation is valid for Newton's Law but it is not ok with Optics, maxwell equations and, EM fields.

→ Newton's Laws is invarient under Galilean Transformation but Maxwell's Laws are not invarient under Galilean Transformation.

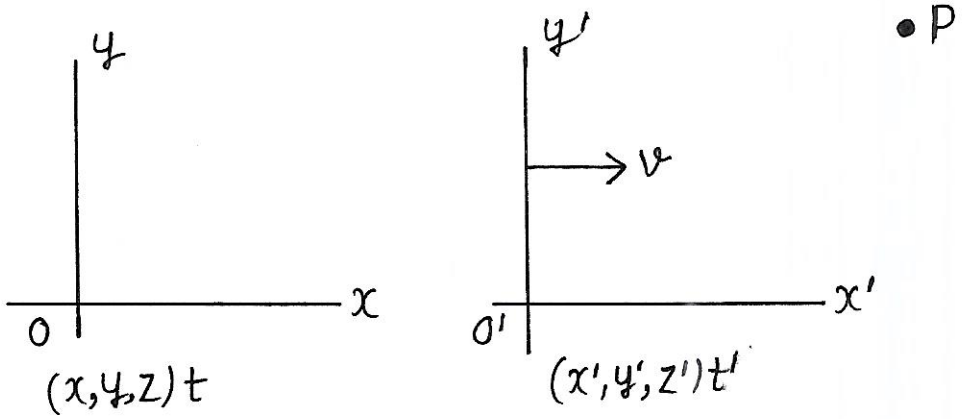
→ Maxwell equations contradict the Relativity (1st postulate). If Maxwell eqn's are authantic and correct laws of physics then physics of Relativity is not correct though.

→ But Relativity is ok in itself than Galilean Transformation needs to be modified. Then A new transformation came up i.e. Lorentz transformation. Then Speed of light is same for All frame of reference.

→ Maxwell equations are invarient under Lorentz transformation.

→ Lorentz Transformation?

1. Principle of Relativity.
2. Constancy of Speed of Light.
3. Homogenety of Space and time. → • Identical Space • All part of Space is same. • Space, itself is same everywhere. • Time is homogenus.
4. Isotopy of Space. → • All direction in Space is same. (is → are)



$$x' = a_{11}x + a_{12}y + a_{13}z + a_{14}t$$
$$y' = a_{21}x + a_{22}y + a_{23}z + a_{24}t$$
$$z' = a_{31}x + a_{32}y + a_{33}z + a_{34}t$$
$$t' = a_{41}x + a_{42}y + a_{43}z + a_{44}t$$

} Homogenous space

$$x' = ax + bt \qquad y' = y,\ z' = z$$
$$t' = dx + ft$$

Event;

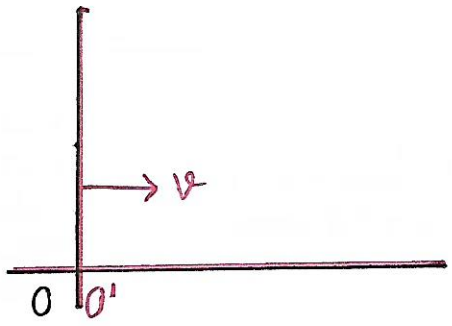
As seen from S, occurs at $(vt, 0, 0)t$

This occurs at O'(Speak as o prime).

So from S', this occurs at $(x'=0, y'=0, z'=0)$

$$0 = a(vt) + bt$$
$$0 = (av + b)t \qquad [\because av + b = 0] \text{ Then } av = -b$$

$$x' = ax - avt$$
$$\boxed{x' = a(x - vt)}$$



$$t = t' = 0$$

A light pulse is radiated from the common origin, and spreads in all direction.

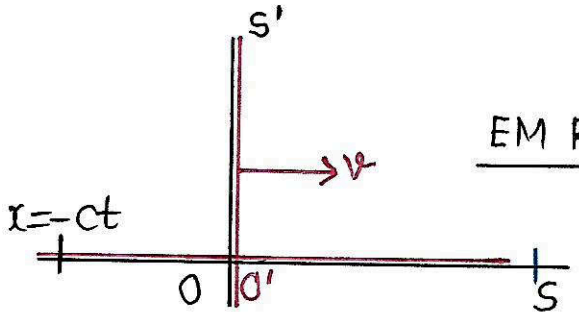
→ Gallilean Transformation has to be transformed because it does not follow the postulate of <u>constancy of speed of light</u>.

$x' = a(x - vt)$

$y' = y$

$z' = z$

$t' = dx + ft$



EM Radiation is generated.

From S:

$t = t$
$x = ct$ } Event 1

$y = 0$

$z = 0$

From S':

$x' = a(x - vt)$

$x' = a(ct - vt)$

$t' = dx + ft = dct + ft$

$y' = 0$

$z' = 0$

$$\frac{x'}{t'} = c$$

$$\boxed{x' = ct'}$$

$$\frac{a(c-v)}{dc+f} = c$$

$$a(c-v) = dc^2 + fc \quad \ldots\ldots\ldots\ldots (1)$$

Event 2:

From S -

$t = t$

$x = -ct$

$y = 0$

$z = 0$

From S' -

$x' = a(x - vt)$

$x' = +a(ct + vt)$

$x' = -a(c+v)t$

$t' = dx + ft = -dct + ft$

$\Rightarrow \frac{x'}{t'} = -c^2$

$\Rightarrow +c^2 = +\frac{a(c+v)}{-dc+f}$

$\Rightarrow dc^2 + fc = a(c+v) \ldots\ldots (2)$

From (1) & (2)

$dc^2 + fc = a(c-v)$

$-dc^2 + fc = a(c+v)$

$2fc = 2ac$

$\boxed{a = f}$

Event 3:

From S -

$t = t, x = 0, y = ct, z = 0$

From S' -

$x' = a(x - vt) = -avt$

$y' = ct$

$z' = 0$

$t' = ft$

distance travelled by the Pulse :

$$D=\sqrt{x'^2+y'^2+z'^2}$$

$$\frac{\text{distance}}{\text{time}} = \text{Speed}$$

$$\frac{D}{t'} = c$$

$$c = \frac{\sqrt{a^2v^2t^2+c^2t^2+0}}{ft}$$

$$c = \frac{t\sqrt{a^2v^2+t^2}}{ft}$$

$$f^2c^2 = a^2v^2+c^2 \quad \ldots\ldots\ldots (3)$$

$$a = f$$

$$a^2c^2 = a^2v^2+c^2$$

$$a^2(c^2-v^2) = c^2$$

$$a^2 = \frac{c^2}{c^2-v^2}$$

$$a^2 = \frac{1}{\left(1-\frac{v^2}{c^2}\right)}$$

$$\boxed{a = f = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}}$$

from (1) & (2)

$$2dc^2 = -2av$$

$$d = \frac{-2av}{2c^2}$$

$$d = \frac{-v}{c^2}$$

$$\boxed{d = -\frac{v}{c^2}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}}$$

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t}{\sqrt{1 - v^2/c^2}} \cdot \left(-\frac{v}{c^2} \cdot \frac{1}{\sqrt{1 - v^2/c^2}} \right) x$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

This is Lorentz transformation Equations.

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}$$

$$y = y'$$

$$z = z'$$

$$t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

This is Inverse Transformation Equations.

$$\frac{v}{c} = \beta$$

$$\frac{1}{\sqrt{1 - v^2/c^2}} = \gamma$$

Hence

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

→ Newton's laws is not invarient under Lorentz transformation.

Length Contraction & Time dilation -

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

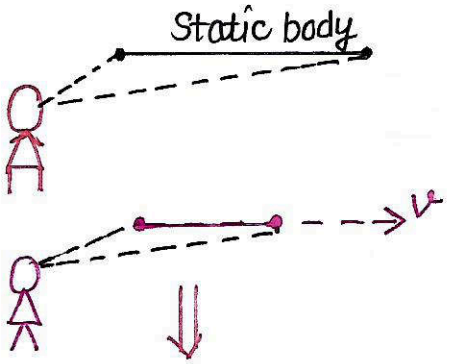
} Transformation Equations

- **Length Contraction -**

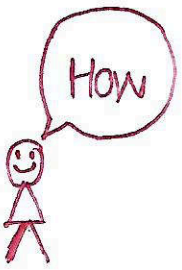
  # S and S' are very defined frames.

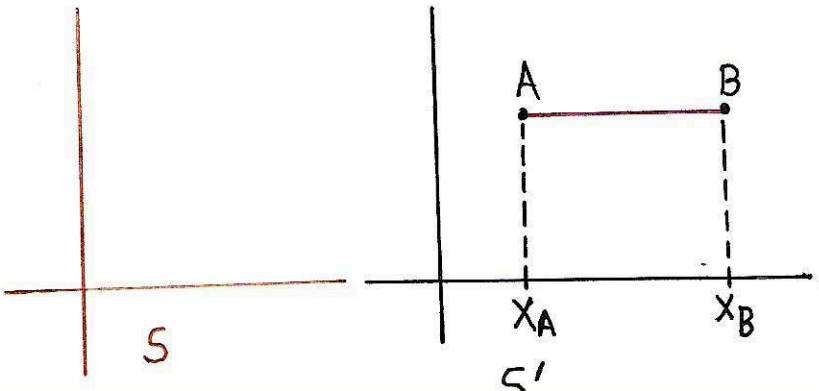
  # x and x' are coincidence axes and, y and y' are parallel to each other.

  # If we have to measure length b/w two points.



How to calculate length of a moving body?





$L_0 = x'_B - x'_A$ ← Rest Length

Event 1 : Measure the length End

$x_A ; t$

Event 2 : Measure the right end

$x_B ; t$

Length of $AB = x_B - x_A$

From S' :

E_1:

$$x_{A'} = \frac{x_A - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

E_2:

$$x'_B = \frac{x_B - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

length of AB in frame S'

$$\boxed{x'_B - x'_A = \frac{x_B - x_A}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

$$t_{A'} = \frac{t_A - \frac{x_A v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t'_B = \frac{t_B - \frac{x_B v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$L_0 = \frac{x_B - x_A}{\sqrt{1 - \frac{v^2}{c^2}}} = x'_B - x'_A$$

$$\text{Rest Length} = \frac{\text{moving Length}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- Rest Length >>> moving Length

Measurements of Lengths as well as of time intervals are affected by relative motion.

⇒ Time Dilation:

S' frame of Reference:

$E_1 : x', t'_1$

$E_2 : x', t'_2$

Time Interval b/w Events E_2 and $E_1 = t'_2 - t'_1$

Proper time interval $= t'_2 - t'_1$

S frame of Reference:

E_1:

$$t_1 = \frac{t'_1 + \frac{x'v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

[Reverse Transformation $t = t' + \frac{x'v}{c^2} \Big/ \sqrt{1 - \frac{v^2}{c^2}}$]

E_2:

$$t_2 = \frac{t'_2 + \frac{x'v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Time interval $= t_2 - t_1$

$$t_2 - t_1 = \frac{t_2' - t_1'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\text{Improper time interval} = \frac{\text{Proper time interval}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Improper time interval >>> Proper time interval

If someone in a moving spacecraft finds that the time interval b/w two events in the spacecraft is t_0, we one the ground would find that the same interval has the longer duration t. This effect is called time dilation because Improper time is greater than proper as found mathematically. (To dilate is to become longer).

#Study twin Paradox ?

Relativity of Simultaneity –

$$\text{Rest Length} = \frac{\text{Moving Length}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\text{Proper time interval} = (\text{Improper time interval}) \times \sqrt{1 - \frac{v^2}{c^2}}$$

Suppose there is a clock (From S Frame) –

Event 1; $t = t_1$
Event 2; $t = t_2$

Time interval $\Delta t = t_2 - t_1$

From S' ;

Event 1; $t_1' = \frac{t_1 - \frac{x_0 v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$

Event 2; $t_2' = \frac{t_2 - \frac{x_0 v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$

$$t_2' - t_1' = \frac{t_2 - t_1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

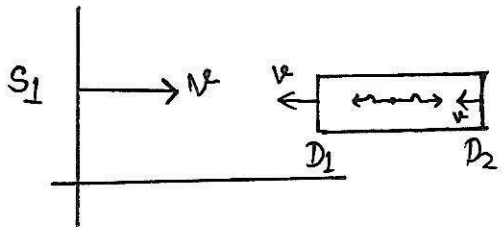
Time $\frac{t_2 - t_1}{\sqrt{1 - \frac{v^2}{c^2}}}$ is elapsed but the clock has advanced only by $t_2 - t_1$.

Moving clocks run slow.

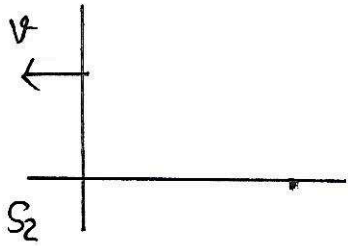
Simulanouity is not universal.
It depends on the frames.



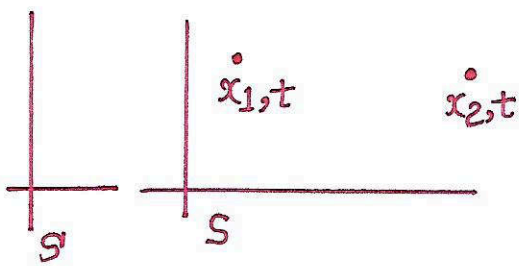
Light hits $D_1 \Rightarrow D_1$ opens.
Light hits $D_2 \Rightarrow D_2$ opens.
Door D_1 and D_2 is opened simaltaniously.



D_2 opens before D_1.
Not simultanious.



D_1 opens before D_2.
Not simultanious.



$$E_1;\ t_1' = \frac{t - \frac{x_1 v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$E_2;\ t_2' = \frac{t - \frac{x_2 v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t_2' - t_1' = \frac{-\frac{v}{c^2}(x_2 - x_1)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$t_2' < t_1'$

↓

Not simulanious.

#Quiz 2 :

1. A moving magnet creates (c)
2. b & d.
3. d.
4. a & c.
5. a & c.
6. b & d.
7. b.
8. c.
9. c.
10. a, b & c.
11. c & d.
12. a, b & c.
13. c, d, e.
14. b & c.
15. b.

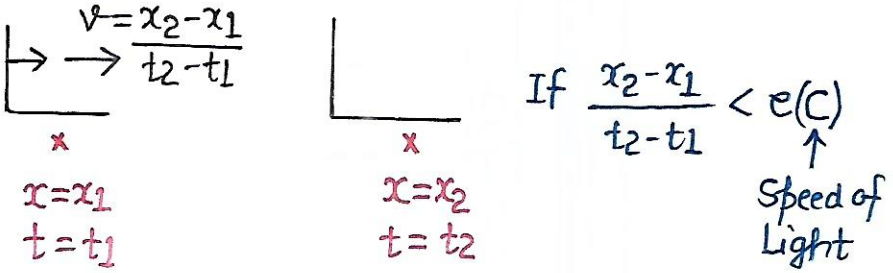
#Synchronization of Time-

- If two events are simulatanious in one frame, they will be simultanious in other frame.

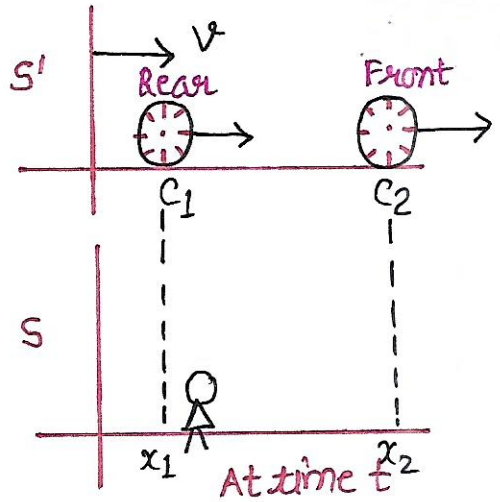
#If two events are occuring at same position-

$$\Delta t' = \frac{\Delta t}{\sqrt{1-\frac{v^2}{c^2}}}$$

If $\Delta t > 0$, $\Delta t'$ is also positive.



- Moving clocks seem slow or we can say, run slow.



- If two clocks showing same time, it means they are synchronized.
- At time t, S observers took the readings on clocks C_1 and C_2.

Event 1 : S Observer notes the reading of C_2

Event 2 : S Observer notes the reading of C_1

S : $E_1 \rightarrow x = x_1, t = t$

$E_2 \rightarrow x = x_2, t = t$

Event 1:

$$t_1' = \frac{t - \frac{x_1 v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t_2' = \frac{t - \frac{x_2 v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t_2' - t_1' = \frac{-\frac{v}{c^2}(x_2 - x_1)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\boxed{t_2' < t_1'}$$

Clock C_1 shows heigher time than C_2 clock

→ Rear clocks leads.
→ Front clock lacks(lags).

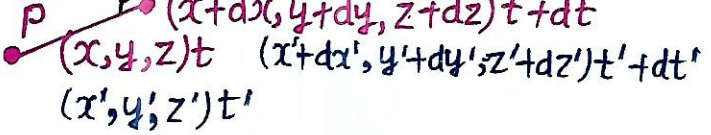
If $\frac{x_2 - x_1}{\sqrt{1 - \frac{v^2}{c^2}}} = L$

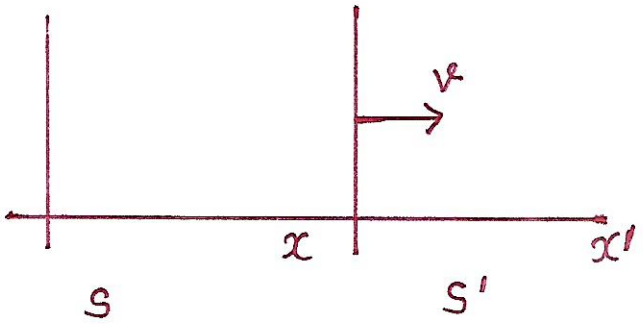
$$t_2' - t_1' = -\frac{v}{c^2} \cdot L$$

$$\boxed{\Delta t = \frac{Lv}{c^2}}$$

#Transformation of Velocities -

#You have frames.
#Transformation of Velocity





$$u_x = \frac{dx}{dt}, \quad u_y = \frac{dy}{dt}, \quad u_z = \frac{dz}{dt}$$

$$u_x' = \frac{dx'}{dt'}, \quad u_y' = \frac{dy'}{dt'}, \quad u_z' = \frac{dz'}{dt'}$$

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = (x - vt)\gamma$$

$$dx' = (dx - v\,dt)\gamma$$

$$t' = \left(t - \frac{xv}{c^2}\right)\gamma$$

$$dt' = \left(dt - \frac{v\,dx}{c^2}\right)\gamma$$

$$u_x' = \frac{dx'}{dt'} = \frac{(dx - v\,dt)\gamma}{\left(dt - \frac{v\,dx}{c^2}\right)\gamma}$$

$$u_x' = \frac{\frac{dx}{dt} - v}{\frac{dt}{dt} - \frac{v\,dx}{c^2 dt}} = \frac{\left(\frac{dx}{dt} - v\right)}{1 - \frac{v}{c^2}\left(\frac{dx}{dt}\right)} = \frac{u_x - v}{1 - \frac{v u_x}{c^2}} \quad \ldots\ldots\ldots (1)$$

$$y' = y$$

$$dy' = dy$$

$$dt' = (dt - v\,dx)\gamma$$

$$u_y' = \frac{dy'}{dt'}$$

$$u_y' = \frac{dy}{(dt - \frac{v\,dx}{c^2})\gamma}$$

$$u_y' = \frac{\frac{dy}{dt}}{\left(1 - \frac{v u_x}{c^2}\right)\gamma}$$

$$u_y' = \frac{\frac{dy}{dt}\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v u_x}{c^2}} = \frac{u_y\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v u_x}{c^2}} \quad \ldots\ldots (2)$$

$$u_x' = \frac{u_x - v}{1 - \frac{v u_x}{c^2}}$$

$$u_z' = \frac{u_z\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v u_x}{c^2}} \quad \ldots\ldots\ldots (3)$$

- Inverse Transformation:

$$u_x = \frac{u^{x'} + v}{1 + \frac{v u_x'}{c^2}}$$

$$u_y = \frac{u_y'\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v u_x}{c^2}}$$

$$u_z = \frac{u_z'\sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{v u_x}{c}}$$

These equations are inverse transformation.

#Problems!

(1) A particular particle created in a nuclear reactor leaves a 1 cm track before delaying. Assuming that the particle moved at 0.995C, calculate the life of the particle (a) lab frame (b) in the frame of particle.

(a) Life time = $\frac{\text{distance}}{\text{Speed}}$

$$\text{Life time} = \frac{1\ \text{cm}}{0.995 \times 3 \times 10^{8}\ \text{cm/s}}$$

$$\text{Life time} = \frac{1}{2.985} \times 10^{-10}\ \text{S} = 0.33 \times 10^{-10}\ \text{S} = 3.35 \times 10^{-11}\ \text{S}$$

(b)

$$t_0 = \left(3.35 \times 10^{-11}\right)\sqrt{1-\left(\frac{v}{c}\right)^2}$$

$$t_0 = 3.35 \times 10^{-11}\sqrt{1-(0.995)^2}$$

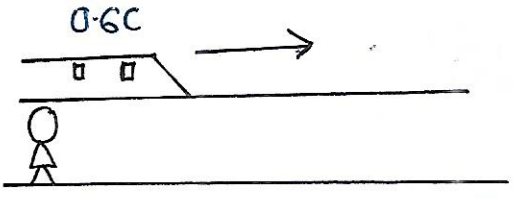
$$t_0 = 3.35 \times 0.09987 \times 10^{-11}$$

$$t_0 = 0.33458 \times 10^{-11}$$

$$\boxed{t_0 = 3.34 \times 10^{-12}\ \text{S}}$$

(2) A person standing on a platform finds that a train moving with velocity 0.6C takes one second to pass by him. Find the length of train as seen by the ~~train~~ person (b) rest length of the train.

(a)



time = 1S
Speed = 0.6C

distance covered by engine.

$$d = 1S \times 0.6 \times 3 \times 10^{8}\ m$$

$$\boxed{d = 1.8 \times 10^{8}\ m}$$

(a) Length of train = 1.8×10^8 m

(b) Rest Length = ?

Rest Length >> Moving Length

$$L_o = \frac{L}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$$L_o = \frac{1.8 \times 10^8}{0.8}$$

$$\boxed{L_o = 2.25 \times 10^8 \text{ m}}$$

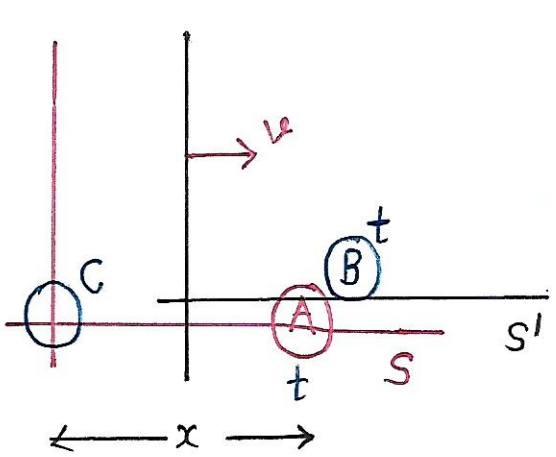
(3) At time t, a particular clock A fixed on the X axes of the frame S agrees with the clock B fixed on the X axis of frame S' and opposite to A.

(a) Find the x coordinates of these clocks as seen by S.

(b) What does the clock C fixed in S' opposite to the origin of S read at the instant as observed in S ?

(c) Show that the rear clock C leads the front clock B by Lv/c^2 where L is the rest sepration between the clocks.

(a)



Event i x,t in S

$$t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = t \quad \text{in } S'$$

$$t\left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1\right]=\frac{xv/c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t\left[1-\sqrt{1-\frac{v^2}{c^2}}\right]=\frac{xv}{c^2}$$

$$x=\frac{c^2}{v}t\left[1-\sqrt{1-\frac{v^2}{c^2}}\right]$$

(b) Event ; C comes opposite to the origin O & S.

S frame ; $x=0, t=t$

S' frame ; $t'=\dfrac{t-\frac{xv}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$

$$t=\frac{t}{\sqrt{1-\frac{v^2}{c^2}}}$$

(c) C leads B by

$$\frac{t}{\sqrt{1-\frac{v^2}{c^2}}}-t=t\left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1\right]$$

From S, the separation between B and C is

$$x=\frac{c^2}{v}t\left[1-\sqrt{1-\frac{v^2}{c^2}}\right]$$

Rest separation $L=\dfrac{\frac{c^2}{v}t\left[1-\sqrt{1-\frac{v^2}{c^2}}\right]}{\sqrt{1-\frac{v^2}{c^2}}}$

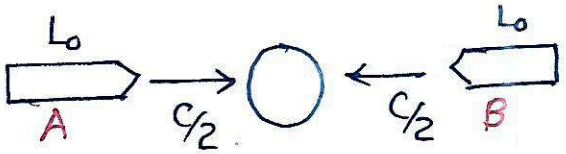
Rest separation $L = \frac{c^2 t}{v}\left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right]$

$$\frac{Lv}{c^2} = t\left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right]$$

LHS = RHS ✓

(4) Two rockets, each of Length L_0 are approaching the earth from opposite distance at the same speed $c/2$. Find the length of one rocket in the frame of other rocket

Rest Length



Velocity of B in the frame of A

$$v = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}} = \frac{\frac{c}{2} + \frac{c}{2}}{1 + \frac{1}{4}} = \frac{4}{5}c$$

Length of B $= L_0\sqrt{1-\frac{v^2}{c^2}} = L_0\sqrt{1-(0.8)^2} = 0.6L_0$

#Position 4 Vectors-

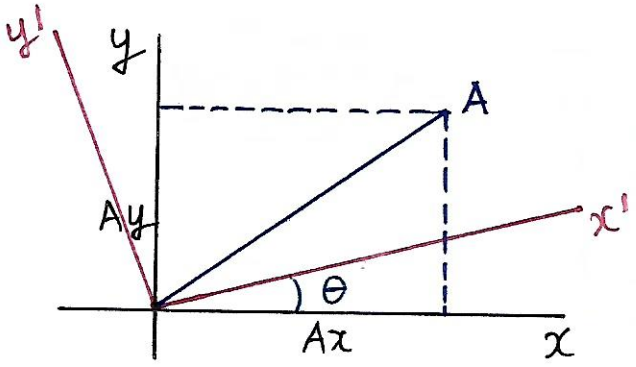
#Geometrical representation of transformation system.

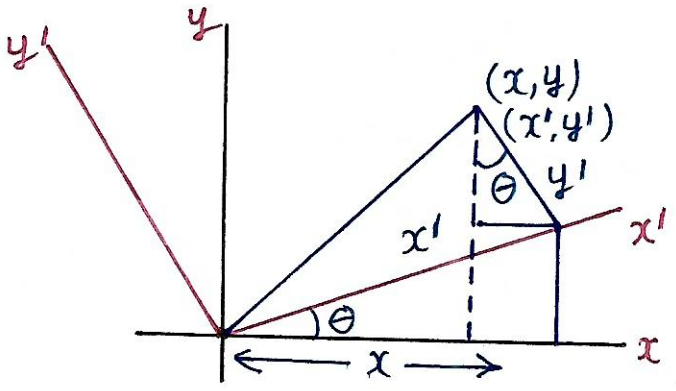
$$t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$





$$x = x'\cos\theta - y'\sin\theta$$

$$y = x'\sin\theta + y'\cos\theta$$

$$x' = x\cos\theta + y\sin\theta$$

$$y' = -x\sin\theta + y\cos\theta$$

$$z' = z$$

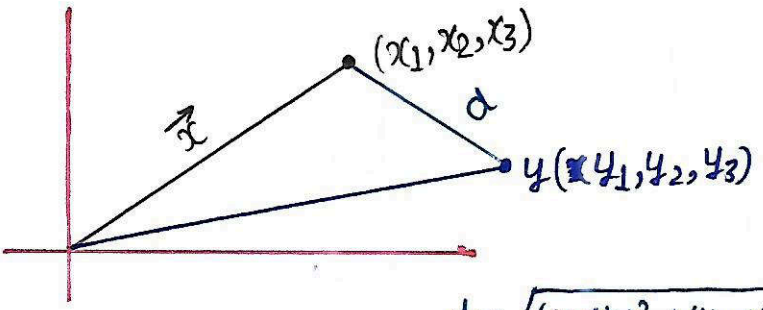
$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$ct' = \frac{ct - \frac{xv}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma ct - x\beta\gamma \qquad \beta = \frac{v}{c}, \quad \gamma = \sqrt{1 - \frac{v^2}{c^2}}$$

$$x' = x\gamma - \beta(ct)\gamma$$
$$y' = y$$
$$z' = z$$

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

→ position 4 Vectors



$$d = \sqrt{(x_1 - y_1)^2 + (y_1 - y_2)^2 + (z_1 - z)^2}$$

x_1, x_2, x_3

$\vec{a}, \vec{b}$

$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$

$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = \sum_i a_i b_i$

$\sqrt{\vec{a} \cdot \vec{a}} = \sqrt{\sum_i a_i a_i} = \sqrt{a_1^2 + a_2^2 + a_3^2}$

⇓

Magnitude of a

(invariant under rotation)

$$(a_1-b_1)^2+(a_2-b_2)^2+(a_3-b_3)^2$$

$ct \Rightarrow x_0, x=x_1, y=x_2, z=x_3$

$$\begin{bmatrix} x_0' \\ x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$x_0^2+x_1^2+x_2^2+x_3^2=(ct)^2+x^2+y^2+z^2$$

*Interval between two events :

$$-c^2(t_1-t_2)^2+(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2$$

* This is not the time interval.
* This is invarient under Lorentz transformation.

$P=mv$?

One from to other frame, things change.
Space and time.
t, x, y, z components.
come to dynamics.
We go to high speed, newton's laws doesn't work.

$$\vec{F}=\frac{d\vec{P}}{dt}$$

$$\vec{P}=m\vec{v}$$

$$\vec{F}=\frac{d}{dt}(m\vec{v})=m\frac{d\vec{v}}{dt}$$

$$\vec{F}=m\vec{a}$$

$$\vec{F}^{ext}=\frac{d\vec{P}}{dt}$$

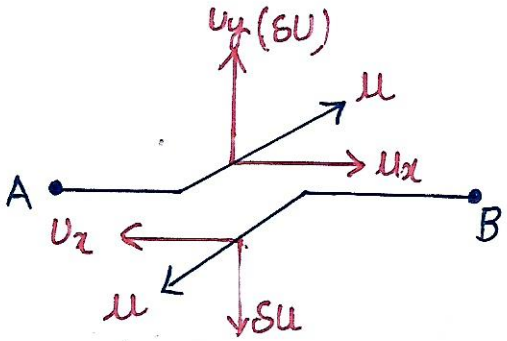
If $\vec{F}^{ext}=0$

$\vec{P}$ = constant

This is called principle of conservation of Linear momentum.

Elastic collision





#Kinetic Energy is conserved.
#Newtonian mechanics.

#Momentum Redefined-

$P = mv$

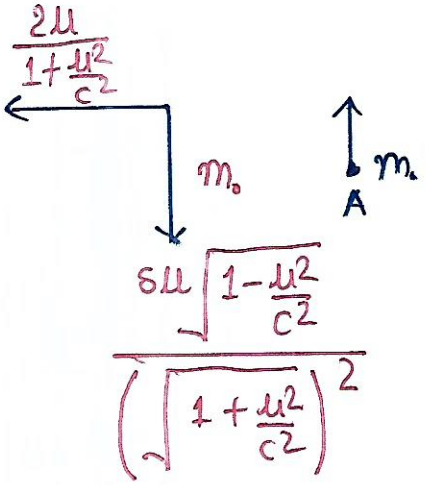
#doesn't work with Lorentz transformation
#valid in S Frame.
#Invalid in S' Frame.

$\vec{P} = m_0\vec{v}\, f(v)$

Where $f(v) \to 1$ as we goes to zero velocity.







S' frame

Before collision $\vec{P} = m_0\left(\frac{2u}{1+\frac{u^2}{c^2}}\right)\delta x\, f\left(\frac{2u}{1+\frac{u^2}{c^2}}\right)$

$$P_x = -m_0 \frac{2u}{1+\frac{u^2}{c^2}} f\left(\frac{2u}{1+\frac{u^2}{c^2}}\right) \leftarrow \text{After collision}$$

$$P_y = 0 \leftarrow \text{Before collision}$$

$$P_{y_B} = -m_0 \frac{\delta u \sqrt{1-u^2/c^2}}{1+\frac{u^2}{c^2}} f\left(\frac{2u}{1+\frac{u^2}{c^2}}\right) \Rightarrow \frac{m_0 \delta u}{\sqrt{1+\frac{u^2}{c^2}}}$$

$$P_{y_m} = m_0 \frac{\delta u}{\sqrt{1-\frac{u^2}{c^2}}} f \frac{\delta u}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$P_{y_m} = \frac{m_0 \delta u}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$f\left(\frac{2u}{1+u^2/c^2}\right) = \frac{c^2-u^2}{c^2+u^2}$$

$$P_{y_B} + P_{y_m} = 0$$

$$P_y = 0 \leftarrow \text{After collision}$$

$$\frac{\omega^2}{c^2} = \frac{4u^2/c^2}{\left(1+\frac{u^2}{c^2}\right)^2} = \frac{4u^2c^2}{(c^2+u^2)^2}$$

$$1-\frac{\omega^2}{c^2} = 1 - \frac{4u^2c^2}{(c^2+u^2)^2}$$

$$1-\frac{\omega^2}{c^2} = \frac{(c^2-u^2)^2}{(c^2+u^2)^2}$$

$$\sqrt{1-\frac{\omega^2}{c^2}} = \frac{c^2-u^2}{c^2+u^2}$$

$$f(\omega) = \frac{1}{\sqrt{1-\frac{\omega^2}{c^2}}}$$

$$f(v) = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\vec{p} = \frac{m_0\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}$$

This is modified version of $\vec{p}$.

Let $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ ← Rest mass

$\vec{p} = m\vec{v}$ ← Relativistic mass

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$m = \frac{m_0}{\sqrt{1-\frac{0^2}{c^2}}} \Rightarrow m = m_0$$

zero velocity

Low velocity $K = \frac{1}{2}mv^2$

$$\vec{P} = m\frac{\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}$$ → Proper velocity

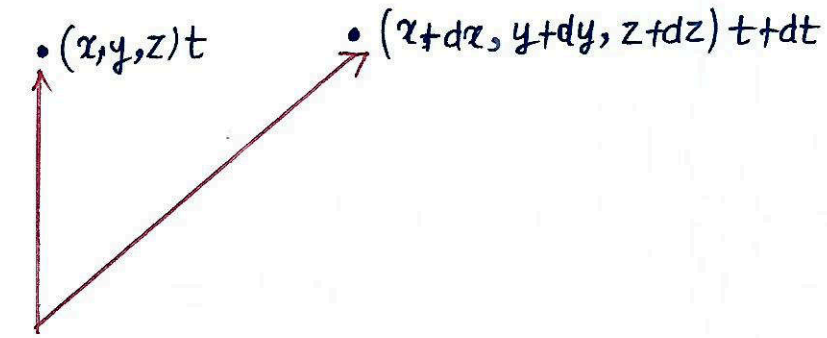
$\vec{v}$ = Ordinary Velocity

$$\vec{P} = m_0\vec{\eta}$$

$$\vec{\eta} = \frac{\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}$$

→ Proper Velocity

$$u_x = \frac{dx}{dt}$$



These are not proper time Interval.

Abhishek

$$\eta_x = \frac{dx}{d\tau}$$

$$\eta_y = \frac{dy}{d\tau}$$

$$\eta_z = \frac{dz}{d\tau}$$

#4 Vectors for Velocity and Momentum!

$\vec{p} = m\vec{v}$

↓

Need to redefined.

Particle of mass m
velocity $\vec{v}$

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\vec{p} = \frac{m_0\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$\Downarrow$

m

$\vec{p} = m\vec{v}$

m_0 → Rest mass

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \text{Relativistic mass}$$

$$\vec{p} = \frac{m_0\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}$$

→ proper Velocity $\left(\eta = \frac{\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}\right)$

$\vec{p} = \vec{\eta} m_0$

$u_x = \frac{dx}{dt}$

$(x,y,z)t$ $\quad$ $(x+dx, y+dy, z+dz)\ t+dt$

S Frame

S'

$$u_y = \frac{dy}{dt}, \quad u_z = \frac{dz}{dt}$$

$$\eta_x = \frac{u_x}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{dx}{dt} \cdot \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$d\tau = dt\sqrt{1-\frac{v^2}{c^2}}$$

$$d\tau = dt\sqrt{1-\frac{u^2}{c^2}}$$

$$\frac{dt}{d\tau} = \frac{dt}{dt} \cdot \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} = \gamma_u$$

$\eta_x = \frac{dx}{d\tau}$ → Frame S

→ Time in particle frame (Proper time Interval)

position 4 vector (ct, x, y, z)

$[c(t+dt), x+dx, y+dy, z+dz]$

$[c(dt), dx, dy, dz]$ } 4 vectors

$$\left[c\frac{dt}{d\tau}, \frac{dz}{d\tau}, \frac{dy}{d\tau}, \frac{dx}{d\tau}\right]$$

$$\left[c\gamma_u, \eta_x, \eta_y, \eta_z\right]$$

$\begin{bmatrix} c\gamma_u \\ \eta_x \\ \eta_y \\ \eta_z \end{bmatrix}$ → Velocity 4 vector For S Frame

$\begin{bmatrix} c\gamma'_{u'} \\ \eta'_x \\ \eta'_y \\ \eta'_z \end{bmatrix}$ → For S' Frame

$$\begin{bmatrix} c\gamma_u' \\ \eta_x' \\ \eta_y' \\ \eta_z' \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\gamma_u \\ \eta_x \\ \eta_y \\ \eta_z \end{bmatrix}$$

.....(1) Velocity 4 Vectors

multiply this equation with mass.

$$\begin{bmatrix} m_0 c\gamma_u \\ m_0\eta_x \\ m_0\eta_y \\ m_0\eta_z \end{bmatrix}$$

→ Linear momentum x component

$$\begin{bmatrix} m_0 c\gamma_u \\ p_x \\ p_y \\ p_z \end{bmatrix}$$ ..... (2) Momentum 4 Vectors $$\begin{bmatrix} m_0 c\gamma_u \\ m_0 u_x \gamma_u \\ m_0 u_y \gamma_u \\ m_0 u_z \gamma_u \end{bmatrix}$$

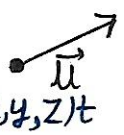
$$\eta = \frac{\vec{u}}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma_u \vec{u}$$

⇒ ($m_0 c\gamma_u$) What this value defines? This will be discussed in next lecture.

$E = mc^2$!-

proper Velocity

$$\vec{\eta} = \gamma \vec{u}$$



$(x+dx, y+dy, z+dz)\ t+dt$

$$\vec{\eta}_x = \frac{dx}{d\tau}, \vec{\eta}_y = \frac{dy}{d\tau}, \vec{\eta}_z = \frac{dz}{d\tau}$$

$$\begin{bmatrix} c\gamma \\ \eta_x \\ \eta_y \\ \eta_z \end{bmatrix}$$ → Velocity 4 Vectors

multiply above equation with mass.

$$\begin{bmatrix} m_0 c\gamma \\ m_0 v_x \\ m_0 v_y \\ m_0 v_z \end{bmatrix} \rightarrow \text{Momentum 4 vectors}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$$

m_0 is rest mass.

$$\vec{p} = \frac{m_0 \vec{u}}{\sqrt{1-\frac{u^2}{c^2}}} = m_0 \vec{\eta}$$

$$\begin{bmatrix} m_0 c\gamma \\ p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} p^{(0)} \\ p^{(1)} \\ p^{(2)} \\ p^{(3)} \end{bmatrix}$$

What is $p^{(0)}$?

Kinetic Energy

$$K = \frac{1}{2} m_0 u^2$$

→ Work done by the force is equal to the change in Kinetic energy at speed zero $K=0$.

$$dW = \vec{F} \cdot d\vec{x}$$

$$dW = \frac{d\vec{p}}{dt} \cdot d\vec{x}$$

$$dW = d\vec{p} \cdot \vec{u} \qquad \left[\because \vec{u} = \frac{d\vec{x}}{dt}\right]$$

$$\boxed{dK = d\vec{p} \cdot \vec{u}}$$

Non Relativistic speed

$$\vec{p} = m_0 \vec{u}$$

$$d\vec{p} = m_0 d\vec{u}$$

$$dK = m_0 \vec{u} \cdot d\vec{u} = \frac{m_0}{2} \vec{u}\, d(u^2)$$

$$u^2 = u \cdot u$$

$$d(u^2) = u\, du + du \cdot u$$

$$d(u^2) = 2\vec{u} \cdot d\vec{u}$$

$$d(u^2) = \frac{1}{2} d(u^2)$$

$$\int_0^K dK = \int_0^u \frac{m_0}{2} d(u^2)$$



$$K = \frac{m_0}{2} u^2 \int_0^u$$

$$\boxed{K = \frac{1}{2} m_0 u^2}$$

$$\vec{p} = m_0 \vec{u} \gamma$$

$$dK = d\vec{p} \cdot \vec{u} = m_0 \vec{u}\left([\vec{u} d\gamma] + [\gamma d\vec{u}]\right)$$

$$dK = m_0 [u^2 d\gamma + \gamma \vec{u} \cdot d\vec{u}]$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} = \left(1 - \frac{u^2}{c^2}\right)^{-1/2}$$

$$d\gamma = \left(-\frac{1}{2}\right)\left(1 - \frac{u^2}{c^2}\right)^{-3/2} \left(-\frac{1}{c^2} 2 \vec{u} \cdot d\vec{u}\right)$$

$$d\gamma = \left(1 - \frac{u^2}{c^2}\right)^{-3/2} \frac{1}{c^2} \vec{u} \cdot d\vec{u}$$

$$\vec{u} \cdot d\vec{u} = c^2 d\gamma \left(1 - \frac{u^2}{c^2}\right)^{3/2}$$

$$\gamma \vec{u} \cdot d\vec{u} = \gamma c^2 d\gamma \frac{\left(1 - \frac{u^2}{c^2}\right)^{3/2}}{\left(1 - \frac{u^2}{c^2}\right)^{1/2}} = c^2 d\gamma \left(1 - \frac{u^2}{c^2}\right) = d\gamma (c^2 - u^2)$$

$$dK = m_0 \left[u^2 d\gamma + d\gamma (c^2 - u^2)\right]$$

$$dK = m_0 c^2 d\gamma$$

$$\int_0^K dK = m_0 c^2 \int_0^u d\gamma$$

$$K = m_0 c^2 [\gamma]_0^u$$

$$\boxed{K = m_0 c^2 \left[\frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} - 1 \right]}$$

Non-relativistic limit

$$K = \frac{1}{2} m_0 u^2$$

$$\hookrightarrow \frac{1}{2} m u^2$$

$$\frac{1}{2} \frac{m_0}{\sqrt{1 - \frac{u^2}{c^2}}} u^2$$

$$K = m_0 c^2 \left[\frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} - 1 \right]$$

Kinetic Energy

Rest mass Energy

$$K + m_0 c^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{u^2}{c^2}}} = mc^2$$

Total Energy

$$\boxed{E = mc^2}$$

4 momentum vector

$$\begin{bmatrix} m_0 c\gamma \\ p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} E/c \\ p_x \\ p_y \\ p_z \end{bmatrix}$$

$$\begin{bmatrix} E/c' \\ p'_x \\ p'_y \\ p'_z \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E/c \\ p_x \\ p_y \\ p_z \end{bmatrix}$$

#Mass Energy Conservation -

Kinetic Energy ← $K = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0c^2$ → Rest Mass Energy

→ Total Energy

If $v << c$

$$K = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0c^2$$

$$K = m_0c^2\left[\left(1-\frac{v^2}{c^2}\right)^{-1/2} - 1\right]$$

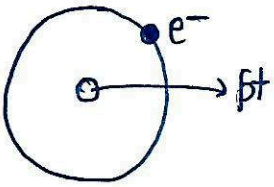
$$K = m_0c^2\left[1 + \frac{1}{2}\cdot\frac{v^2}{c^2} + \ldots\ldots - 1\right] = \frac{1}{2}m_0v^2$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \ldots\ldots \quad [\because (x<1)]$$

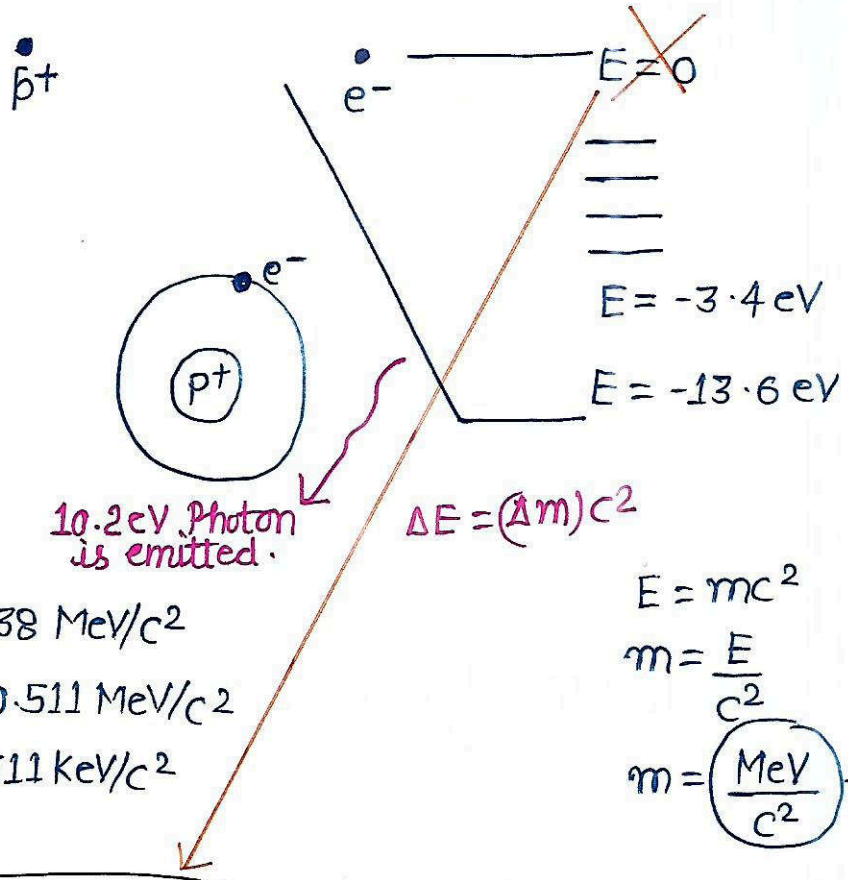
$$\boxed{K = \frac{1}{2}m_0v^2}$$

Hydrogen Atom

(p) and (e^-)



Ground State Energy $E = -13.6$ eV



$m_p = 938$ MeV/c^2

$m_e = 0.511$ MeV/c^2

$m_e = 511$ keV/c^2

$E = mc^2$

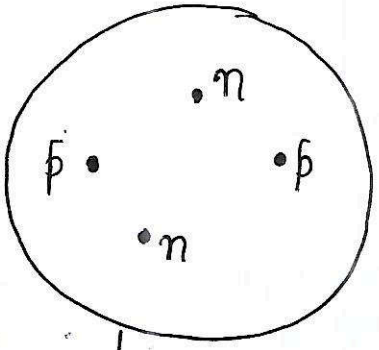
$m = \frac{E}{c^2}$

$m = \left(\frac{\text{MeV}}{c^2}\right)$ → Unit

$m_H = 938.511$ MeV/c^2

$m(H) = 938.511 - 13.6$ eV/c^2

α particle :



Rest mass = $2m_p + 2m_n$

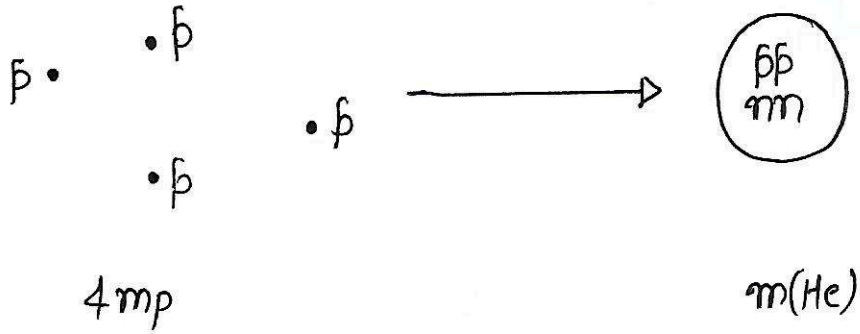


Rest mass energy = $\frac{2m_p + 2m_n}{c^2}$

$= \frac{2(m_p + m_n)}{c^2}$

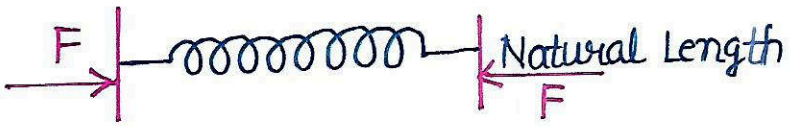
→ Rest mass energy of α Particle = $2(m_p - m_n)c^2 - 28$ MeV

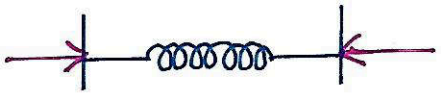
Rest mass of He = $2(m_p + m_n) - 28 \frac{MeV}{c^2}$



$4m_p$ $\qquad\qquad$ $m(He)$

$4m_p > m(He)$

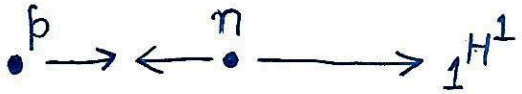




Large energy of compressed spring.
Hot object(A) has large mass than cold object(A).

→ Rest mass energy inside an object.

Specific object.
Hoter body have higher rest mass energy than the cold body.
Compressed spring has higher rest mass energy because its total energy is higher.



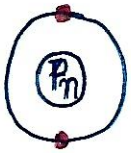
Hydrogen Nucleus

No. of protons = 1
No. of e^- = 1
No. of $_0n^1$ = 1

A = 1
Z = 1

Nucleons = Protons and Neutrons

$_1H^2$



Heavy Hydrogen
Deutron

$$m\left({}_1^2H\right) < m_p + m_n$$

Fission Reactors

→ 21 Reactors in India

$$_{92}U^{235} + {}_0n^1 \longrightarrow {}_{92}U^{236} \longrightarrow X + Y + e + \bar{\nu} + \text{Energy}$$

$$(\Delta m)c^2 = \Delta E$$

$$m_p + m_n \longrightarrow D + 2.2\,\frac{MeV}{c^2}$$

$$(\Delta m)c^2 = 2.2\,MeV$$

→ If Particle is completely distroyed.

Electron-Positron Annihilation

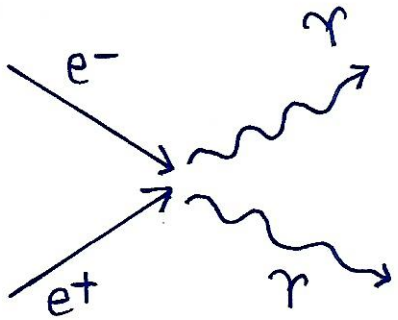
$e^+ \longrightarrow$ mass of e^-

charge = $+e$

$$^{22}_{11}Na \longrightarrow {}^{22}_{10}Ne + e^{+} + \nu$$

$Z = 11$
$A = 22$
$p^{+} = 11$
$e^{-} = 11$
$_{0}n^{1} = 11$

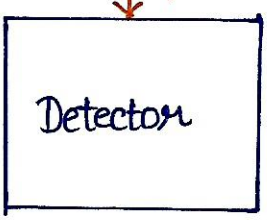
$$e^{+} + e^{-} \longrightarrow \gamma + \gamma$$



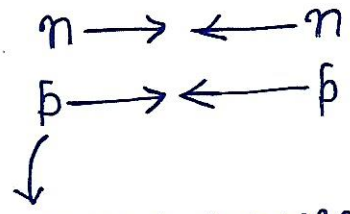
$E_1 = m_0c^2 = 0.51\ MeV$
$E_1 = m_0c^2 = 0.51\ MeV$
$E = m_0c^2 + m_0c^2$
$E = 2m_0c^2$
$E = 2m_0c^2 = 1.02\ MeV$
$E = 2m_0c^2\gamma$
$h\nu = 2m_0c^2\gamma$

- direct conversion of Particle into energy
- Annihilation process.

$\gamma > 1022\ KeV$



LHC – Large Hadron Collider



↓

938 MeV Rest mass energy

$p + p = p + p + p + n = p + p + \underbrace{\pi^+ + \pi^-}_{\text{Pions}}$

$$\begin{bmatrix} E/c \\ p_x \\ p_y \\ p_z \end{bmatrix} \longleftarrow \text{Identical} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E/c \\ p_x \\ p_y \\ p_z \end{bmatrix}$$

$$\begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix} = \uparrow \qquad \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

$-c^2t^2 + x^2 + y^2 + z^2 \Rightarrow$ invariant

$(\text{Magnitude})^2$

$E^2 = p^2c^2 + m^2c^4$

$-\left(\frac{E}{c}\right)^2 + \underbrace{p_x^2 + p_y^2 + p_z^2}_{\text{Magnitude}} \Rightarrow$ invariant

$E^2 - p^2c^2$ is invariant.

$E = E_1 + E_2$

$\vec{p} = \vec{p_1} + \vec{p_2}$ ← Linear momentum

$E_1, p_1 \mid E_2, p_2 \mid$

#Energy Momentum Relations!

$$\begin{bmatrix} E/c \\ p_x \\ p_y \\ p_z \end{bmatrix}$$

$$= \left(\frac{-E^2}{c^2} + p^2\right)$$

$\Rightarrow E^2 - p^2c^2$ is invarient.

For a single particle.

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\vec{p} = \frac{m_0 \vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$E^2 - p^2c^2 = \frac{m_0^2 c^4}{1 - \frac{v^2}{c^2}} - \frac{m_0^2 v^2 c^2}{1 - \frac{v^2}{c^2}}$$

$$E^2 - p^2c^2 = \frac{m_0^2 c^2 (c^2 - v^2)}{1 - \frac{v^2}{c^2}}$$

$$E^2 - p^2c^2 = \frac{m_0^2 c^2 \cancel{(c^2 - v^2)}}{\frac{\cancel{c^2 - v^2}}{c^2}}$$

$$E^2 - p^2c^2 = m_0^2 c^4$$

$$E^2 - p^2c^2 = m_0^2 c^4 \quad \Rightarrow E^2 = p^2c^2 + m_0^2 c^4$$

$$\boxed{E = \sqrt{p^2c^2 + m_0^2 c^4}}$$

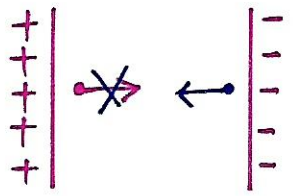
→ Relativistic or Newtonian Mechanics.

$$E = m_0c^2 + K$$

$$K >>> m_0c^2$$

$$E >>> m_0c^2$$

$$pc >>> m_0c^2$$



100V

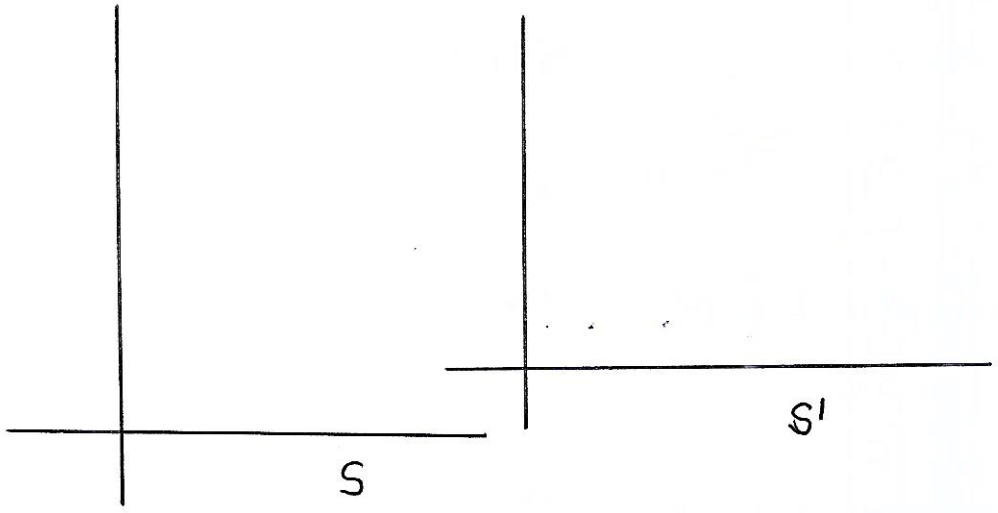
$K = 100\,eV$

e^- $m_0c^2 = 511\,keV = 511000\,eV$

→ Accelerating an e^- in Linac at 1MeV.

Proton 1 MeV Kinetic Energy

↳ Rest mass energy



- S' moves in +x direction with respect to S.
- x' axis and x are coinside.
- y' axis is parallel to y axis.
- z' axis is parallel to z axis

When axis cross

$t=0$

$t'=0$

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}, \quad y' = y, \quad z' = z$$

$$t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\begin{bmatrix} E'/c \\ p_{x'} \\ p_{y'} \\ p_{z'} \end{bmatrix} = \begin{bmatrix} ? \end{bmatrix} \begin{bmatrix} E/c \\ p_x \\ p_y \\ p_z \end{bmatrix}$$

$$-\left(\frac{E'}{c}\right)^2 + p_{x'}^2 + p_{y'}^2 + p_{z'}^2 = \text{?} \; -\left(\frac{E}{c}\right)^2 + p_x^2 + p_y^2 + p_z^2$$

→ When I am changing a frame, I am not changing the magnitude of the vector.

→ Length of the vector doesn't change after rotation.

$$E'^2 - p'^2 c'^2 = E^2 - p^2 c^2$$

$$-c^2 t'^2 + x'^2 + y'^2 + z'^2 = c^2 t^2 + x^2 + y^2 + z^2$$

Conservation of Energy momentum:

- Energy and momentum are mixed up.
- Energy momentum are constant if there are no forces on the system.

→ If a system is invarient, frames S and S' are here.

→ If anything constant, single frame is there.

- All the 4 components remain constant.

The Massless Particle -

Problem 1. A particle has K.E. which is 1/4 of its total energy. Find the speed of the particle.

Kinetic Energy $K = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0 c^2$

($\frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$: Total energy; $m_0 c^2$: Rest mass energy)

$$K = m_0 c^2 \gamma - m_0 c^2$$

$$K = m_0 c^2 (\gamma - 1)$$

$$\frac{1}{4} = \frac{m_0 c^2 (\gamma - 1)}{m_0 c^2 \gamma}$$

$$\frac{1}{4} = \frac{\gamma - 1}{\gamma} = 1 - \frac{1}{\gamma}$$

$$\frac{1}{\gamma} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{3}{4}$$

$$1 - \frac{v^2}{c^2} = \frac{9}{16}$$

$$\frac{v^2}{c^2} = \frac{7}{16}$$

$$\frac{v}{c} = \sqrt{\frac{7}{16}} = 0.66$$

$$v = 0.66c$$

$$v = 2 \times 10^8 \text{ m/s}$$

Problem 2: Find the speed of an electron having kinetic energy 100eV.

Kinetic Energy $K = 100\,eV$

Rest mass energy $= 511\,KeV$

$$K = \frac{1}{2}mv^2 = 100\,eV$$

$$v^2 = \frac{200\,eV}{511\,KeV/c^2}$$

$$\frac{v^2}{c^2} \Rightarrow \frac{200\,eV}{511 \times 10^3\,eV}$$

$$\frac{v^2}{c^2} = \frac{200}{511 \times 10^3} = \frac{2}{5110}$$

$$\frac{v}{c} = \sqrt{\frac{2}{5110}}$$

$$\frac{v}{c} = 0.01978$$

$$v = 0.01978 \times 3 \times 10^8\ m/s$$

$$v = 0.059 \times 10^8$$

$$v = 5.9 \times 10^6\ m/s$$

Problem 3: Find the speed on an electron having kinetic energy 1MeV.

$$K = 1\,MeV = 1000\,KeV$$

$$m_0c^2 = 511\,KeV$$

$$K = m_0c^2(\gamma - 1)$$

$$1000\,KeV = 511\,KeV \times (\gamma - 1)$$

$$\gamma - 1 = \frac{1000}{511}$$

$$\gamma = \frac{1511}{511}$$

$$\frac{1}{\gamma} = \sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{\frac{1511}{511}}$$

$$1 - \frac{v^2}{c^2} = \left(\frac{511}{1511}\right)^2$$

$$1 - \frac{v^2}{c^2} = 0.1144$$

$$\frac{v^2}{c^2} = 1 - 0.1144$$

$$\frac{v^2}{c^2} = 0.8856$$

$$\frac{v}{c} = \sqrt{0.8856} = 0.941$$

$$v = 0.94c$$

$$v = 0.94 \times 3 \times 10^8 \text{ m/s}$$

$$v = 2.82 \times 10^8 \text{ ms}^{-1}$$

Problem 4: Two particles of same rest mass m_0 moves towards each other with equal speed v. They make perfectly inelastic collision and make one composite particle. Find the rest mass of this particle.



Before collision



After collision

Linear momentum is = 0

Total energy before collision

$$E = \frac{2 \times m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Total Energy After collision

$$E = M_0 c^2$$

- Total energy is conserved even if it is inelastic collision.

$$2m_0 c^2 \gamma = M_0 c^2$$

$$M_0 = \frac{2m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$M_0 > 2m_0$$

Problem 5: A particle with rest mass m_1 moves with the speed $c\sin\theta$ in positive x direction and another particle of rest mass m_2 moves with a speed $c\cos\theta$ in negative x direction. If the total linear momentum of the system is zero, find the ratio m_2/m_1.



$$p_1 = m_1 c\sin\theta\,\gamma = \frac{m_1 c\sin\theta}{\sqrt{1-(\sin\theta)^2}} = \frac{m_1 c\sin\theta}{\cos\theta}$$

$$p_2 = \frac{m_2 c\cos\theta}{\sqrt{1-(\cos\theta)^2}} = \frac{m_2 c\cos\theta}{\sin\theta}$$

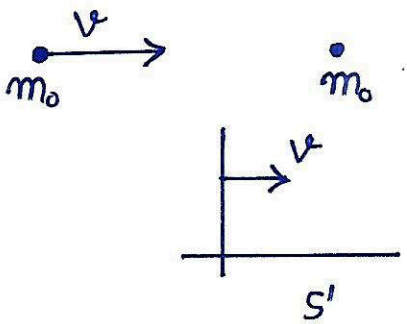
$$\frac{m_1 \cancel{c}\sin\theta}{\cos\theta} = \frac{m_2 \cancel{c}\cos\theta}{\sin\theta}$$

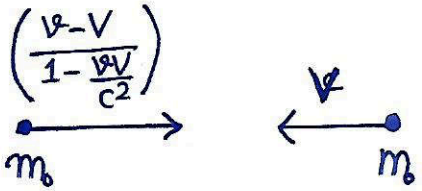
$$\frac{m_1}{m_2} = \frac{\cos\theta}{\sin\theta}\cdot\frac{\cos\theta}{\sin\theta} = \frac{\cos^2\theta}{\sin^2\theta}$$

$$\frac{m_2}{m_1} = \frac{\sin^2\theta}{\cos^2\theta}$$

$$\boxed{\frac{m_2}{m_1} = \tan^2\theta}$$

Problem 6: A particle of rest mass m_0 moves at speed v towards a stationary identical particle. Find the velocity of the frame in which the total linear momentum is zero





S'

$$p_1 = \frac{m_0 u}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$p_2 = \frac{m_0 V}{\sqrt{1-\frac{V^2}{c^2}}}$$

$$\frac{v-V}{1-\frac{vV}{c^2}} = V$$

$$V - \frac{vV^2}{c^2} = v - V$$

$$\frac{vV^2}{c^2} - 2V + v = 0$$

$$vV^2 - 2c^2V + vc^2 = 0$$

$$V = \frac{2c^2 \pm \sqrt{4c^4 - 4v^2c^2}}{2v}$$

$$V = \frac{c^2 \pm \sqrt{c^4 - v^2c^2}}{v}$$

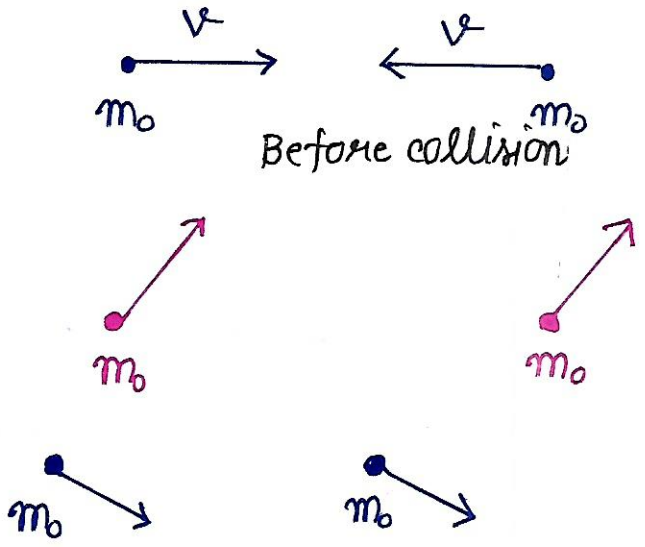
$$V = \frac{c^2 \pm c^2\sqrt{1-\frac{v^2}{c^2}}}{v}$$

$$V = \frac{c^2}{v}\left[1 \pm \sqrt{1-\frac{v^2}{c^2}}\right]$$

$$V = \frac{c^2}{v}\left[1 - \sqrt{1-\frac{v^2}{c^2}}\right]$$

Zero momentum Frame (ZMF)

Problem 7: Two protons move towards each other with speed v and collide. As a result a new proton and anti proton is created. Find the minimum value of v for this to happen.



After Collision

* Linear momentum must be conserved.
* Total Linear momentum is zero.

Before collision

$$E_K = 2m_0c^2\left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1\right]$$

$$2m_0c^2\left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1\right] = 2m_0c^2$$

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 2$$

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{1}{2}$$

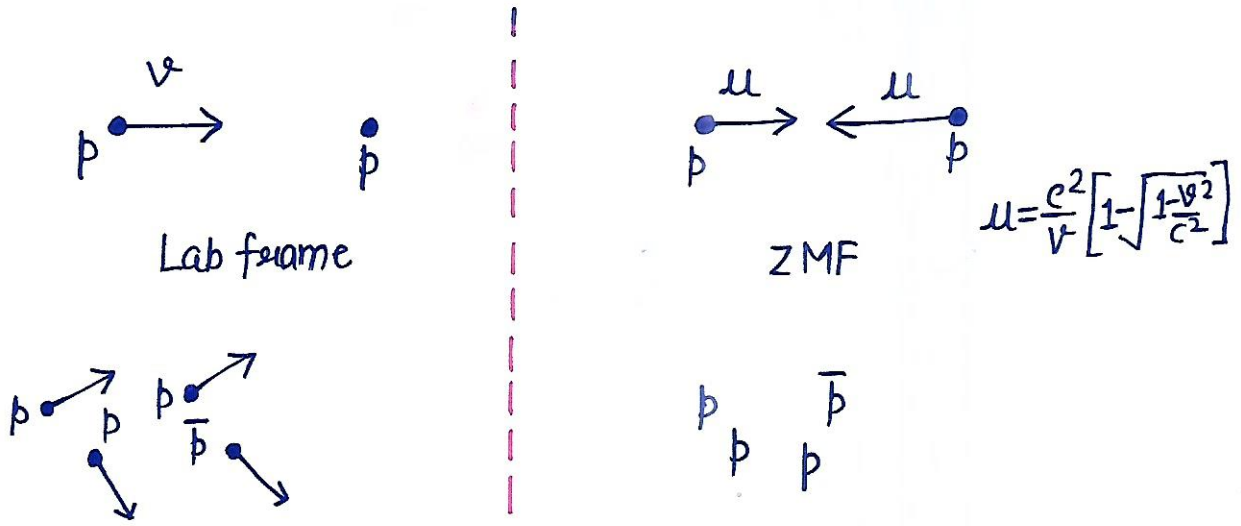
$$1-\frac{v^2}{c^2} = \frac{1}{4}$$

$$\frac{v^2}{c^2} = \frac{3}{4}$$

$$v^2 = \frac{3}{4}c^2$$

$$v = \frac{\sqrt{3}}{2}c$$

Problem 8: A proton moves at speed v towards another proton at rest. As a result a new proton and an anti-particle is created. Find the minimum value of v for this to happen.



Total Energy $= E$

Total linear momentum $= \vec{p}$

$$E^2 - p^2c^2 = (E_1 + E_2)^2 - |(\vec{p}_1 + \vec{p}_2)|^2 c^2$$

$$E^2 - p^2c^2 = E_1^2 + E_2^2 + 2E_1E_2 - p_1^2c^2$$

$$E^2 - p^2c^2 = m_0^2c^4 + m_0^2c^4 + 2m_0c^2 \cdot m_0c^2$$

$$E^2 - p^2c^2 = 2m_0^2c^4 + \frac{2m_0^2c^4}{\sqrt{1-\frac{v^2}{c^2}}} \sqrt{1-\frac{v^2}{c^2}} = 16m_0^2c^4$$

$$E^2 - p^2c^2 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 7 \Rightarrow \sqrt{1-\frac{v^2}{c^2}} = \frac{1}{7}$$

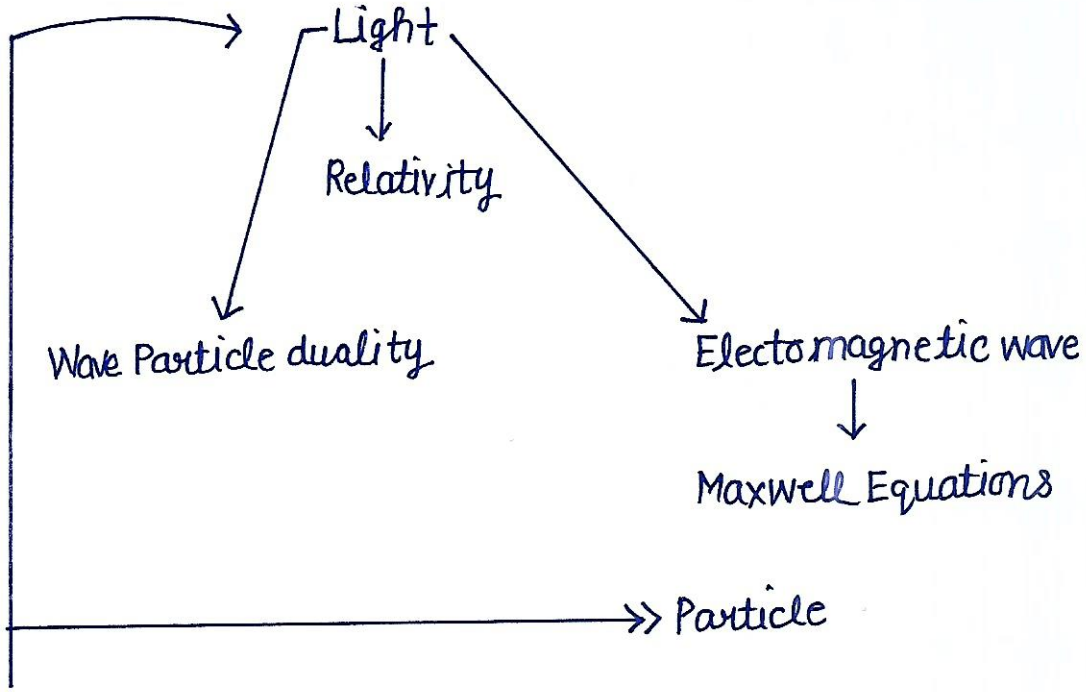
$$1 - \frac{v^2}{c^2} = \frac{1}{49}$$

$$\frac{v^2}{c^2} = \frac{48}{49}$$

$$v = \sqrt{\frac{48}{49}}\, c$$

$$\boxed{v = \frac{4\sqrt{3}}{7} c}$$

#Photon :



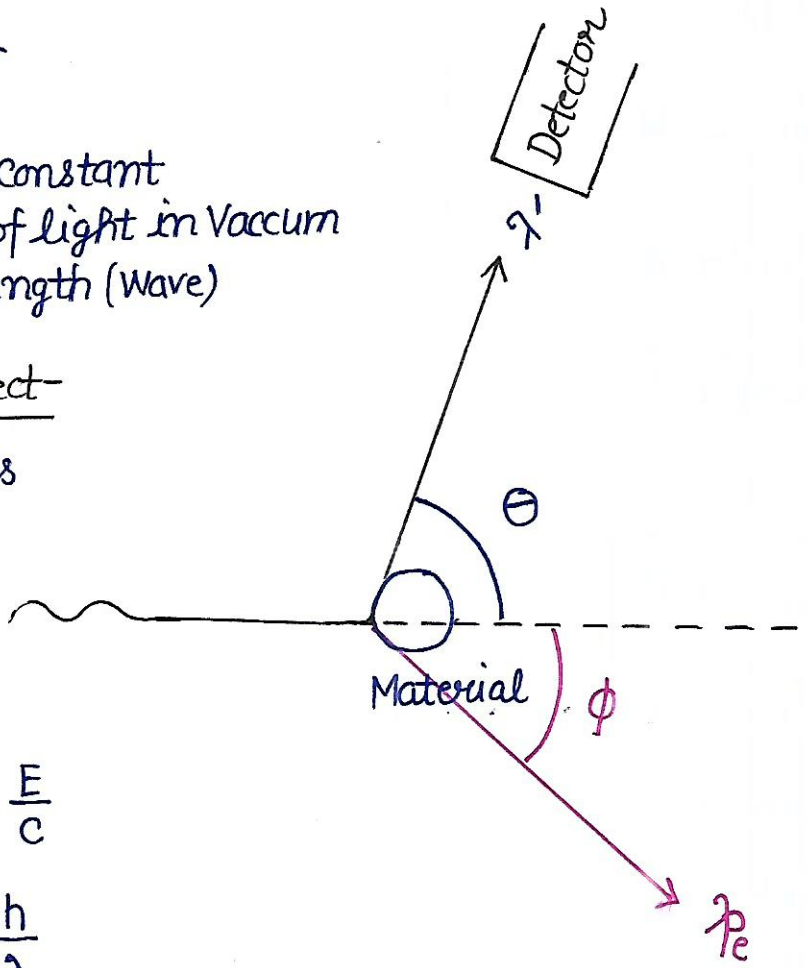
Energy of a photon

$$E = \frac{hc}{\lambda}$$

h = Plank Constant
c = Speed of light in Vaccum
λ = Wavelength (Wave)

#Compton Effect-

X rays



$$p = \frac{E}{c}$$

$$p = \frac{h}{\lambda}$$

Energy of photon $E = \frac{hc}{\lambda}$

Energy of $e^- = m_0c^2$

Momentum of Photon $= \frac{h}{\lambda}$

Momentum of $e^- = 0$

Energy of photon $= \frac{hc}{\lambda'}$

Energy of e^- $E_e = \sqrt{p_e^2c^2 + m_0^2c^4}$

Momentum of Photon $= \frac{h}{\lambda'}$

Momentum of $e^- = p_e$

$$\frac{hc}{\lambda} + m_0c^2 = \frac{hc}{\lambda'} + E_e$$

$$\frac{h}{\lambda} = \frac{h}{\lambda'}\cos\theta + p_e\cos\phi$$

$$0 = \frac{h}{\lambda'}\sin\theta - p_e\sin\phi$$

$$E_e = \sqrt{p_e^2c^2 + m_0^2c^4}$$

Eliminate ϕ, p_e, E_e

$$\boxed{\lambda' - \lambda = \frac{h}{m_0c}(1 - \cos\theta)}$$

For a particle:

$$E^2 = p^2c^2 + m_0^2c^4$$

For Photon, $E = \frac{hc}{\lambda}$, $p = \frac{h}{\lambda}$

$$E^2 = p^2c^2$$

so $m_0^2c^4 = 0$, $m_0 = 0$

Photon is said to have zero rest mass.

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{If } m_0 = 0$$

For a finite energy $v = c$

$$p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- If a particle has zero rest mass, it must move with velocity c.

Problem 9:

Pions or π-mesons

π^0 $\quad$ π^+ $\quad$ π^-

π^0

Unstable

Lifetime of $\pi^0 \simeq 10^{-17}$ sec.

Rest mass = 135 MeV/c^2

A π^0 particle moving with kinetic energy 200 MeV decays in two γ-photons. One of the photons moves along the original direction of π^0 particle and the other opposite to it. Find the energies of the photon.

π^0 $\longrightarrow$

KE = 200 MeV

Rest mass energy = 135 MeV

Total Energy = 335 MeV

γ (E_2) $\quad$ γ (E_1)

Total energy = $E_1 + E_2$

$$E_1 + E_2 = 335\,MeV \quad \ldots\ldots\ldots (1)$$

$$p^2c^2 = E^2 - m_0^2c^4$$

$$p^2c^2 = (335)^2 - (135)^2$$

$$p^2c^2 = 200 \times 470\,MeV^2$$

$$p^2c^2 = 94000\,MeV^2$$

$$p = \frac{E_1}{c} - \frac{E_2}{c}$$

$$E_1 - E_2 = pc$$

$$E_1 - E_2 = \sqrt{94000}\ MeV$$

$$E_1 - E_2 = 306.6\,MeV$$

$$E_1 + E_2 = 335\ MeV$$

$$2E_1 = 641.6\ MeV$$

$$E_1 = 320.8\ MeV$$

~~$E_2 = -320.8 + (E_1 + E_2)$~~

$$E_2 = 14.2\ MeV$$

$$E_1 + E_2 = 335\ MeV$$

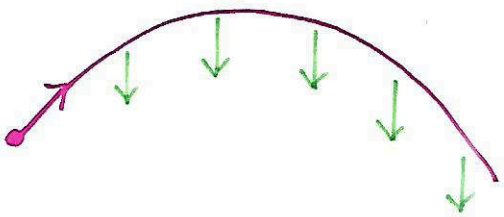
- Magnetism is a Relativistic Phenomena !

#Acceleration need not be in direction of Force-

$$\vec{F} = m\vec{a}$$

↑ Rate of change of velocity

$$\vec{F} = m\frac{d\vec{v}}{dt}$$



$$v = u + at$$

$$v = u + \frac{F}{m}t$$

- In Relativity, this concept doesn't work.

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{P} = \frac{m_0 \vec{v}}{\sqrt{1-\frac{v^2}{c^2}}} = m_0(\vec{v}\gamma)$$

$$\frac{d\vec{p}}{dt} = m_0\left[\gamma\frac{d\vec{v}}{dt} + \vec{v}\frac{d\gamma}{dt}\right]$$

$$\frac{d\gamma}{dt} = \frac{d}{dt}\left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right] = -\frac{1}{2}\frac{1}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}\left(-\frac{2v}{c^2}\frac{dv}{dt}\right)$$

$$E = m_0 c^2 \gamma$$

$$\frac{dE}{dt} = m_0 c^2 \frac{d\gamma}{dt}$$

$$\frac{d(m_0c^2+K)}{dt} = \frac{dK}{dt} = \vec{F}\cdot\frac{d\vec{r}}{dt}$$

$$\frac{d\gamma}{dt} = \frac{1}{m_0c^2}\frac{dE}{dt}$$

$$\frac{d\gamma}{dt} = \frac{1}{m_0c^2}(F\cdot\vec{v})$$

$$\vec{F} = m_0\gamma\frac{dv}{dt} + \vec{v}\frac{1}{m_0c^2}\vec{F}\cdot\vec{v}\, m_0$$

$$\vec{F} = m_0\gamma\frac{dv}{dt} + m_0 v\frac{\vec{F}\cdot\vec{v}}{m_0c^2}$$

$$\frac{d\vec{v}}{dt} = \frac{1}{m_0\gamma}\left[\vec{F} - (\vec{F}\cdot\vec{v})\frac{\vec{v}}{c^2}\right]$$

$$\frac{d\vec{v}}{dt} = \frac{\vec{F}}{m_0\gamma} - \frac{(\vec{F}\cdot\vec{v})\vec{v}}{m_0c^2\gamma}$$

Acceleration is not in the direction of force, in general.

If Force is parallel to velocity.

$$\vec{F} \parallel \vec{v}$$

$$\frac{d\vec{v}}{dt} = \frac{\vec{F}}{m_0\gamma} - \frac{(\vec{F}\cdot\vec{v})\vec{v}}{m_0c^2\gamma}$$

$$\frac{dv}{dt} = \frac{F}{m_0\gamma} - \frac{Fv^2}{m_0c^2\gamma}$$

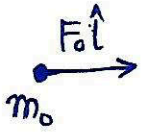
$$= \frac{F}{m_0\gamma}\left[1 - \frac{v^2}{c^2}\right]$$

$$= \frac{F}{m_0\gamma^3} = \frac{F}{m_0\gamma^3} = \frac{F}{m_0}\left(1-\frac{v^2}{c^2}\right)^{3/2}$$

$$\vec{F} \perp \vec{v}$$

$$\frac{d\vec{v}}{dt} = \frac{F}{m_0\gamma}$$

Problem 1:
A constant Force $F_0\hat{i}$ acts on a particle of mass m_0. At $t=0$, the particle is at the origin and at rest. Find the velocity of the particle as a fn of time.



$$\frac{d\vec{v}}{dt} = \frac{F}{m_0}\left(1-\frac{v^2}{c^2}\right)^{3/2}$$

$$\vec{F}_0 = \frac{d\vec{p}}{dt}$$

$$d\vec{p} = \vec{F}_0 dt$$

$$\vec{p} = F_0 t$$

$$\frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}} = F_0 t$$

$$m_0^2 v^2 = F_0^2 t^2\left(1-\frac{v^2}{c^2}\right)$$

$$m_0^2 v^2 + \frac{F_0^2 t^2 v^2}{c^2} = F_0^2 t^2$$

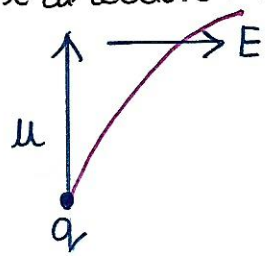
$$v^2 = \frac{F_0^2 t^2}{m_0^2 + \frac{F_0^2 t^2}{c^2}}$$

$$v = \frac{F_0 t}{\sqrt{m_0^2 + \frac{F_0^2 t^2}{c^2}}}$$

$$v = \frac{F_0 c t}{\sqrt{m_0^2 c^2 + F_0^2 t^2}}$$

$$\boxed{v < t \quad \text{for all } t.}$$

Problem 2: A particle having charge q and mass m_0 is projected with speed u in y direction in a uniform electric field $\vec{E}$ that exists in x direction. Find velocity as function of time.



$$\vec{F} = q\vec{E}$$

$$\frac{d\vec{P}}{dt} = q\vec{E}$$

$$\frac{dP_x}{dt} = q\vec{E}$$

$$\frac{dP_y}{dt} = 0$$

$$P_x = q\vec{E} \cdot t$$

$$P_y = P_0$$

$$\frac{m_0 v_x}{\sqrt{1 - \frac{v^2}{c^2}}} = qEt$$

$$\frac{m_0 v_y}{\sqrt{1-\frac{v^2}{c^2}}} = p_0$$

$$\boxed{\vec{v} = \frac{\vec{p}}{m_0 \gamma} = \frac{\vec{p}c^2}{E}}$$

$$E = m_0 c^2 \gamma$$

$$\vec{v}_x = \frac{p_x c^2}{E} = \frac{qEtc^2}{\sqrt{m_0^2 c^4 + p^2 c^2}}$$

$$\vec{v}_x = \frac{p_x c^2}{E} = \frac{qEtc^2}{\sqrt{m_0^2 c^4 + [(qEt)^2 + p_0^2]c^2}}$$

$$\vec{v}_y = \frac{p_y c^2}{E} = \frac{p_0 c^2}{\sqrt{m_0^2 c^4 + [(qEt)^2 + p_0^2]c^2}}$$

Problem 3: A particle with charge q and mass m_0 is projected in a uniform magnetic field $\vec{B}$. The initial velocity $\vec{u}$ is perpedicular to the $\vec{B}$. Describe the motion.

$$\frac{d\vec{v}}{dt} = \frac{\vec{F}}{m_0 \gamma} - \frac{(\vec{F}\cdot\vec{v})\vec{v}}{m_0 c^2 \gamma}$$

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$\vec{B} = B\hat{k}$$

$\vec{F}$ in x-y plane

$$\frac{d\vec{p}}{dt} = q(v_x\hat{i} + v_y\hat{j}) + B\hat{k}$$

$$= -q v_x B\hat{j} + q v_y B\hat{i}$$

$$\frac{dp_x}{dt} = q v_y B$$

$$\frac{dp_y}{dt} = -q v_x B$$

$$\vec{v} = \frac{pc^2}{E}$$

$$\frac{dv_x}{dt} = \frac{dp_x}{dt}\frac{c^2}{E}$$

$$\frac{dv_x}{dt} = (qv_yB)\frac{c^2}{E}$$

$$\frac{dv_y}{dt} = +(-qv_xB)\frac{c^2}{E}$$

$$\frac{qBc^2}{E} = \omega = \frac{qB}{m_0\gamma} = \frac{qB}{m_0}\sqrt{1-\frac{v^2}{c^2}}$$

$$\frac{dv_x}{dt} = \omega v_y$$

$$\frac{dv_y}{dt} = -\omega v_x$$

Non-relativistic speeds

$$\left(\omega = \frac{qB}{m_0}\right)$$

Transformation of Force if you change Frame !

$$\vec{F} = \frac{d\vec{p}}{dt}$$



$$\begin{bmatrix} dE'/c \\ dp'_x \\ dp'_y \\ dp'_z \end{bmatrix} = \begin{bmatrix} & & & \\ & & C & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} dE/c \\ dp_x \\ dp_y \\ dp_z \end{bmatrix}$$

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} & & & \\ & & C & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$dp_x' = \frac{dp_x - v\frac{dE}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$dt' = \frac{dt - \frac{v\,dx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$F_x' = \frac{dp_x'}{dt'} = \frac{dp_x - \frac{v}{c^2}dE}{dt - \frac{v}{c^2}dx}$$

$$= \frac{\frac{dp_x}{dt} - \frac{v}{c^2}\frac{dE}{dt}}{1 - \frac{v}{c^2}\frac{dx}{dt}} = \frac{F_x - \frac{v}{c^2}(\vec{F}\cdot\vec{u})}{\left(1 - \frac{v^2}{c^2}\right)u_x}$$

$$\frac{dE}{dt} = \frac{d}{dt}(m_0c^2 + K) = \frac{dK}{dt} = \vec{F}\cdot\frac{d\vec{r}}{dt} = F\cdot\vec{u}$$

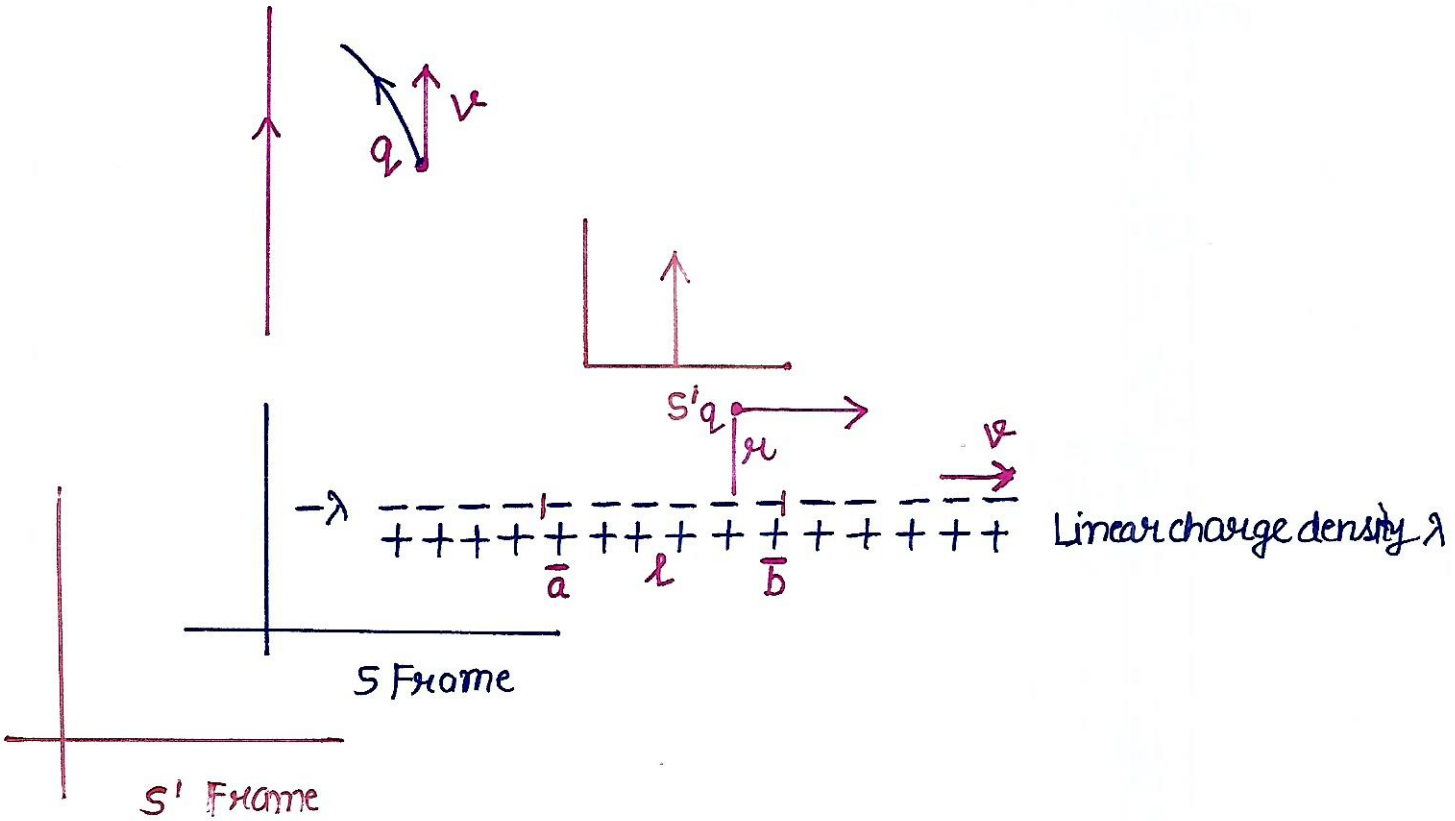
$$F_y' = \frac{dp_y'}{dt'} = \frac{dp_y\sqrt{1 - \frac{v^2}{c^2}}}{dt - \frac{v}{c^2}dx}$$

$$F_y' = \frac{\frac{dp_y}{dt}\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v u_x}{c^2}}$$

$$F_y' = \frac{F_y\sqrt{1 - v^2/c^2}}{1 - \frac{v u_x}{c^2}}$$

#Magnetic Field | A consequence of Relativity-

$$F = (q \times v) B$$



$$\lambda_+' = \frac{\lambda}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\lambda_-' = \lambda\sqrt{1-\frac{v^2}{c^2}}$$

$$\lambda' = \lambda_+' - \lambda_-' = \frac{\lambda}{\sqrt{1-\frac{v^2}{c^2}}} - \lambda\sqrt{1-\frac{v^2}{c^2}}$$

$$= \lambda\left[\frac{1-\left(1-\frac{v^2}{c^2}\right)}{\sqrt{1-\frac{v^2}{c^2}}}\right]$$

$$= \frac{\lambda\frac{v^2}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$



$$E = \frac{\lambda'}{2\pi\epsilon_0 x}$$

$$E = \frac{\lambda'}{2\pi\epsilon_0 y}$$

$$q\vec{E} = \frac{q\lambda'}{2\pi\epsilon_0 y}\hat{j}$$

$$F_x = 0$$

$$F_y = \frac{q\lambda\frac{v^2}{c^2}}{2\pi\epsilon_0 y\sqrt{1-\frac{v^2}{c^2}}}$$

$$F_z = 0$$

$$F_x' = 0$$

$$F_y' = \frac{q\lambda\frac{v^2}{c^2}}{2\pi\epsilon_0 y\sqrt{1-\frac{v^2}{c^2}}}$$

$$F_z' = 0$$

$$F_x' = \frac{F_x - \frac{v}{c^2}(\vec{F}.u)}{1-\frac{vu_x}{c^2}}$$

$$F_x = \frac{F_x' + \frac{v}{c^2}(F'.\vec{u}')}{1+\frac{vu_x}{c^2}}$$

$$F_y' = \frac{F_y\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{vu_x}{c^2}}$$

$$F_y = \frac{F_y'\sqrt{1-\frac{v^2}{c^2}}}{1+\frac{vu_x}{c^2}}$$

Again

$$F_x = 0$$

$$F_y' = \frac{q\lambda\frac{v^2}{c^2}}{2\pi\epsilon_0 y}$$

$$F_x = F_x'$$

$$F_y = F_y'\sqrt{1-\frac{v^2}{c^2}}$$

$\lambda v = I$ Electric current

$$F_y = \frac{qIv}{c^2 \cdot 2\pi\epsilon_0 y}$$

$$F_y = \frac{qIv}{2\pi\epsilon_0 c^2 y}$$

$$F_y = \frac{\mu_0}{2\pi} \cdot \frac{qIv}{y}$$

$$F_y = \frac{q\mu_0 I}{2\pi y} v$$

$$F_y = qvB$$

And

$\vec{B} = -B\hat{k}$

$\vec{v} = v\hat{i}$

$$\vec{F} = q\vec{v} \times \vec{B}$$

Magnetism is a relativistic Phenomena.

#Quiz 3

Q:1- Mass of the electron can be written as

$511\ KeV/c^2$, $9.1 \times 10^{-31} kg$, and $0.511\ MeV/c^2$

(a) (b) (d)

Q:2- The Expression for the Kinetic energy of a particle is

(b) $K = \frac{1}{2} m_0 v^2$

(d) K = Total energy − Rest mass energy

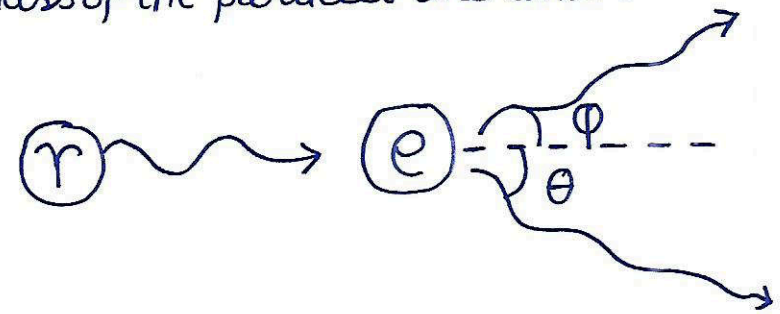
Because $E_T = E_K + E_P$

$E_K = E_T - E_P$

(c) $K = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0 c^2$

Q:3- In the two particle collision

Total Elastic collision, in which the total momentum, total Kinetic energy and the mass of the particles are conserved.



$p = m_0 v$

$K = \frac{1}{2} m_0 v^2$

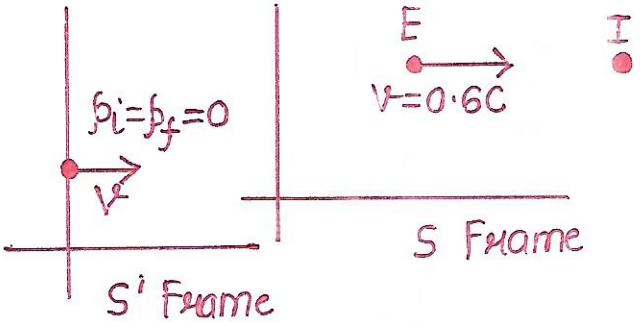
Mass conserved.

(a) The total rest mass always remains the same because it is elastic collision.

(d) The rest mass energy plus the Kinetic energy of the system remains the same.

(c) The Kinetic energy may be converted into rest mass energy.

Q:4- As seen from the Frame S, a particle of rest mass energy E moves at velocity 0.6C towards an identical particle at rest. The total linear momentum of the two particles is zero in another S' Frame. The velocity of S' with respect to S;



$$V=\frac{c^2}{v}\left[1-\sqrt{1-\frac{v^2}{c^2}}\right]$$

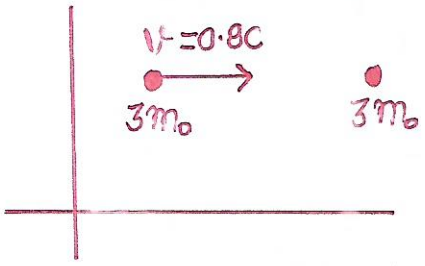
$$V=\frac{c^2}{0.6c}\left[1-\sqrt{1-\frac{(0.6)^2c^2}{c^2}}\right]$$

$$V=\frac{c^2}{0.6}\left[1-\sqrt{1-0.36}\right]=\frac{c}{0.6}\left[1-\sqrt{0.64}\right]=\frac{c}{0.6}\left[1-0.8\right]$$

$$V=\frac{c}{0.6}(0.2)$$

$$\boxed{V=\frac{c}{3}}$$

Q:5- A particle of rest mass $3m_o$ moves along the +ve x axis with a speed 0.8C. Another particle of same rest mass is at rest at the origin. The momentum 4 vectors of the two particles system is -



4 momentum vectores

$$\begin{bmatrix} m_0 c\gamma_u \\ p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} m_0 c\gamma_u \\ p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} 5m_0 c \\ 4m_0 c \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 3m_0 c \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$3m_0\left(n_x \Rightarrow \frac{\vec{u}}{\sqrt{1-\frac{v^2}{c^2}}}\right) \Rightarrow \frac{3m_0 \times (0.8c)}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{3m_0(0.8c)}{\sqrt{1-\frac{0.64c^2}{c^2}}} = \frac{3m_0(0.8)c}{\sqrt{0.6}\times\sqrt{0.6}}$$

$$= \frac{3m_0(0.8)c}{0.6} = \frac{3m_0 c}{3}$$

$$= 4m_0 c$$

$$u=0 \Rightarrow \quad 3m_0\left(n_x \Rightarrow \frac{\vec{u}}{\sqrt{1-\frac{v^2}{c^2}}}\right) \Rightarrow \frac{0\times 3m_0}{\sqrt{1-0}} = 0$$

$$\Rightarrow \frac{m_0 c}{\gamma_u} = \frac{3m_0 c}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{3m_0 c \times 10}{6} = 5m_0 c$$

$$\begin{bmatrix} m_0 c\gamma_u \\ p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} 8m_0 c \\ 4m_0 c \\ 0 \\ 0 \end{bmatrix}$$

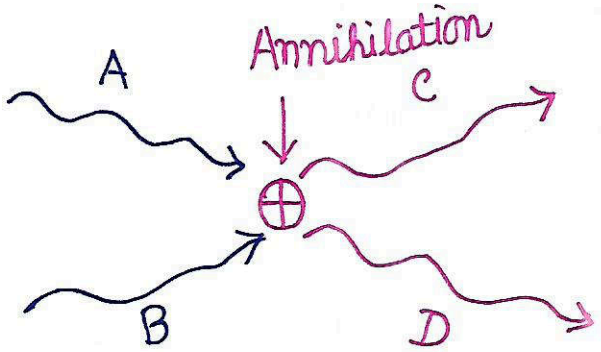
Q:6- Two particles A and B, each at rest mass m_0, move toward each other. Each of the particle has speed v. They collide and two new particles C and D are created, each having rest mass m_0. The reaction is :-

$$A + B \longrightarrow A + B + C + D$$

The total energy of the system before the collision is same as the total energy after the collision.

$$p^2c^2 + m_0^2c^4 = p^2c^2 + m_0^2c^4$$

$$m_0^2v^2c^2 + m_0^2c^4 = m_0^2v^2c^2 + m_0^2c^4$$



Each have rest mass = m_0
velocity = v

$$E_1 = E_2 \Rightarrow E_1 + E_2 = E_1 + E_2$$

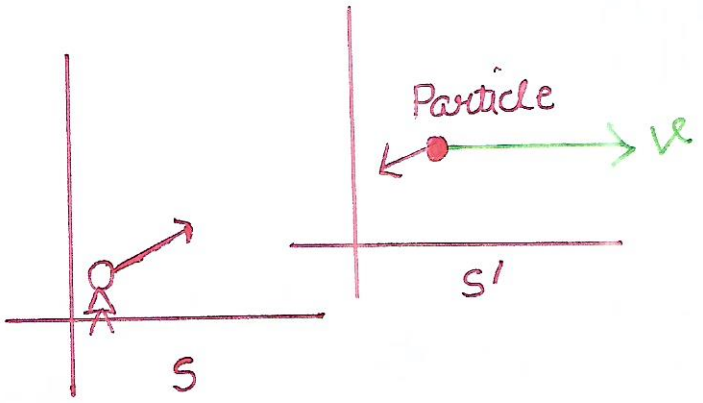
Linear momentum

$$\left.\begin{aligned} p_A &= m_A v_A = m_0 v \\ p_B &= m_B v_B = m_0 v \end{aligned}\right\} 2m_0v \leftarrow \text{Before collision}$$

$$\left.\begin{aligned} p_C &= m_C v_C = m_0 v \\ p_D &= m_D v_D = m_0 v \end{aligned}\right\} 2m_0v \leftarrow \text{After collision}$$

$$\underbrace{\frac{1}{2}m_0v^2 + \frac{1}{2}m_0v^2}_{\text{Total Kinetic Energy Before the collision}} = \underbrace{\frac{1}{2}m_0v^2 + \frac{1}{2}m_0v^2}_{\text{Total Kinetic Energy After the collision}}$$

Q:7- A particle is observed from S and S'. Which of the following quantities of the particles surely does not change as one goes from S to S'.



- The Linear momentum in y direction of the particle does not change.
- The total energy of the particle does not change.

Q:8- The mass (rest mass) of a proton is m_p, of a neutron is m_n, of an electron is m_e, of nitrogen atom (having 7 proton, 7 neutrons and 7 electrons) is m_N and of a nitrogen molecule N_2 is m_{N_2}.

Rest mass energy of nitrogen molecule $N_2 = 7(m_p - m_n)c^2 - 7m_e c^2$

$$m_{N_2} \leq 2M_N$$

$$m_N = 7m_p + 7m_n + 7m_e$$

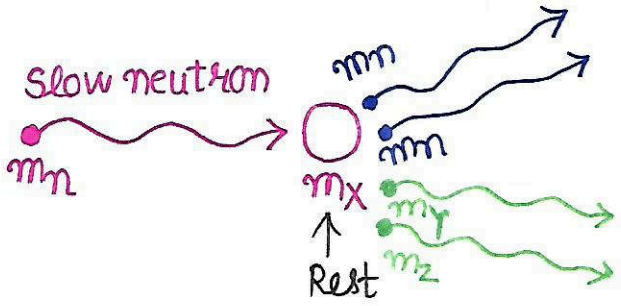
Q:14- Which of the following can be a possible momentum 4 vector for a photon for same appropriate value of α, β, γ, E

$$\begin{bmatrix} m_0 c\gamma_u \\ p_x \\ p_y \\ p_z \end{bmatrix} = ?$$

Question 9: In a Fusion reaction, a slow neutron hits a nucleus X at rest and this results in formation of two nuclei Y and Z together with two neutrons. Thus

$$n + X \longrightarrow Y + Z + 2n$$

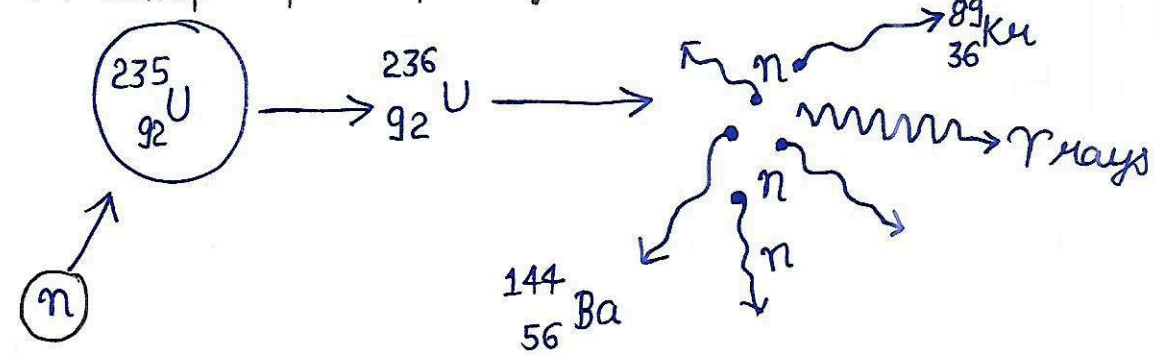
The total E_K of the product is 200 MeV. Rest masses of the neutron, X, Y and Z are m_n, m_X, m_Y and m_Z respectively.



$$m_n + m_X \longrightarrow m_Y + m_Z + 3n + 200\,MeV$$

- **Uranium 235 Fission-**

An example of one of many reactions in the Uranium 235 fission process.



#Impact by slow neutron with energy on order of an eV.

$$^{1}_{0}n + ^{235}_{92}U \longrightarrow ^{236}_{92}U \longrightarrow ^{144}_{56}Ba + ^{89}_{36}Kr + {}_{0}n^{1} + {}_{0}n^{1} + {}_{0}n^{1} + \gamma\ rays$$

177 MeV

e.g. $_{0}n^{1} + ^{21}_{2}X \longrightarrow ^{22}_{2}X \longrightarrow ^{10}_{1}X + ^{10}_{1}X + {}_{0}n^{1} + {}_{0}n^{1} + 200\,MeV$

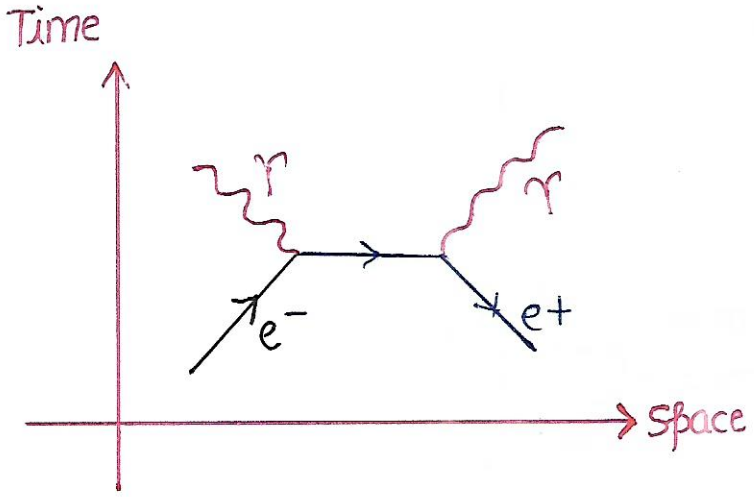
Wrong Example

$m_X > m_Y + m_Z + m_n$

m_X

Question 10-

When a positron and an electron with negligible E_K annihilate each other,



Both the annihilating e^- and e^+ particle has a rest mass energy of about 0·511 MeV. If their K·E. are relatively negligible, this total rest mass energy appears as the photon energy of the photon produced.

Question 11- For the 11 to 13, use the following information. Two protons collide with each other. As seen from frame S.

Total energy of system before collision $= E_1$

After collision $= E_2$

Total momentum of system before collision $= p_1$

After collision $= p_2$

As seen from the S' moving with respect to S, the above quantities are E_1', E_2', p_1', p_2'.

$E_1 = E_2$

$p_1 = p_2$

$E_1^2 - p_1^2c^2 = E_2^2 - p_2^2c^2$

#Transformation of EB Field-

→ Electric Field and Magnetic Field depend on Frame.

→ Pure Electric Field may become EB Field in some other frame and Pure magnetic Field may become EB field in some other frame.

→ Clue of above situation, comes from the definition of magnetic field.

$$F = (q \times v)\vec{B}$$

→ Fields are frame dependant.

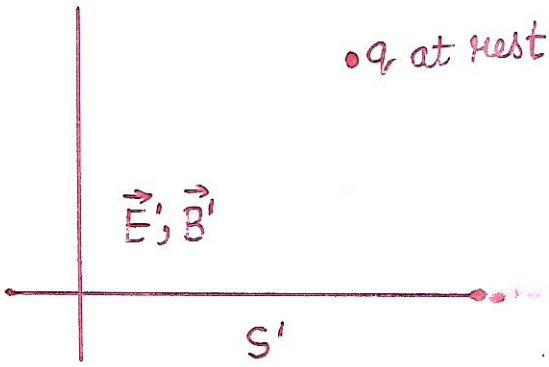
→ Space and time has their own definition as velocity and momentum.

↓

Energy

→ If you change the frame space, time and Energy, momentum get mixed up.

→ Transformation of EB Field.



$$\vec{F}' = q\vec{E}'$$
$$\vec{F}_x' = q\vec{E}_x'$$
$$\vec{F}_y' = q\vec{E}_y'$$
$$\vec{F}_z' = q\vec{E}_z'$$

$$F_x' = \frac{F_x - (\vec{F}\cdot\vec{u})v}{1 - \frac{u_x v}{c^2}}$$

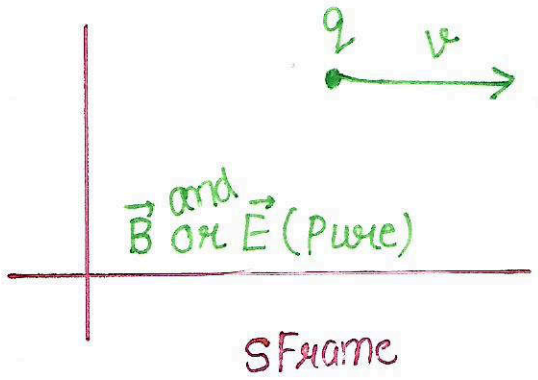
$$F_y' = \frac{F_y\sqrt{1 - v^2/c^2}}{1 - \frac{u_x v}{c^2}}$$

$$F_x = \frac{F_x' + (\vec{F}\cdot\vec{u}')v}{1 - \frac{u_x' v}{c^2}}$$

$$F_y = \frac{F_y'\sqrt{1 - v^2/c^2}}{1 - \frac{u_x' v}{c^2}}$$

Soln Trick

$$\begin{cases} F_x = F_x' = qE_x' \\ F_y = F_y'\sqrt{1-v^2/c^2} = qE_y'\sqrt{1-v^2/c^2} \\ F_z = qE_z'\sqrt{1-v^2/c^2} \end{cases} \quad \ldots\ldots\ldots\ldots\ldots (1)$$



$$\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$$

$$\vec{F} = q(E_x\hat{i}+E_y\hat{j}+E_z\hat{k}) + q(v_x\hat{i}+v_y\hat{j}+v_z\hat{k})\times(B_x\hat{i}+B_y\hat{j}+B_z\hat{k})$$

$v_y\hat{j}$ and $v_z\hat{k}$ $\rightarrow$ 0

$$= q(E_x\hat{i}+E_y\hat{j}+E_z\hat{k}) + q[(v)B_y\hat{k} - (vB_z\hat{j})]$$

$$\left.\begin{aligned} \vec{F}_x &= qE_x \\ \vec{F}_y &= qE_y - qvB_z \\ \vec{F}_z &= qE_z + qvB_y \end{aligned}\right\} \text{ component in S Frame} \quad \ldots\ldots\ldots\ldots (2)$$

Equate eqn(1) and eqn(2)

$$E_x' = E_x$$

$$qE_y'\sqrt{1-v^2/c^2} = qE_y - qvB_z$$

$$E_y' = \frac{E_y - vB_z}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\cancel{q}E_z'\sqrt{1-v^2/c^2} = \cancel{q}E_z + \cancel{q}\overset{v}{v}B_y$$

$$E_z'\sqrt{1-v^2/c^2} = E_z + vB_y$$

$$E_z' = \frac{E_z + vB_y}{\sqrt{1-\frac{v^2}{c^2}}}$$

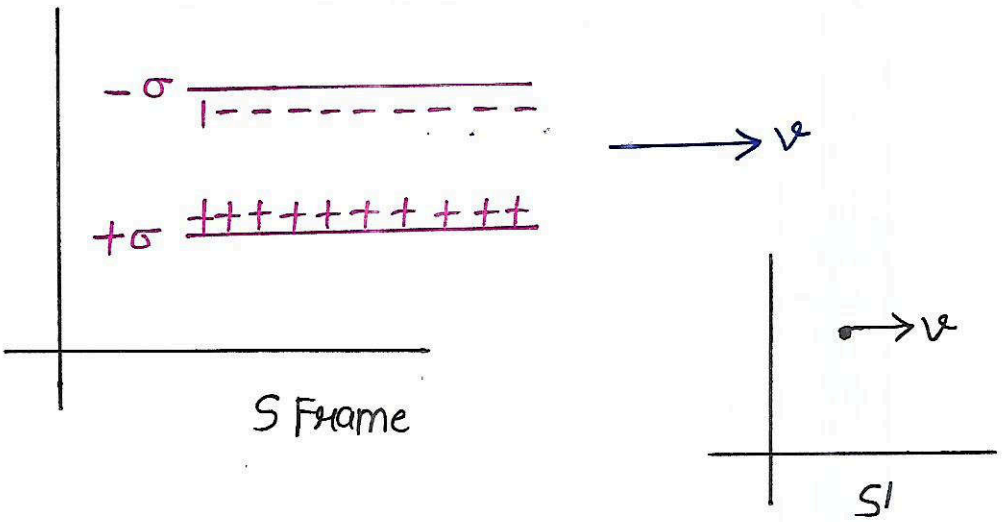
For Magnetic Field

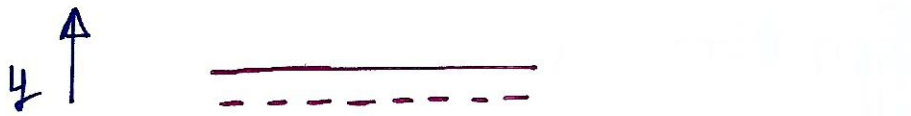
$$B_x' = B_x$$

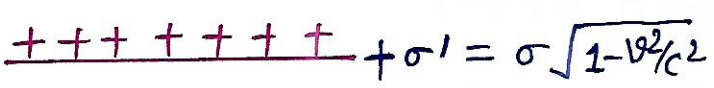
$$B_y' = \frac{B_y + \frac{v}{c^2}E_z}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$B_z' = \frac{B_z - \frac{v}{c^2}E_y}{\sqrt{1-\frac{v^2}{c^2}}}$$

- Problem 1: A parallel plate capacitor has surface charge densities $+\sigma$ and $-\sigma$ on the two plates. The capacitor moves parallel to the plate with a speed v. Find Electric field and magnetic field in the capacitor.







S' Frame

$$\vec{E}' = \frac{\sigma}{\epsilon_0}\hat{j} = \frac{\sigma\sqrt{1-v^2/c^2}}{\epsilon_0}\hat{j}$$

$$\vec{E}_x' = 0$$

$$\vec{E}_y' = \frac{\sigma}{\epsilon_0}\sqrt{1-\frac{v^2}{c^2}}$$

$$\vec{E}_z = 0$$

$$B_x' = B_y' = B_z' = 0$$

$$E_x = E_x'$$

$$E_y = \frac{E_y' + vB_z'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$E_z = \frac{E_z' - vB_y'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$B_x = B_x'$$

$$B_y = \frac{B_y' - \frac{v}{c^2}E_z'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$B_z = \frac{B_z' + \frac{v}{c^2}E_y'}{\sqrt{1-\frac{v^2}{c^2}}}$$

} Inverse Transformation

$$E_x = 0$$

$$E_y = \frac{\frac{\sigma}{\epsilon_0}\sqrt{1-\frac{v^2}{c^2}}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\sigma}{\epsilon_0}$$

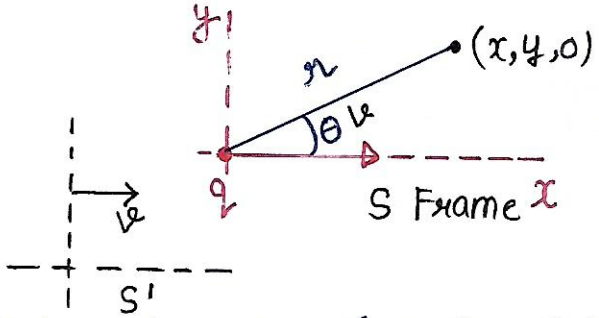
$$E_z = 0$$

$$B_x = 0$$

$$B_y = 0$$

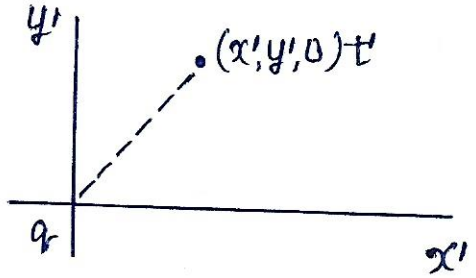
$$B_z = \frac{\frac{v}{c^2}\frac{\sigma}{\epsilon_0}\sqrt{1-\frac{v^2}{c^2}}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\not{c}^2\epsilon_0 c^2}\cdot \sigma v = \mu_0 \sigma v = \mu_0 v \sigma$$

- **E, B Fields due to moving charge:**



$x = r\cos\theta$
$y = r\sin\theta$

At $t=0$, charge is ~~not~~ at the origin



$$\vec{E} = \frac{q\vec{r}'}{4\pi\epsilon_0 r'^3} = \frac{q(x'\hat{i}+y'\hat{j})}{4\pi\epsilon_0[x'^2+y'^2]^{3/2}}$$

$$E_x' = \frac{qx'}{4\pi\epsilon_0[x'^2+y'^2]^{3/2}} \qquad B_x' = 0$$

$$E_y' = \frac{qy'}{4\pi\epsilon_0[x'^2+y'^2]^{3/2}} \qquad B_y' = 0$$

$$B_z' = 0$$

$$E_x' = \frac{q(x-vt)}{4\pi\epsilon_0\sqrt{1-\frac{v^2}{c^2}}\left(\left[\frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}}\right]^2 + y'^2\right)^{3/2}}$$

$$E_x' = \frac{qx}{4\pi\epsilon_0\sqrt{1-\frac{v^2}{c^2}}\left[\frac{x^2}{\sqrt{1-\frac{v^2}{c^2}}} + y^2\right]^{3/2}}$$

$$E_x' = \frac{qr\cos\theta}{4\pi\epsilon_0\sqrt{1-\frac{v^2}{c^2}}\left[\frac{r^2\cos^2\theta + r^2\cos^2\theta\sin^2\theta\left(1-\frac{v^2}{c^2}\right)}{1-\frac{v^2}{c^2}}\right]^{3/2}}$$

$$E_x' = \frac{qr\cos\theta\left(1-\frac{v^2}{c^2}\right)^{3/2}}{4\pi\epsilon_0\sqrt{1-\frac{v^2}{c^2}}\left[1-\frac{v^2}{c^2}\sin^2\theta\right]^{3/2} r^3}$$

$$E_y' = \frac{qy\left(1-\frac{v^2}{c^2}\right)^{3/2}}{4\pi\epsilon_0 r^3\left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{3/2}} \qquad E_z' = 0$$

$$E_y' = \frac{qr\sin\theta\left(1-\frac{v^2}{c^2}\right)^{3/2}}{4\pi\epsilon_0 r^3\left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{3/2}}$$

$$E_x = E_x' = \frac{q r \cos\theta \left(1-\frac{v^2}{c^2}\right)}{4\pi\epsilon_0 r^3 \left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{3/2}}$$

$$E_y = \frac{E_y' + vB_z'}{\sqrt{1-\frac{v^2}{c^2}}}$$

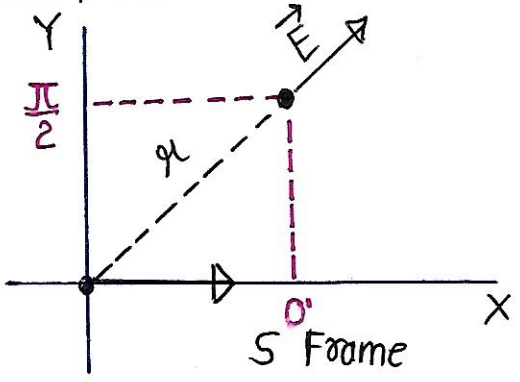
$$E_y = \frac{q r \sin\theta \left(1-\frac{v^2}{c^2}\right)^{3/2} + (v \times 0)}{4\pi\epsilon_0 r^3 \left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{3/2} \sqrt{1-\frac{v^2}{c^2}}}$$

$$E_y = \frac{q r \sin\theta \left(1-\frac{v^2}{c^2}\right)}{4\pi\epsilon_0 r^3 \left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{3/2}}$$

$$E_z = 0$$

$$\vec{E} = \frac{q\left(1-\frac{v^2}{c^2}\right)\vec{r}}{4\pi\epsilon_0 r^3 \left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{3/2}}$$

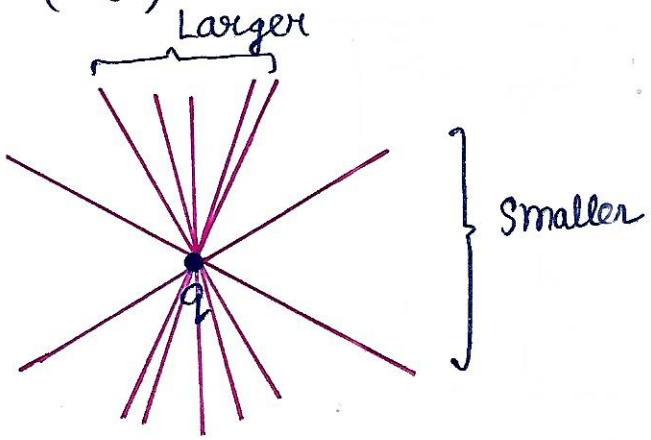
- Electric field is in the direction of r.



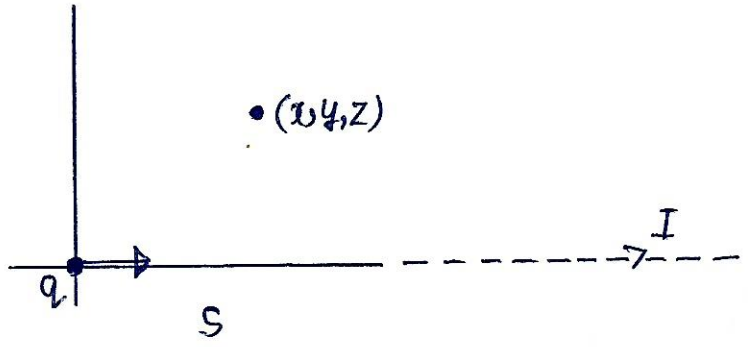
$$\vec{E} = \frac{q\left(1-\frac{v^2}{c^2}\right)}{4\pi\epsilon_0 r^2\left[1-\frac{v^2}{c^2}\sin^2\theta\right]^{3/2}}$$

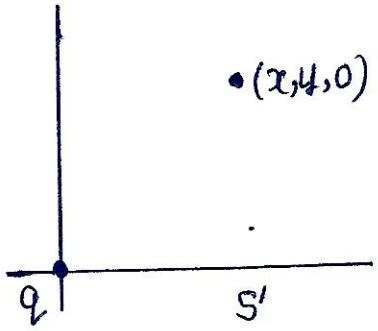
$\theta = 0^\circ, E = \dfrac{q\left(1-\frac{v^2}{c^2}\right)}{4\pi\epsilon_0 r^2}$ (Smaller) Backward and forward directions

$\theta = \dfrac{\pi}{2}, E = \dfrac{q}{4\pi\epsilon_0 r^2\left(1-\frac{v^2}{c^2}\right)^{1/2}}$ (Larger) Field in transverse side



Electromagnetic Field Tensors!





$B'_x = B'_y = B'_z = 0$

$$B_x = B_x'$$

$$B_y = \frac{\left(B_y' - \frac{v}{c^2} E_z'\right)}{\sqrt{1-\frac{v^2}{c^2}}}$$

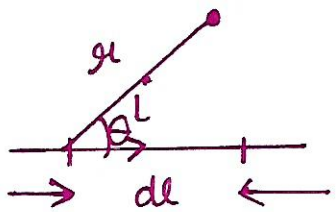
$$B_z = \frac{B_z' + \frac{v}{c^2} E_y'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$B_x = 0$$

$$B_y = 0$$

$$B_z = \frac{\frac{v}{c^2} E_y'}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\frac{v}{c^2} q r \sin\theta \left(1-\frac{v^2}{c^2}\right)^{3/2}}{4\pi\epsilon_0 r^3 \left[1-\frac{v^2}{c^2}\sin^2\theta\right]^{3/2} \sqrt{1-\frac{v^2}{c^2}}}$$

$$B_z = \frac{q v r \sin\theta \left(1-\frac{v^2}{c^2}\right)}{4\pi\epsilon_0 c^2 r^3 \left[1-\frac{v^2}{c^2}\sin^2\theta\right]^{3/2}}$$



Bio-Sevart's Law

$$d\vec{B} = \frac{\mu_0 i \, d\vec{l} \times \vec{r}}{4\pi r^3}$$

$$\mu_0 = \frac{1}{\epsilon_0 c^2}$$

$$d\vec{B} = \frac{\mu_0 i(dl) r \sin\theta}{4\pi r^3} \quad \ldots\ldots\ldots\ldots (1)$$

$$B_z = \frac{qvr\sin\theta \cdot \mu_0}{4\pi r^3}$$

$$qv \approx i\,dl$$

a single charge moving with velocity v, can be treated as current element.

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix} \quad \leftarrow L$$

2nd Rank Tensor → 16 Components

$$\begin{bmatrix} a^{11'} & a^{12'} & a^{13'} & a^{14'} \\ a^{21'} & a^{22'} & a^{23'} & a^{24'} \\ a^{31'} & a^{32'} & a^{33'} & a^{34'} \\ a^{41'} & a^{42'} & a^{43'} & a^{44'} \end{bmatrix} = [L^T] \begin{bmatrix} a^{11} & a^{12} & a^{13} & a^{14} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & a^{44} \end{bmatrix} [L]$$

S' Frame — S Frame

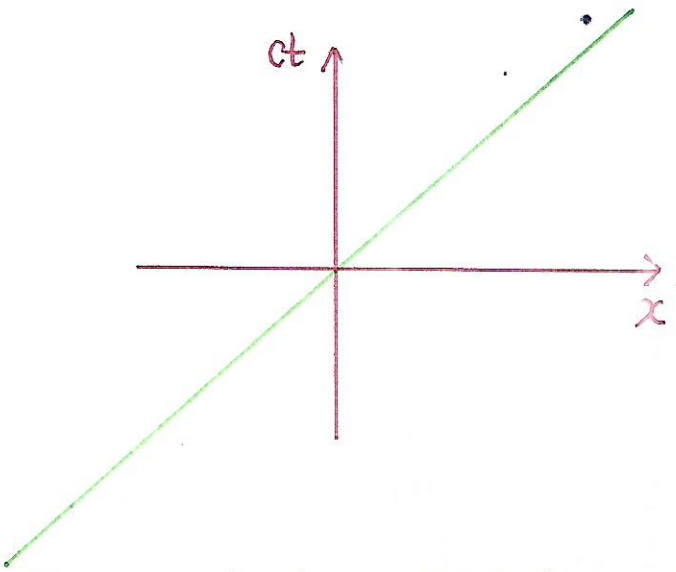
Electromagnetic Field Tensors

$$\begin{bmatrix} 0 & \frac{E_x'}{c} & \frac{E_y'}{c} & \frac{E_z'}{c} \\ -\frac{E_x'}{c} & 0 & B_z' & -B_y' \\ -\frac{E_y'}{c} & -B_z' & 0 & B_x' \\ -\frac{E_z'}{c} & B_y' & -B_x' & 0 \end{bmatrix} = [L^T] \begin{bmatrix} 0 & \frac{E_x}{c} & \frac{E_y}{c} & \frac{E_z}{c} \\ -\frac{E_x}{c} & 0 & B_z & -B_y \\ -\frac{E_y}{c} & -B_z & 0 & B_x \\ -\frac{E_z}{c} & B_y & -B_x & 0 \end{bmatrix} [L]$$

S' — S

#Minkowski Diagram-

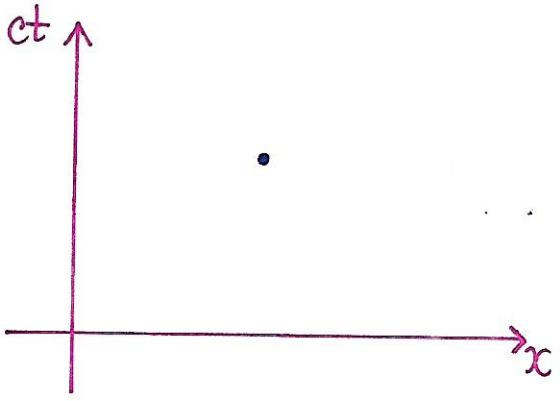
The minkowski diagram, also known as spacetime diagram, was developed in 1908 by Hermann Minkowski and provides an illustration of the property of space and time in (this course) Spacial theory of relativity. It allows a qualitiative understanding of the corresponding phenomena like time dilation and length contraction without mathematical equations.



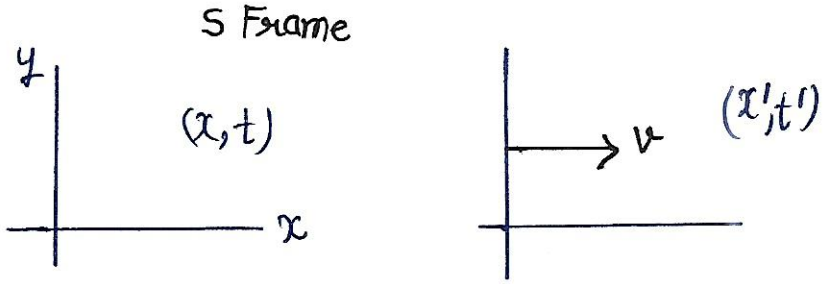
The green line (World path) of a photon, which is at location $x=0$ at time $ct=0$.

- **Geometry in Lorentz transformation:**

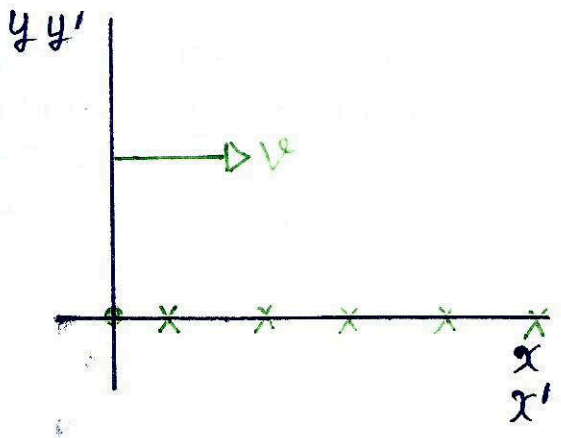
→ Published in 1908



Event x, t



Event occurs at $t'=0$

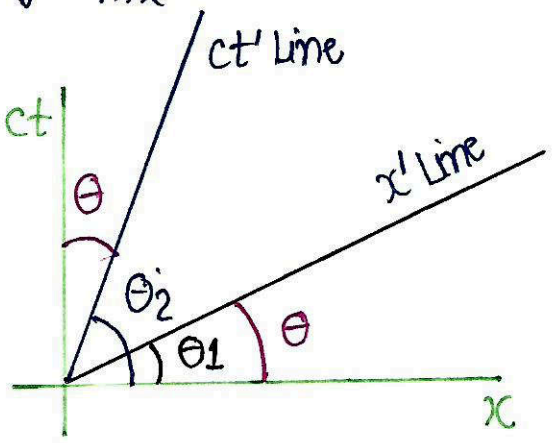


$$x = \frac{x'+vt'}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{x'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$ct = \frac{c\left(t'+\frac{vx'}{c^2}\right)}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\cancel{c}\frac{vx'}{\cancel{c^2}}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\beta x'}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \left(\because \beta = \frac{v}{c}\right)$$

$ct = \beta x$

$y = mx$



$\tan\theta_1 = \beta = v/c$

Events occurs at $x'=0$

$$x = \frac{x'+vt'}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{vt'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$ct = \frac{c\left(t'+\frac{vx'}{c^2}\right)}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{ct'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\frac{ct}{x} = \frac{c}{v}$$

$$ct = \frac{c}{v}x$$

$$ct = \frac{1}{\beta}x$$

$$\tan\theta_2 = \frac{1}{\beta}$$

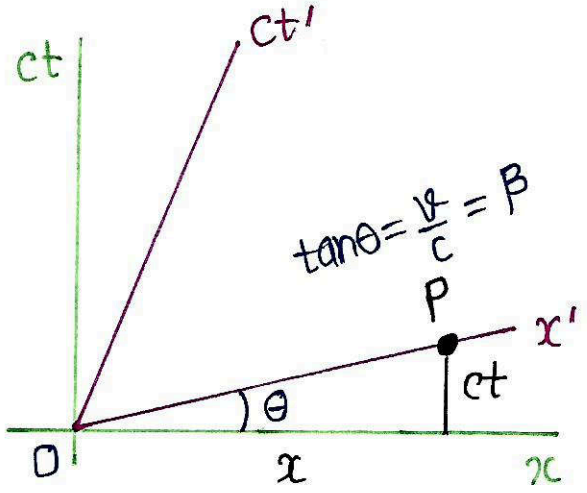
$$\cot\theta_2 = \beta = \tan\theta_1$$

$$\tan\left(\frac{\pi}{2}-\theta_2\right) = \tan\theta_1$$

$$\frac{\pi}{2}-\theta_2 = \theta_1$$

$$-\theta_2 = \frac{-\pi}{2}+\theta_1$$

$$\theta_2 = \frac{\pi}{2}-\theta_1$$



Point P ⟶ Event ⟶ $t' = 0$

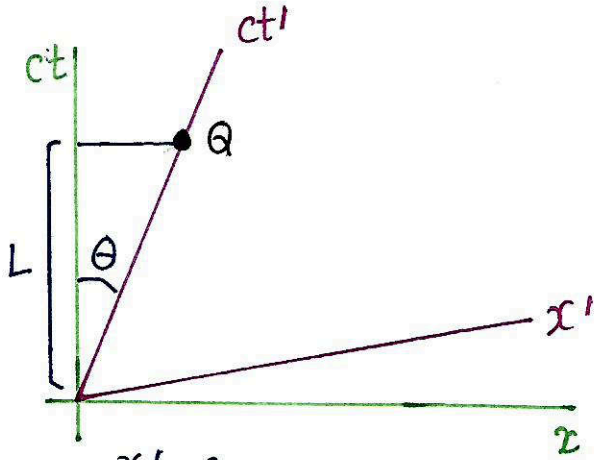
$x' = ?$

$$x = \frac{x' - vt'}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{x'}{\sqrt{1-\beta^2}}$$

$$x = OP\cos\theta = OP \times \frac{1}{\sqrt{1+\beta^2}}$$

$$\frac{x'}{\sqrt{1-\beta^2}} = OP\frac{1}{\sqrt{1+\beta^2}}$$

$$\boxed{x' = OP\sqrt{\frac{1-\beta^2}{1-\beta^2}}}$$



$x' = 0$

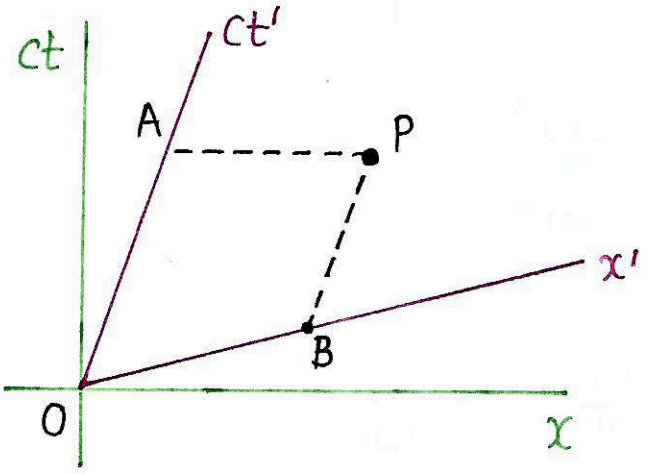
$ct' = ?$

$$ct = \frac{\left(t' + \frac{vx'}{c^2}\right)c}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$ct = (OQ)\cos\theta = OQ\,\frac{1}{\sqrt{1+\beta^2}}$$

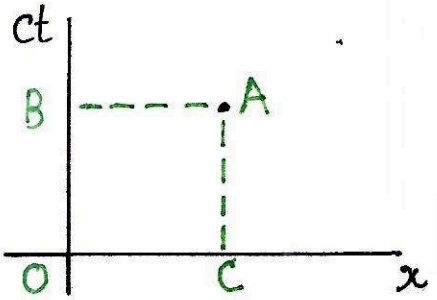
$$\frac{ct'}{\sqrt{1-\beta^2}} = OQ\,\frac{1}{\sqrt{1+\beta^2}}$$

$$\boxed{ct' = OQ\sqrt{\frac{1-\beta^2}{1+\beta^2}}}$$



$$ct' = OA\sqrt{\frac{1-\beta^2}{1+\beta^2}}$$

$$x' = OB\sqrt{\frac{1-\beta^2}{1+\beta^2}}$$



Hots!

- If the force is always perpendicular to the instantaneous direction of velocity.

Then

$$\vec{F} \cdot \vec{u} = 0$$

Therefore

$$\vec{F} = m\vec{a} = m_0 \vec{a} \gamma_u$$

$m_0 \gamma_u$ is transverse mass.

- A charged particle experience a force when moving through a magnetic field. It follows the circular motion of the charged particle as well as other motion that results from a charged particle entering a magnetic field.

$$F_c = \frac{mv^2}{r}$$

Reduced magnetic field

$$qvB = \frac{mv^2}{r}$$

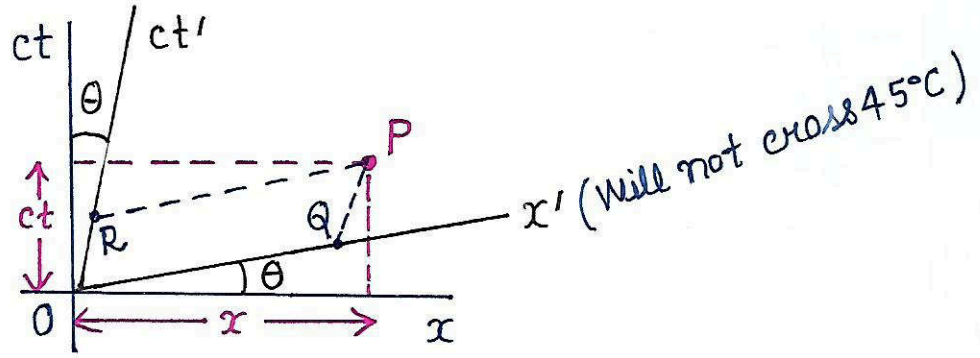
$$r = \frac{mv}{qB}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v} \frac{mv}{qB} = \frac{2\pi m}{qB}$$

$$f = \frac{1}{T}$$

- Electromagnetic Field is a 4 Vector and a tensor of Rank 2 represent by a 4X4 matrix.

#Geometry of time dilation-



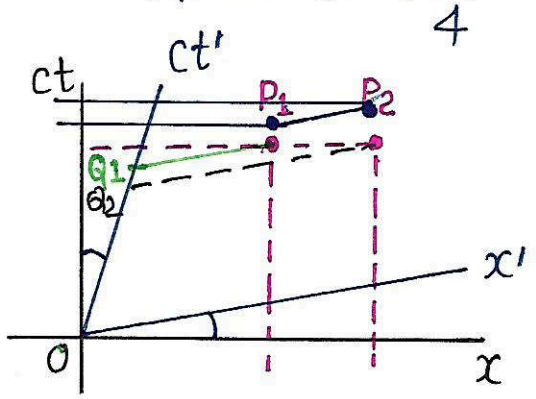
In S' Frame

$$\tan\theta = \frac{v}{c}$$

$$x' = OQ\sqrt{\frac{1-\beta^2}{1+\beta^2}}$$

$$ct' = OR\sqrt{\frac{1-\beta^2}{1+\beta^2}}$$

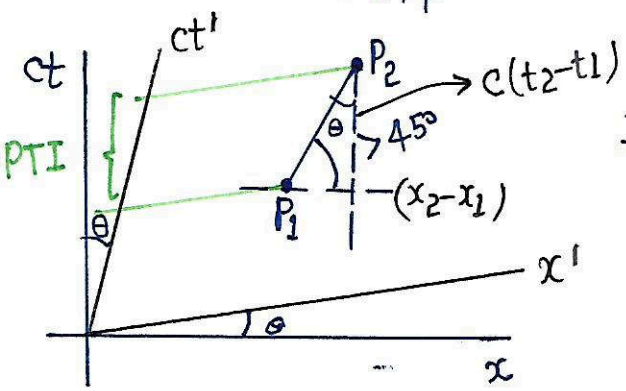
$v << c$, $\beta << 1$, $\theta < \frac{\pi}{4}$



Two events are simultanious in S.

$$ct_1' = OQ_1\sqrt{\frac{1-\beta^2}{1+\beta^2}}$$

$$ct_2' = OQ_2\sqrt{\frac{1-\beta^2}{1+\beta^2}}$$



I donot have any frame, If P_1P_2 has slope $> 45°$, these events are not simultanious.

$$t_1' \neq t_2'$$

$$c(t_2-t_1) > (x_2-x_1)$$

such pair of events is called time like.

Proper time Interval between two events

$$ct_2' - ct_1' = P_1P_2\sqrt{\frac{1-\beta^2}{1+\beta^2}}$$

$$t_2' - t_1' = \frac{P_1P_2}{c}\sqrt{\frac{1-\beta^2}{1+\beta^2}} \quad \ldots\ldots\ldots (1)$$

$$ct_2 - ct_1 = P_1P_2\cos\theta$$

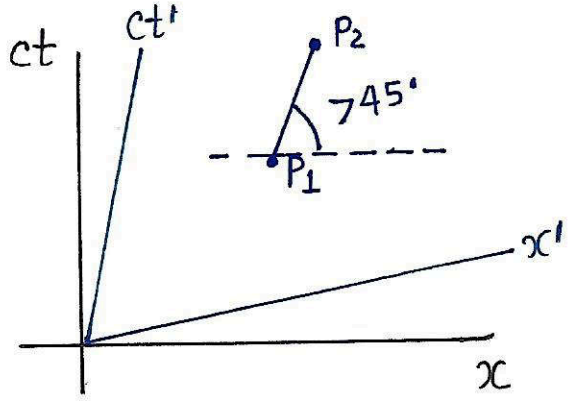
Improper time Interval $t_1 - t_2 = \dfrac{P_1P_2\cos\theta}{c}$

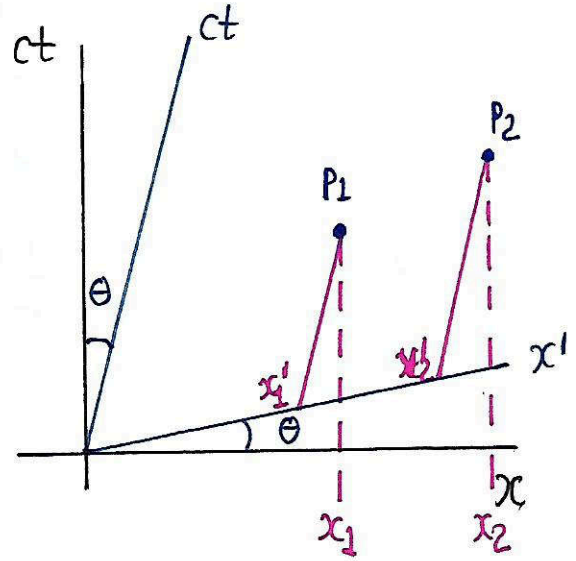
$$t_1 - t_2 = \frac{P_1P_2}{c}\frac{1}{\sqrt{1+\beta^2}} \quad \ldots\ldots\ldots (2)$$

Eq^n(1) devided by Eq^n(2).

$$\boxed{(\Delta t)_{Prop} = (\Delta t)_{Imp}\sqrt{1-\beta^2}}$$

Geometry of Length contraction-





Spacelike events

In S'

$$L_0' = x_2' - x_1'$$

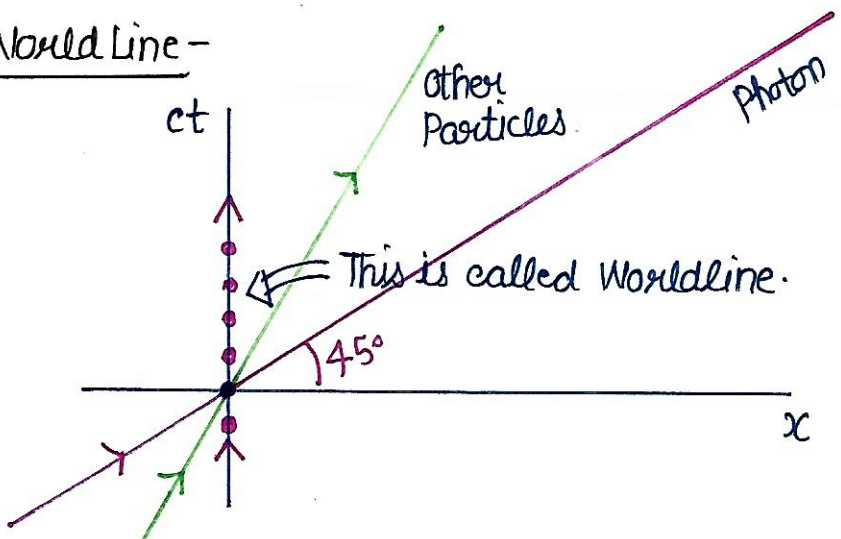
$$L_0' = P_1 P_2 \sqrt{\frac{1-\beta^2}{1+\beta^2}}$$

$$L_0 = P_1 P_2 \cos\theta$$

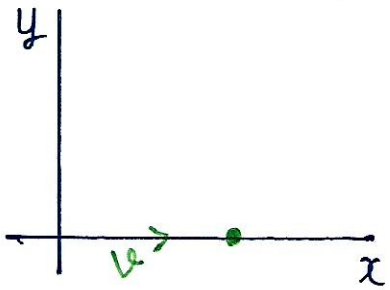
$$L_0 = P_1 P_2 \frac{1}{\sqrt{1+\beta^2}} \qquad \left[\because \tan\theta = \frac{v}{c} = \beta\right]$$

$$\boxed{L' = L_0 \sqrt{1-\beta^2}}$$

World Line -



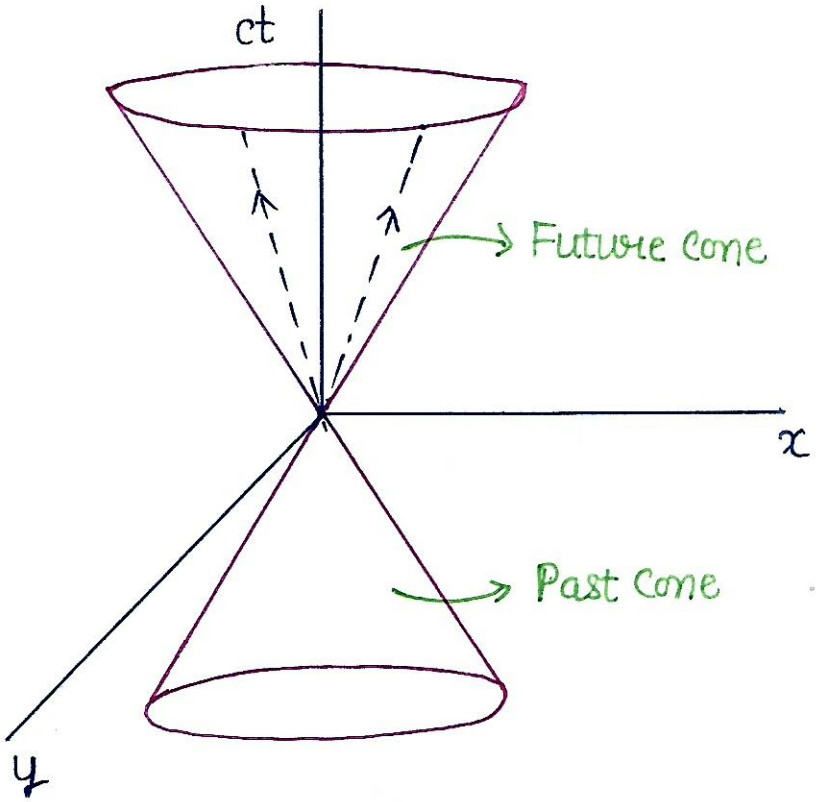
E = Particle is at $x=0$, at $t=0$.



$$x = vt$$

$$t = \frac{x}{v}$$

$$ct = \frac{c}{v}(x)$$

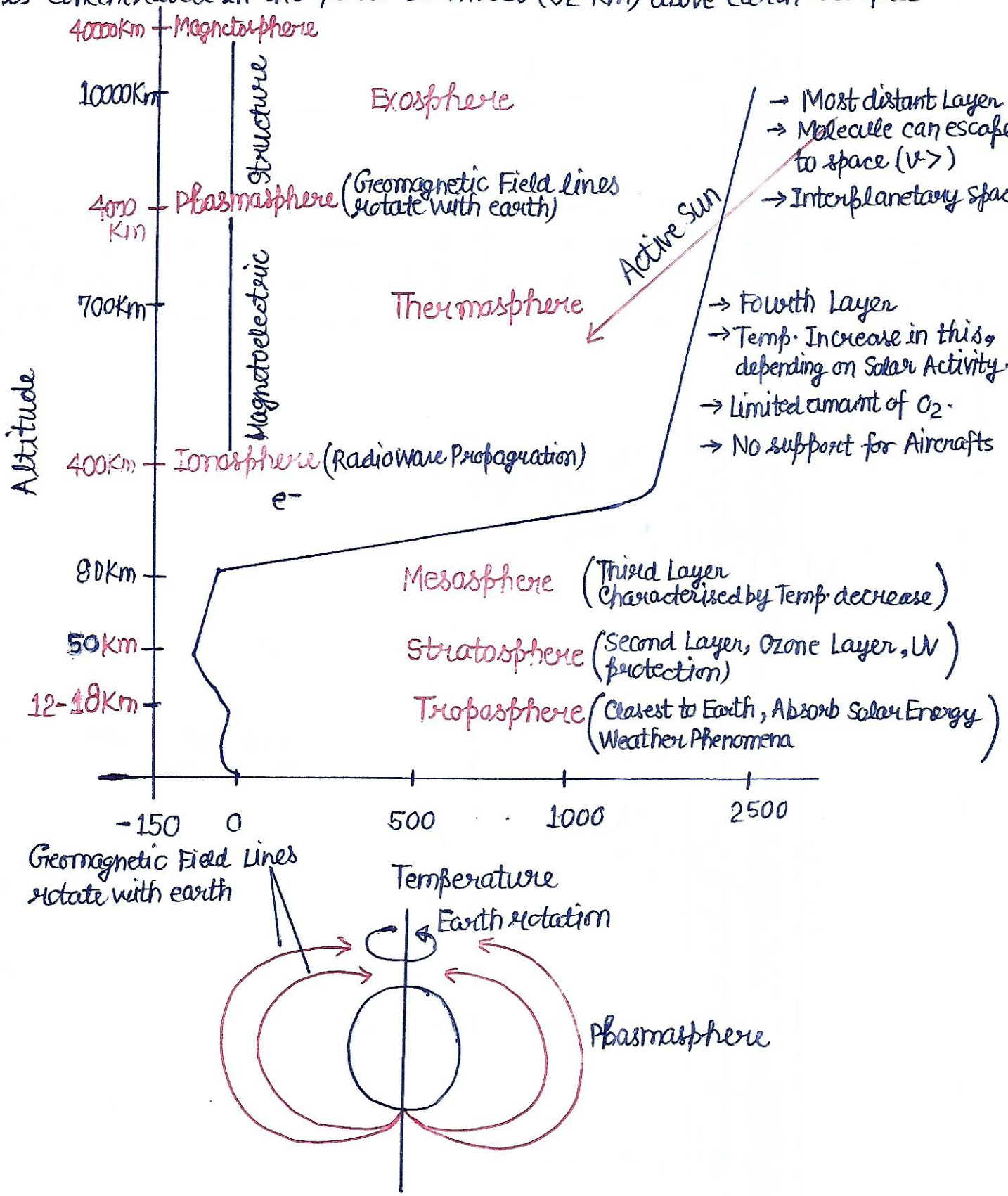

ct
Future Cone
x
Past Cone
y

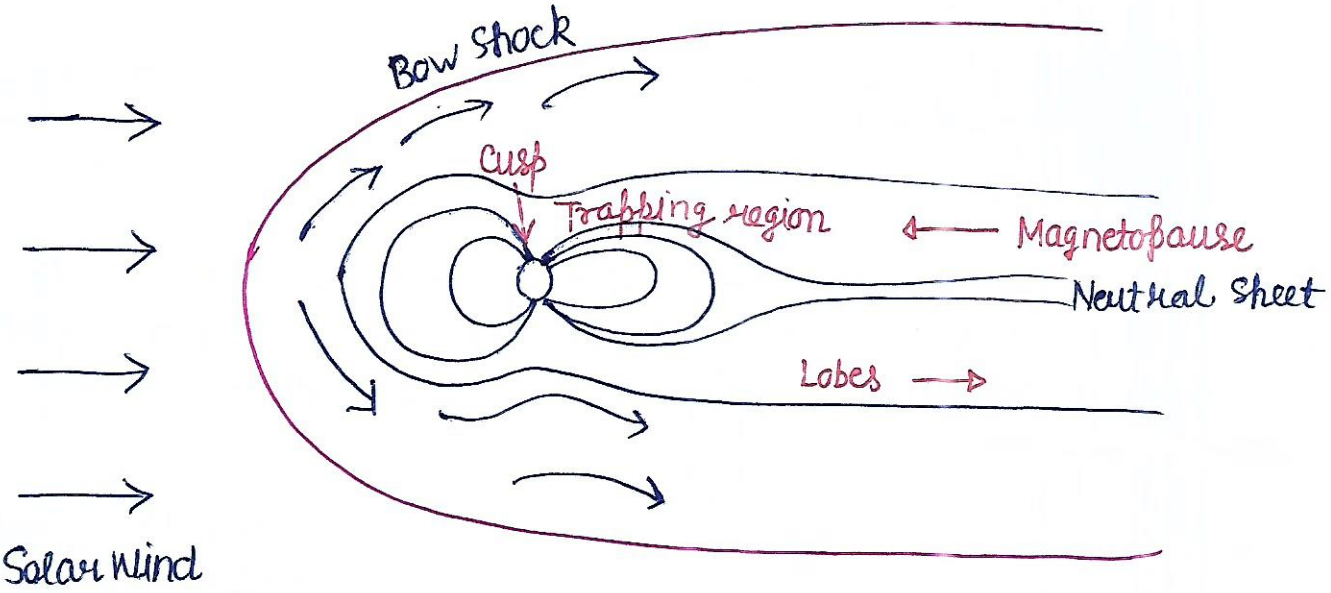
Environmental Physics

- Essentials of Environment
- Solar and Terrestrial
- Environmental Pollution and degradation
- Environmental Change and Remote Sensing
- Global and Regional Climate

• Atmospheric Structure and its thermodynamics :

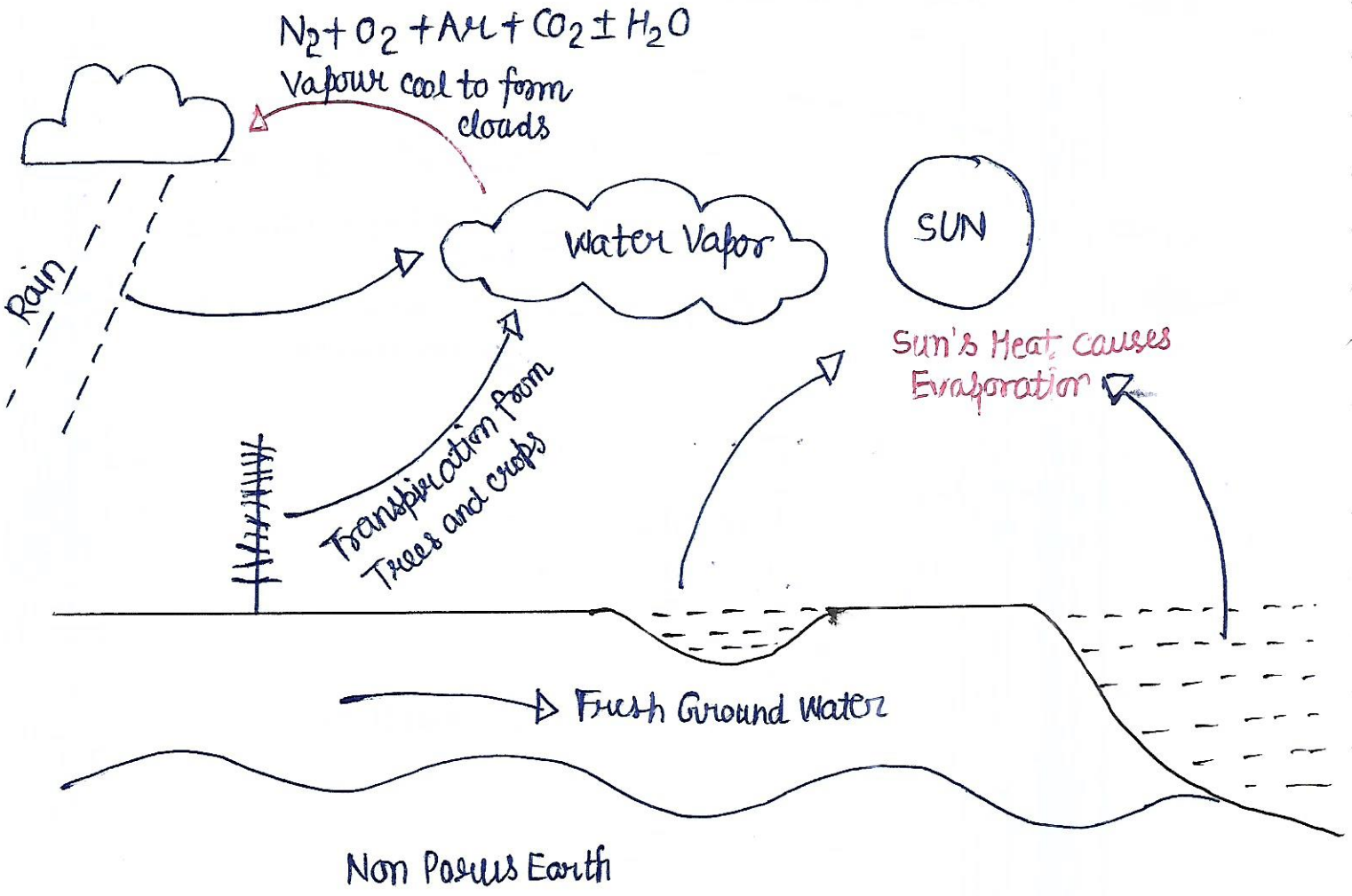
The gaseous area surrounding the planet is devided into several concentric strata or layers. About 99% of the total atmospheric mass is concentrated in the first 20 miles (32 Km) above earth surface.





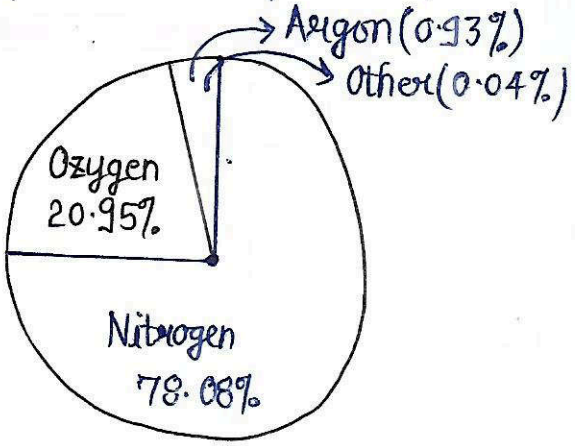
Outside the plasmapause, magnetic field lines are unable to corotate because they are influenced strongly by $\vec{E}$ of solar wind origin. The magnetosphere is a cavity in which Earth's magnetic field is constrained by the solar wind and Interplanetary magnetic field. The outer boundary of this is called magnetopause.

Thermodynamics:



Composition of Air-

The Air in our atmosphere is composed of molecules of different gases.

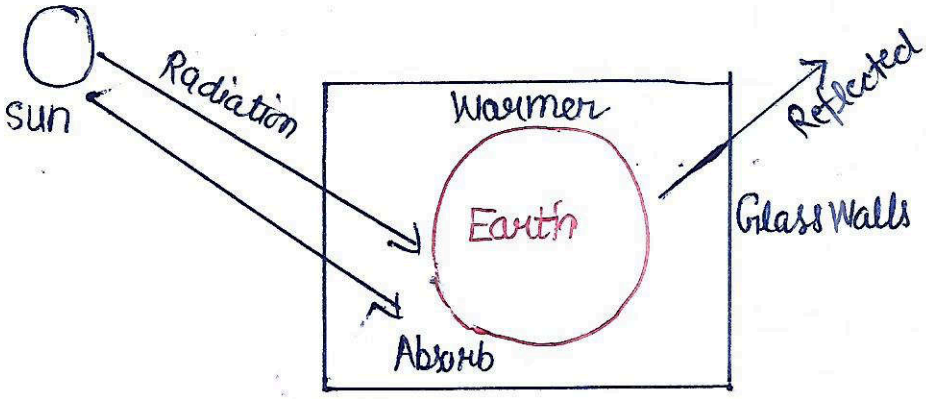


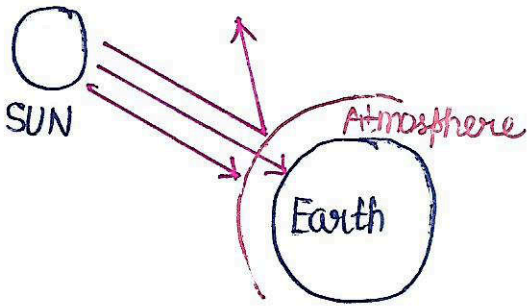
Nitrogen	78.08%
Oxygen	20.95%
Argon	0.93%
Other	0.04%

→ $CO_2 + O_3 + H_2 + Ne + He + CH_4 + Kr + N_2O$

Greenhouse Effect-

The greenhouse Effect is a process that occurs when gases in Earth's atmosphere trap the Sun's Heat. This process makes Earth warmer than it would be without an atmosphere. The greenhouse effect is one of the things that makes Earth a comfortable place to live.

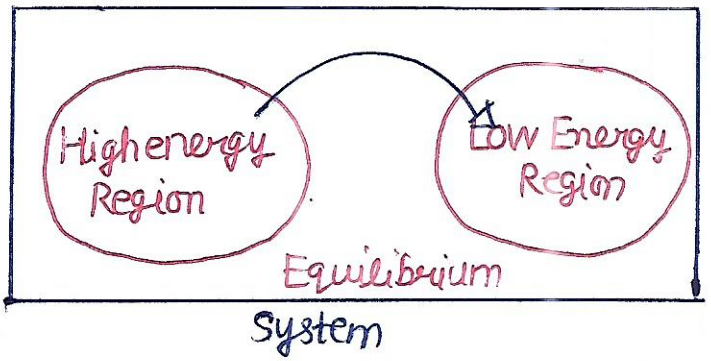




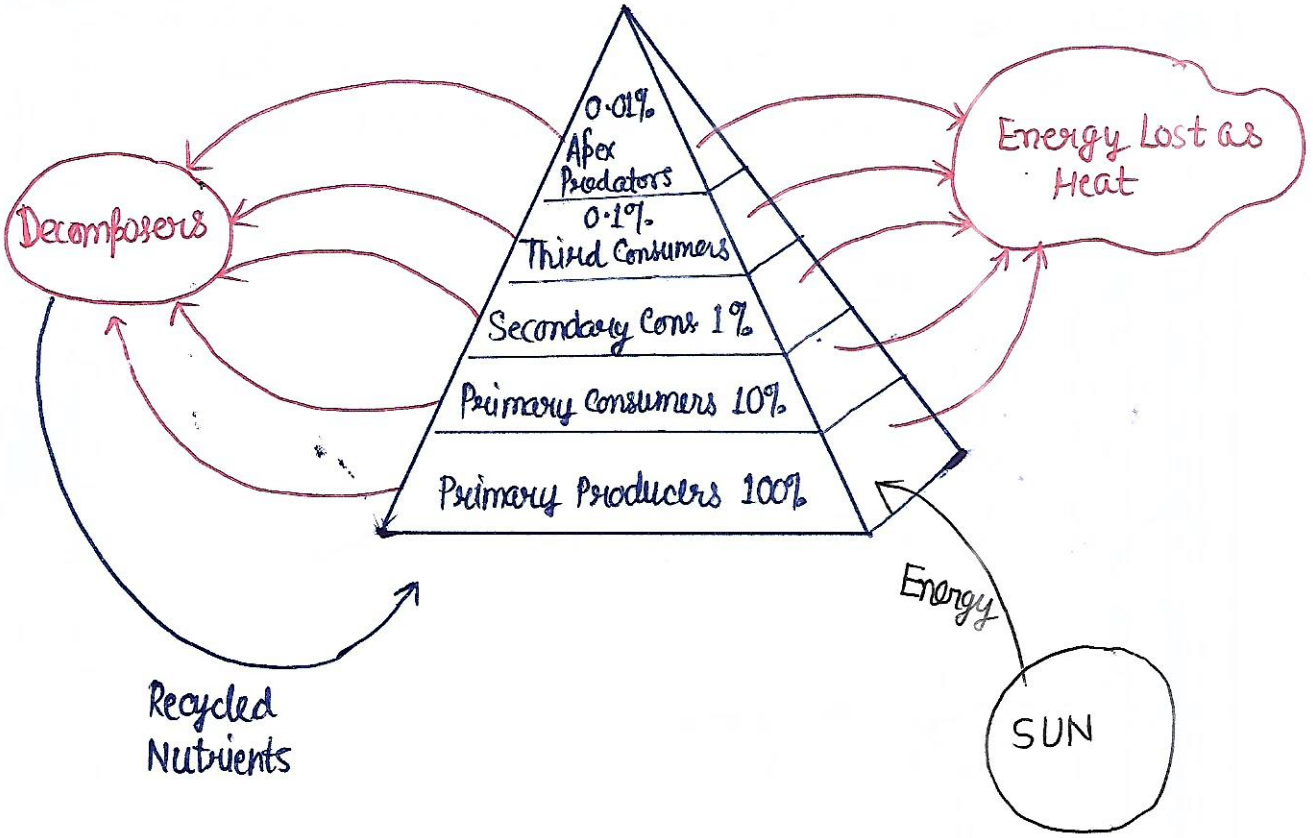
#Transport Of Matter-

Transport properties of matter express the way a current of a certain quantity is created by an imposed gradient of another conjugate quantity. For e.g. A gradiant of temperature causes an entropy current which is perceived as a flow of heat.

Those molecule which have high energy content in a particular region moves in those region in which heat energy content is less to set up the thermal equilibrium.



- Energy transfer in Nature !



A pyramid of energy represents how much energy, initally from the sun, is retained or stored in the form of new biomass at each trophic level in an ecosystem.

- The Momentum Conservation!

In a system, the total momentum is constant. This fact is known as law of conservation of momentum.

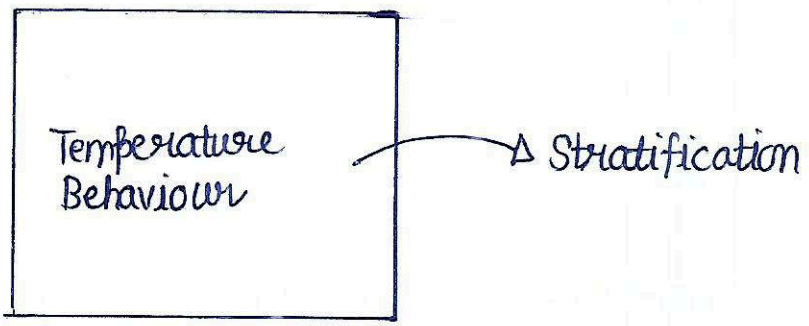
$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$ — Momentum Conservation

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$ — Energy conservation

- Stratification of Atmosphere!

In general, Air pressure and density decreases with Altitude in the atmosphere. However temperature has a very complicated profile with Altitude, and may remain relatively constant or even increase with Altitude in some regions. Because the general pattern of the atmosphere's temperature/Altitude Profile, or lapse rate, is constant and measurable by means of Instruments balloon surroundings, the temperature behaviour provides a useful metric to distinguish atmospheric layers. In this way, Earth's atmosphere can be devided into 5 main layers, is called Atmospheric Stratification.

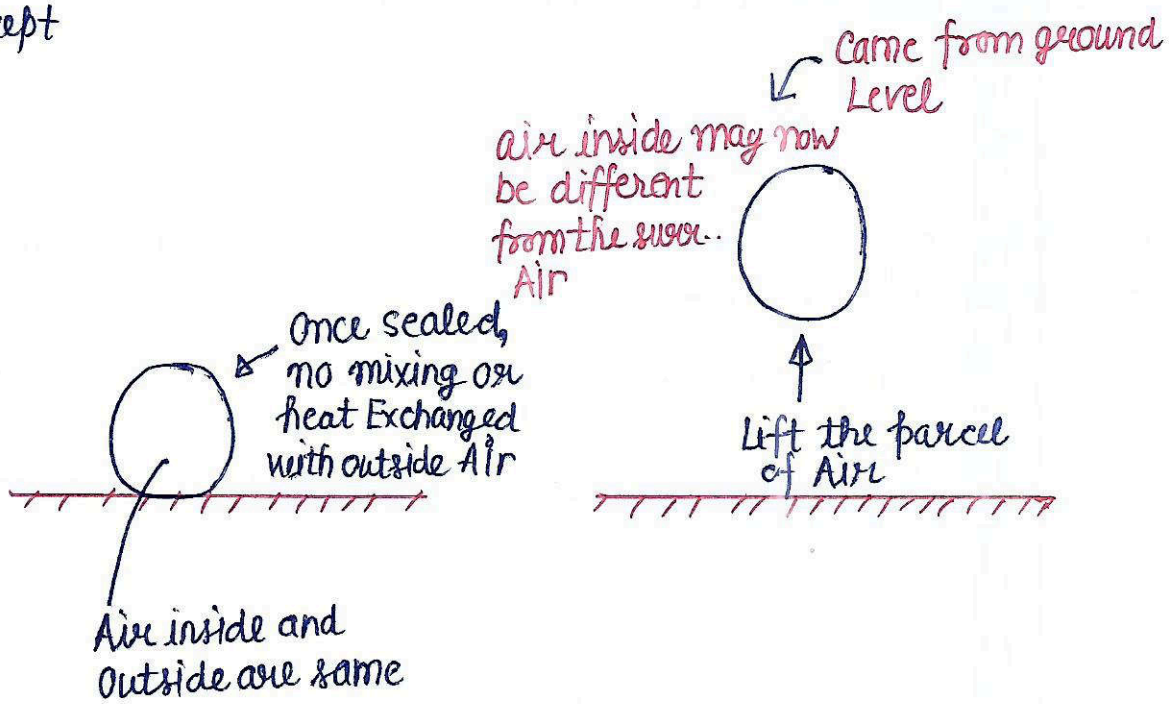
Exosphere! 700 to 10000 Km
Thermosphere! 80 to 700 Km
Mesosphere! 50 to 80 Km
Stratosphere! 50 to 12 Km
Troposphere! 0 to 12 Km

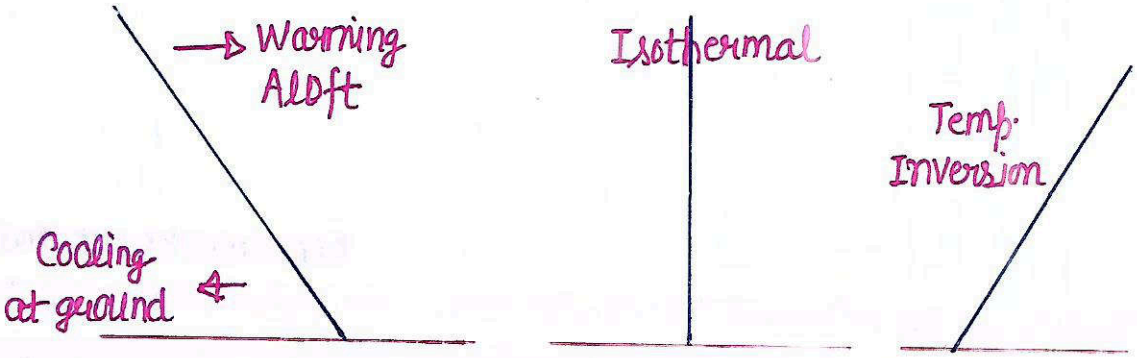


• Stability of Atmosphere :

→ We will learn when the atmosphere is stable and when is unstable.

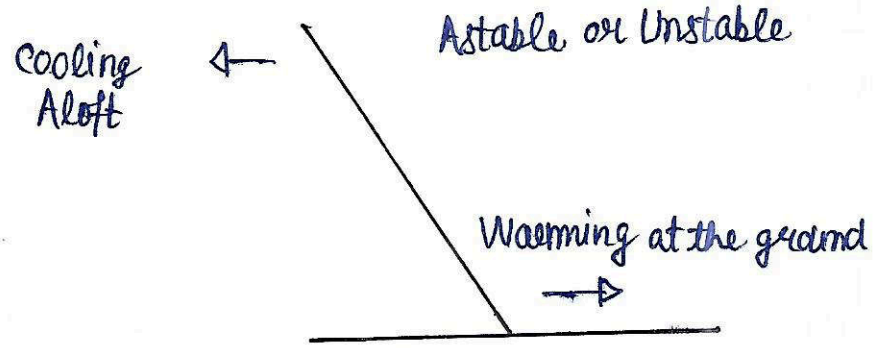
→ Concept





→ Warming the air above the ground and/or cooling the air next to the ground will make the atmosphere more stable.

→ The ground and the air above it cool during the night. The atmosphere is usually most stable early in the morning.



→ Sunlight warms the ground and the next air during day. → Unstable.

#Laws of Motion:

Newton's Laws of motion are three physical laws.

1st-

In an inertial frame of Reference, an object either remains at rest or continues to move at a constant velocity, unless acted upon by a force.

2nd-

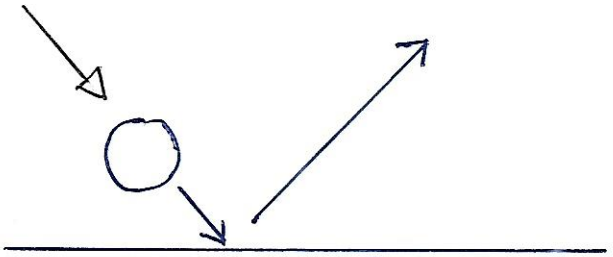
$$\vec{F} = m\vec{a}$$

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt}$$

The rate of change of the momentum of a body is directly proportional to the force applied.

3rd-

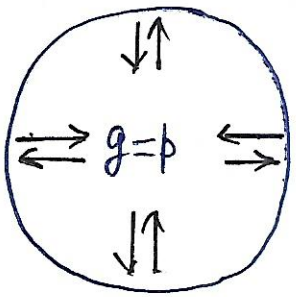
When one body exerts a force on a second body, the second body simultaniously exerts a force on the first body, equal in magnitude and opposite in direction.



Every Action has a equal and Opposite reaction.

#Hydrostatic Equilibrium:

In the atmosphere, the pressure of the air decreases with increasing Altitude. This pressure difference causes an upward force called the pressure gradient force. The force of gravity balance this out, keeping the atmosphere bound to Earth and maintaining pressure difference with Altitude.



$$\frac{Gmdm}{r^2} = -4\pi r^2 dp$$

UNIT-2

SOLAR AND TERRESTRIAL

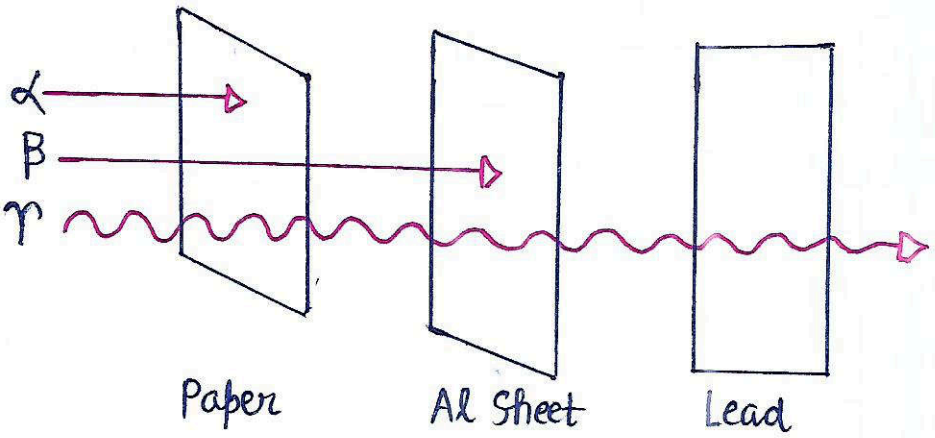
Radiation-

Radiation is the emission or transmission of energy in the form of wave or particle through space or through a material medium.

- Electromagnetic Radiation such as Radiowaves, Microwaves, IR, Visible, UV, X-rays, and γ rays
- Particle Radiation, such as α, β and neutron radiation, cosmic radⁿ.
- Acoustic Radiation such as Ultrasonic, Sound and seismic waves.
- Gravitational Radiation.

Types

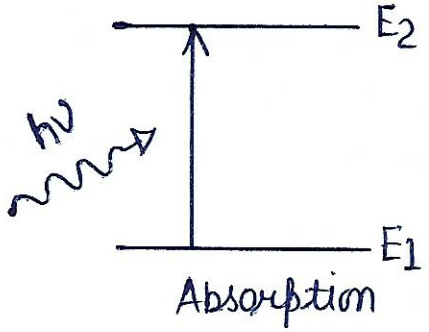
- Ionizing Radiation (Radiation more than 10eV)
  - UV
  - X Rays
  - γ rays
  - α radⁿ
  - β
  - Neutron
  - Cosmic
- Non Ionizing Radiation
  - UV Light
  - Visible
  - IR
  - Microwave
  - Radiowave
  - Thermal Radⁿ
  - Blackbody Radⁿ



Interaction of Light with Matter:

The Interaction of light with matter can take many forms.

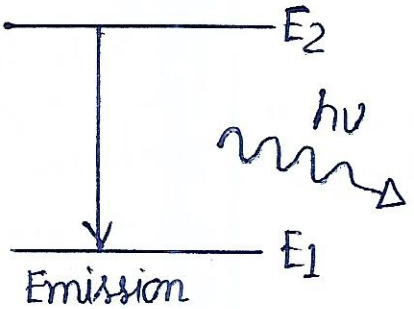
(1) Absorption and Emission of Photons; Photons are annihilated or created.



Absorption

$E_2 - E_1 = h\nu$

must satisfy Bohr relation

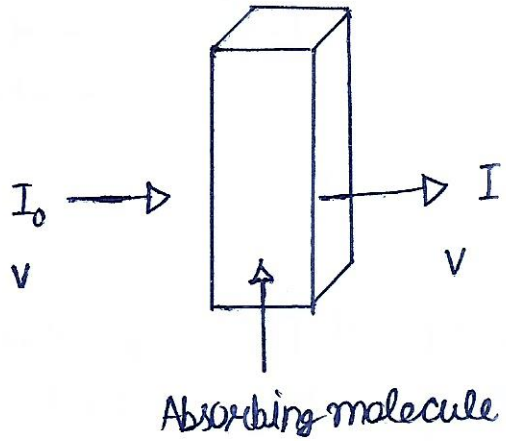


Emission

$E_2 - E_1 = h\nu$

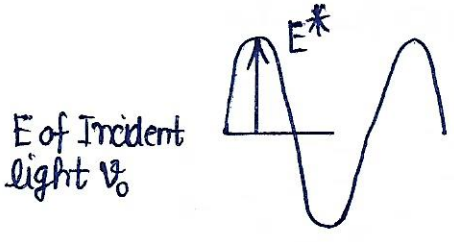
Emission could be induced or spontaneous.

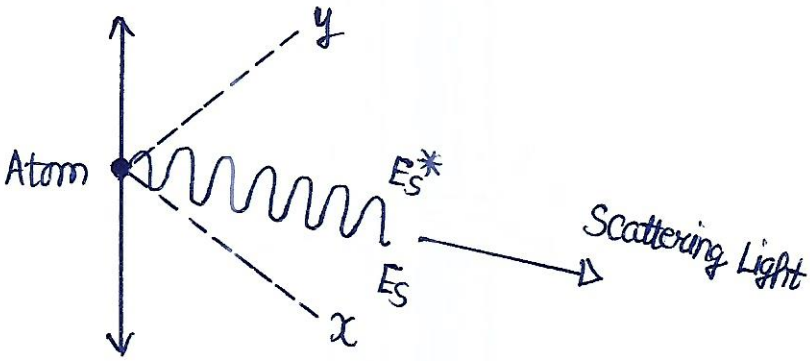
Most absorbing meterials satisfy the Beer Lambert Law.



$$E_S^* = \frac{|e|a \sin\phi}{e^2 r}$$

(2) Light Scattering.

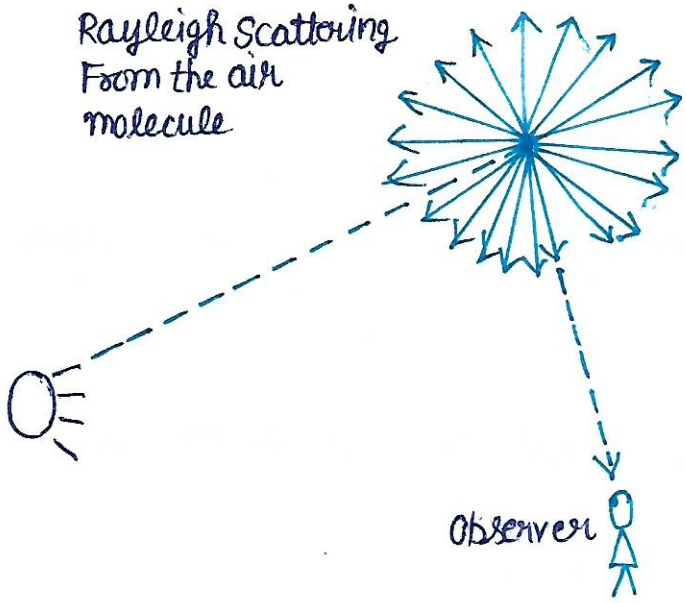




(a) Rayleigh Scattering
(b) Brillioin Scattering
(c) Raman Scattering

#Rayleigh Scattering :

The blue colour of the sky is caused by the scattering of sunlight off the molecule of the atmosphere. This scattering is called Rayleigh Scattering, is more effective at short wavelengths (the blue end of the visible spectrum). Therefore the light scattered down to the earth at the large Angle with respect to the direction of the sun's light is predominantly in the blue end of the spectrum.

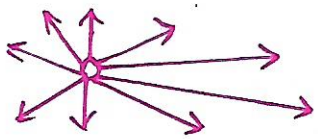


$$I \propto \frac{1}{\lambda^4}$$

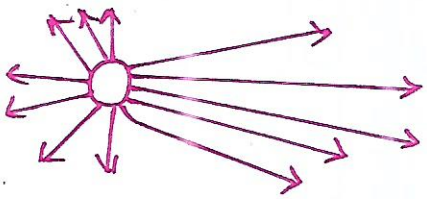
The Strong wavelength dependance of Rayleigh Scattering hence the short wavelength, gives us the blue sky.

#Mie Scattering :

The scattering from molecules and very tiny particles ($< \frac{1}{10}$ wavelength) is predominantly Rayleigh Scattering. For particles sizes larger than a wavelength, Mie Scattering predominates. This scattering produces a pattern like an anteena lobe, with a sharper and more intense forward lobe for larger particles.

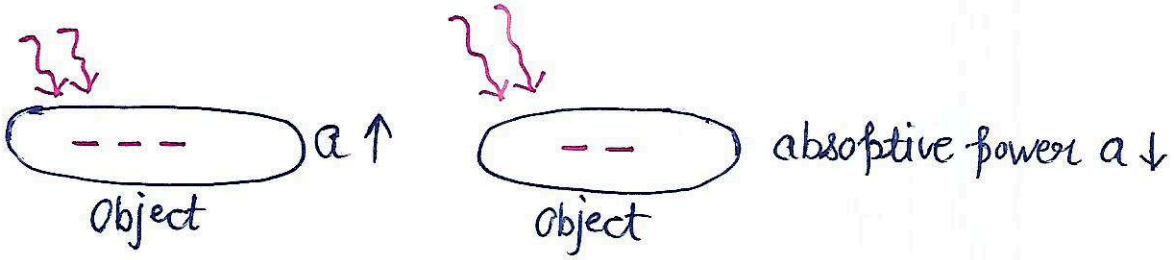


Larger particle

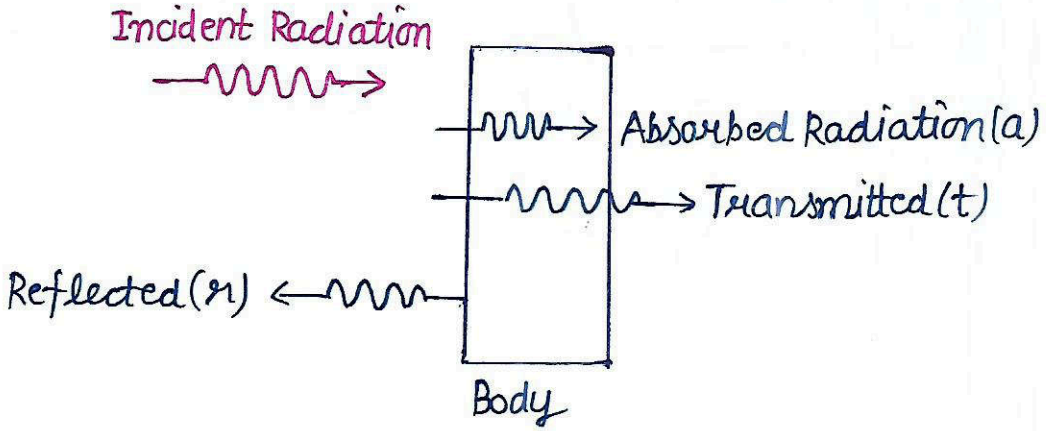


Laws of Radiation:

- Kirchoff's Law-
  - Good absorbers are good emitters



All bodies radiates energy in the form of Photons. Where these photons reach another surface, they may either be absorbed, reflected or transmitted. The behaviour of a surface with incident radiation is described by the following quantities.



Real bodies radiates less effectively than blackbodies. The measurement of this emissitivity (e) defined by

$$e = \frac{E}{E_b}$$

E ← Radiated power from Real bodies

E_b ← from Blackbody Radiation.

$e = 1$ for Blackbodies

$0 < e < 1$ for Real bodies

$$e = \frac{E}{E_b}$$

$$E = eE_b \quad \ldots\ldots\ldots (1)$$

Power absorbed per unit area $= aE_b$

$$E = aE_b \quad \ldots\ldots\ldots (2)$$

From eqn (1) & (2)

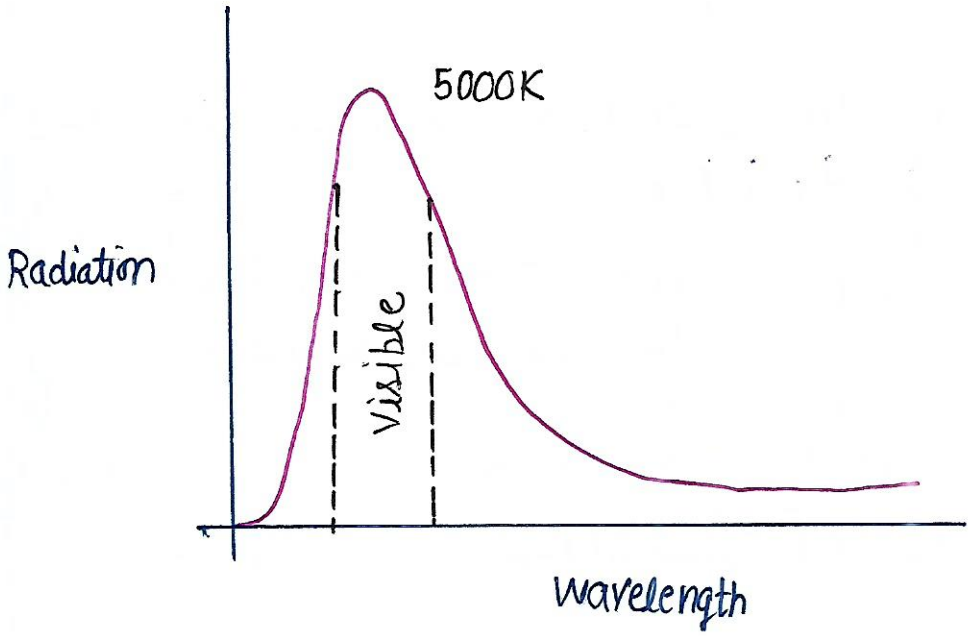
$$eE_b = aE_b$$

$$\boxed{e = a}$$

- **Plank's Law-**

From the assumption that the electromagnetic modes in a cavity were quantized with the quantum energy. Equal to Plank's constant times the frequency, Plank derived a radiation formula. The avarge energy per quantum is the energy of quantum times the probability that it will be occured.

$$\langle E \rangle = \frac{h\nu}{e^{h\nu/KT} - 1}$$



#Wein's Displacement Formula -

When the temperature of the blackbody radiator increases, the overall radiated energy increases and the peak of the radiation curve moves to shorter wavelength. When the maximum is evaluated from Plank's law, the product of the peak wavelength and the temperature is found to be constant.

$$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^{+5}} \cdot \frac{d\lambda}{e^{hc/\lambda KT} - 1}$$

For short wavelength or high frequency $e^{hc/\lambda KT} >> 1$

$$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^{+5}} \frac{d\lambda}{e^{-hc/\lambda KT}}$$

$$u_\lambda d\lambda = (8\pi hc) \lambda^{-5} e^{-hc/\lambda KT} d\lambda$$

$8\pi hc \rightarrow A$

$\frac{hc}{K} = B$

$$\boxed{u_\lambda d\lambda = A\lambda^{-5} e^{-B/\lambda T} d\lambda}$$

This relationship is called Wein's displacement Law.

#Solar Spectra -

Sunlight is a portion of the electromagnetic radiation given off by the Sun, in particular IR, Visible and UV light. On Earth, sunlight is filtered through Earth Atmosphere, and is obvious as daylight when the sun is above horizon.

The intensity of sunlight can be measure using a sunshine recorder, pyranometer or pyrheliometer. To calculate the amount of sunlight reaching the ground, both the eccentricity of Earth elliptical orbit and

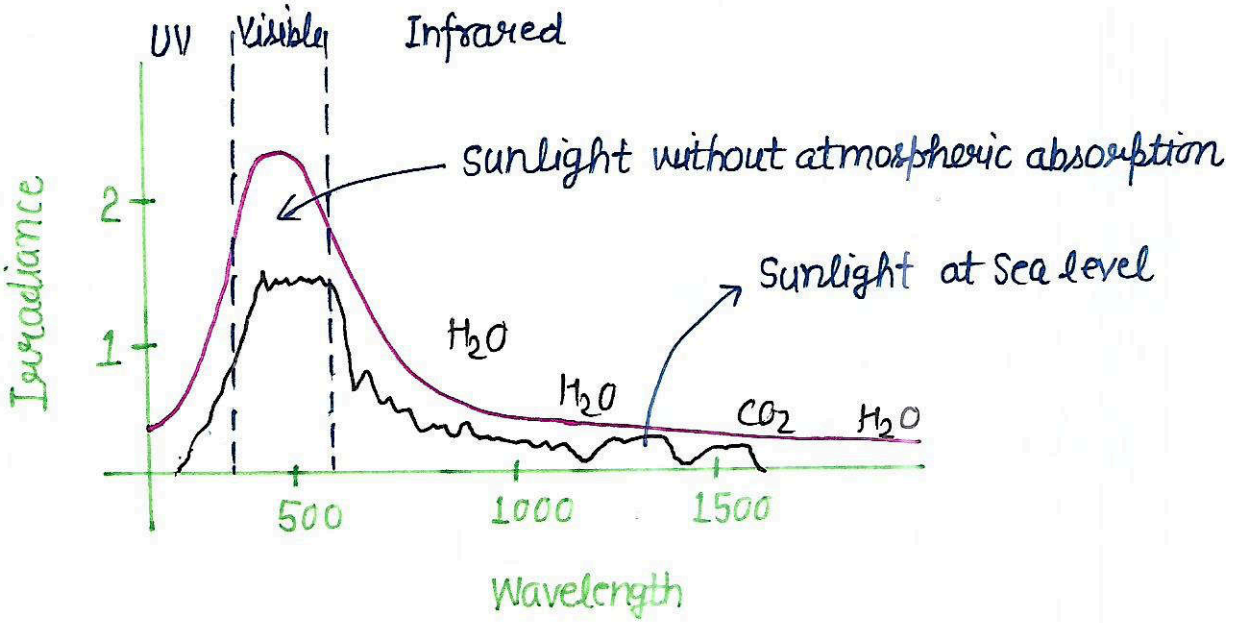
the attenuation by Earth's atmosphere have to be taken into accounts.

$$E_{ext} = E_{SC}\left(1 + 0.033412 \cos\left[2\pi \frac{dn-3}{365}\right]\right)$$

different for each month.

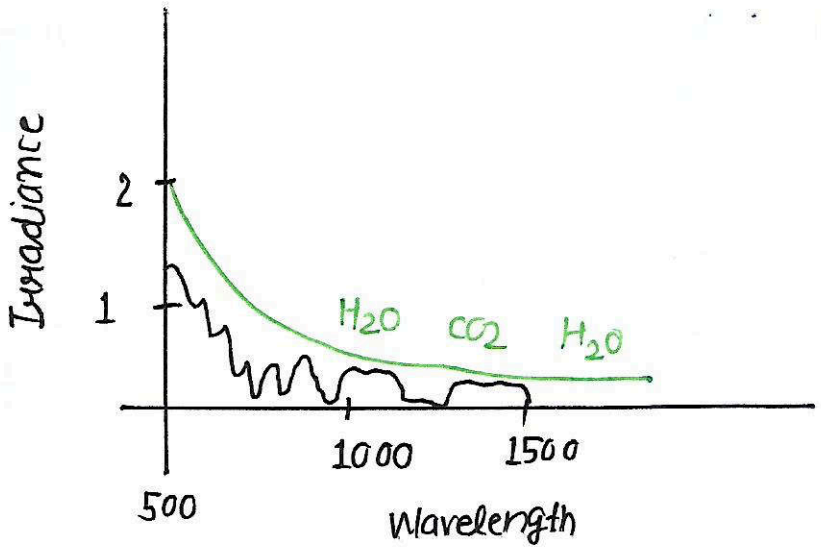
E_{SC} → Solar(E) Illumination Constant

E_{ext} → Extraterrestrial Solar Illumination



Terrestrial Source of Radiation-

Electromagnetic radiation emitted by Earth's surface and atmosphere is called terrestrial or longwave radiation. Since these bodies exhibit radiation temp. in the range 200-300 K, terrestrial radiation lies in the Infrared portion of the EM Spectrum.



#UV Radiation-

Everyone is exposed to UV radiation from the sun and an increasing no. of people are exposed to artificial source used in industry, commerce and recreation. Emissions from the sun including visible light, heat and UV radiation.

The UV region covers the wavelength range 100-400 nm and devided into three bands.

(1) UVA (315-399 nm)
(2) UVB (280-315 nm)
(3) UVC (100-280 nm)

As sunlight passes through the atmosphere, all UVC and approx. 90% UVB radiation is absorbed by ozone, water vapour, O_2 and CO_2. UVA radiation is less affected by the atmosphere. Therefore, the UV radiation reaching the Earth's surface is largly composed of UVA with small UVB component.

Factors that influence the UV Levels :

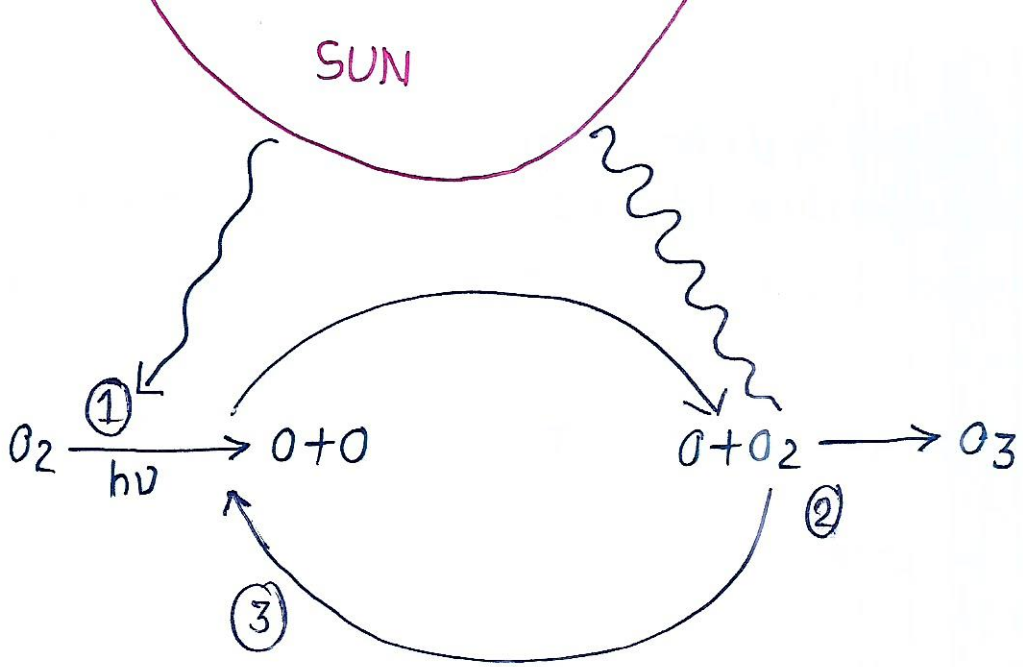
- Sunlight
- Latitude
- Cloud cover
- Altitude
- Ozone
- Ground Reflection

#Ozone Depletion :

The ozone shield or Ozone Layer is a region of Earth Stratosphere that absorbs most of the Sun's UV radiation.

$$O_2 + h\upsilon_{UV} \longrightarrow 2O$$
$$O + O_2 \longrightarrow O_3$$

(O_3) → Ozone



Ozone depletion consists of two related events observed since the 1970's, a steady lowering of about four percent in the total amount of ozone in Earth Atmosphere, and a much larger Springtime decrease in Stratospheric ozone around Earth polar regions. The phenomena is referred to as Ozone hole.

Effect of UV deplation;

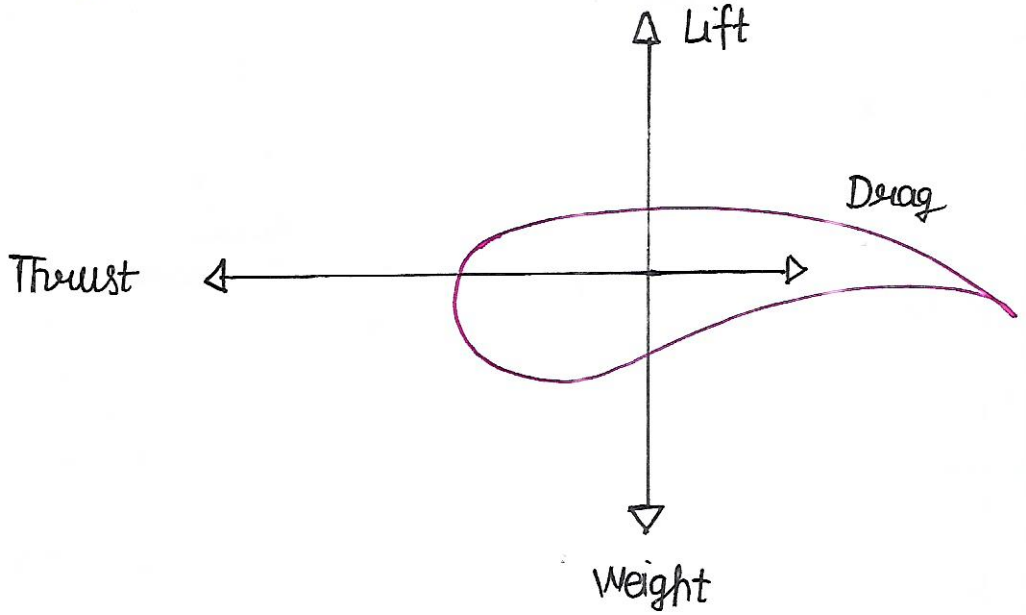
(1) Effect on Human Health
(2) Effect on Plants
(3) Effect on Marine Ecological system.
(4) Effect on Biochemical Cycle
(5) Effect on Materials.

IR Absorption -

IR Spectroscopy is a universal, analytical technique for the structure elucidation and quantification of a large variety of Organic, Inorganic and biological samples. The technique is based on the measurement of IR radiation absorbed or emitted by a sample. IR Spectroscopy is used in the broad range of Environmental Applications. It is viable tool in remote measurement of Gases, liquid, and Solids and as fast and reliable method.

Elementary Fluid Dynamics :

Fluid dynamics is a subdiscipline of Fluid Mechanics that describes the flow of fluids : liquids and gases. It has several subdisciplines including aerodynamics and hydrodynamics.



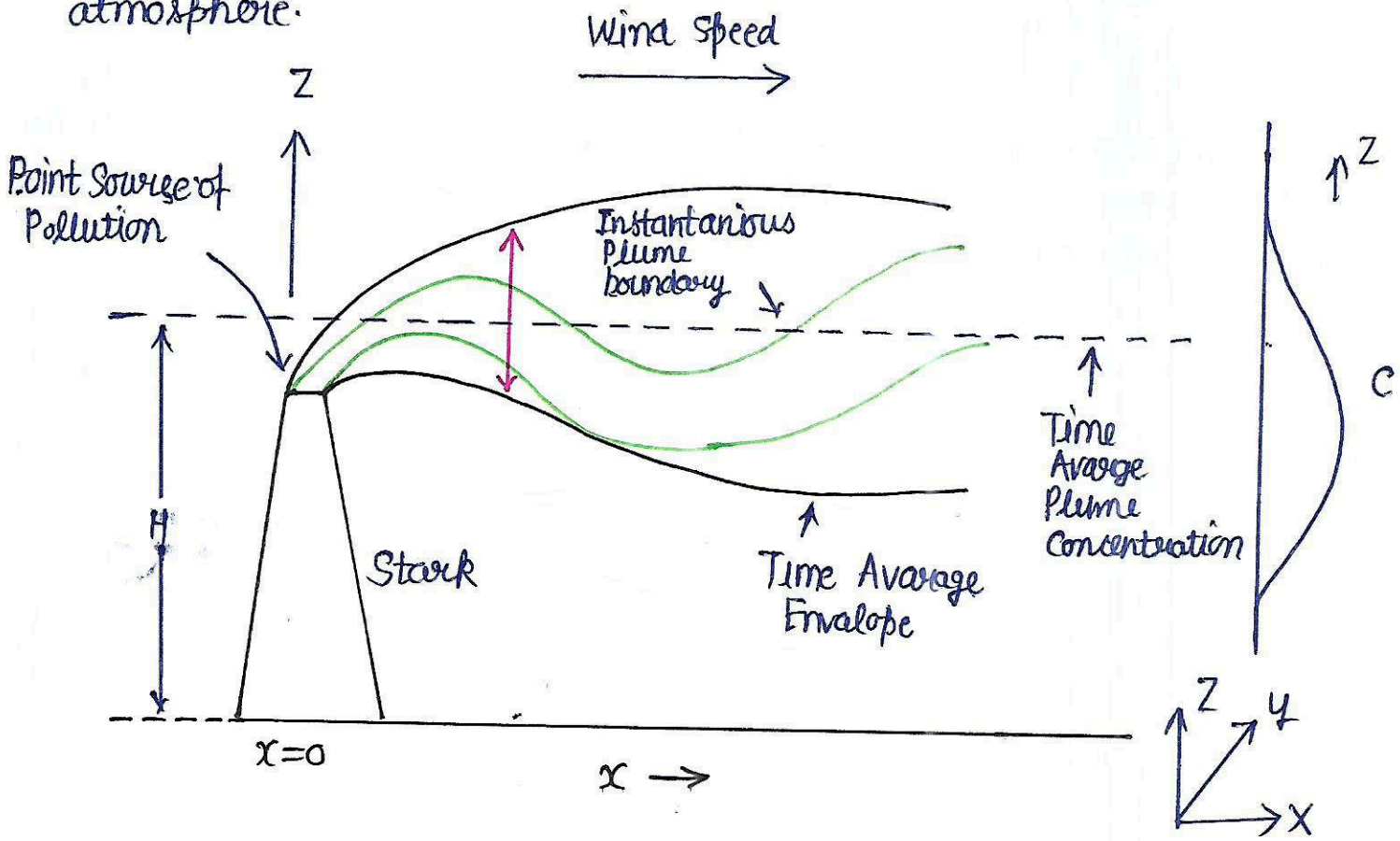
Understanding the motion of air around an object (often called a flow field) enables the calculation of forces and moments acts on the object. In many aerodynamics problem, the forces of interest are the fundamental forces of flight : lift, drag, thrust and weight. Of these, lift and drag are aerodynamics forces i.e. forces due to air flow over the solid body. Calculation of these quantities is often found upon the assumptions that the flow field behaves like a continuum.

A system should be designed also considering the surroundings.

Atmospheric Diffusion-

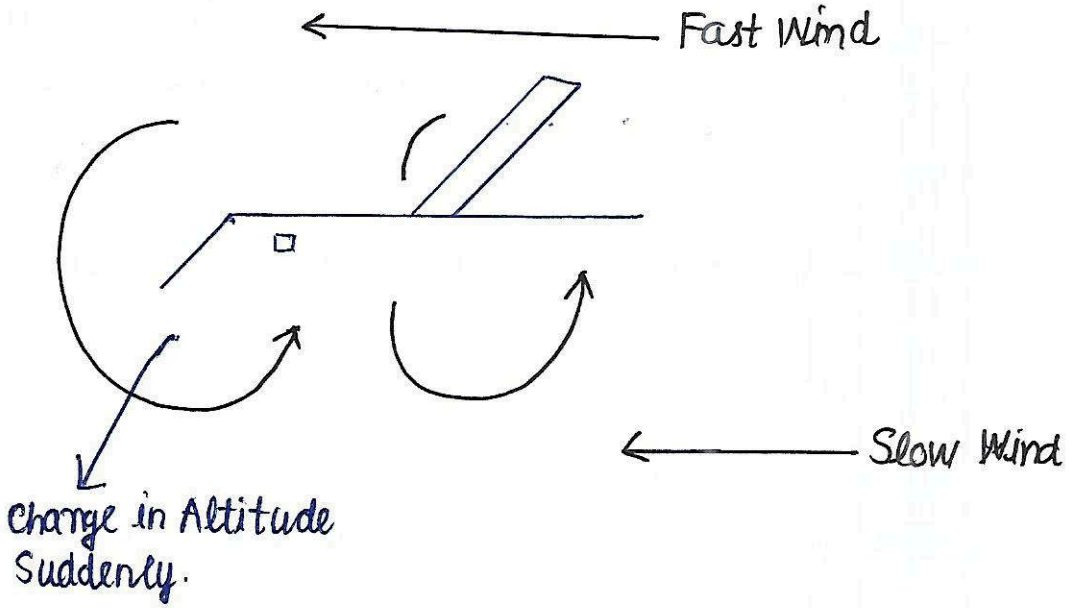
- Behaviour of gases and particles in turbulent Flow (in the atmosphere) is referred to as atmospheric diffusion.

- Goals of diffusion models is to describe mathematically the spatial and temporal distribution of contaminants released into atmosphere.



Turbulence and Turbulent diffusion –

Turbulence is fluid motion characterized by chaotic changes in pressure and Flow velocity. Turbulence is commonly observed in everyday phenomena such as fast flowing rivers, Storm clouds, or smoke from the chimney.



Turbulent diffusion is the transport of mass, heat or momentum within a system due to random and chatic time dependant motions. It occurs when turbulent fluid system reaches critical conditions in response to shear Flow.

Eulerian Approach to turbulent diffusion forces on a infinisitimal Volume at a specific space and time in a fixed frame of reference.

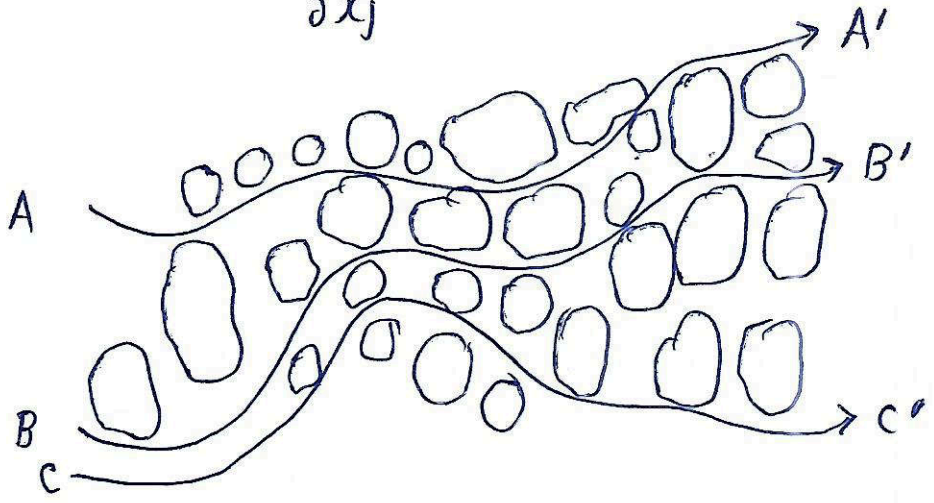
$$\frac{\partial c_i}{\partial t} + \frac{\partial}{\partial x_i}(u_j, c_j) = D_i \frac{\partial^2 c_i}{\partial x_j \partial x_j} + R_i(c_1, \ldots c_N, T) + S_i(x,t)$$

$$i = 1, 2, \ldots, N$$

Where c_i = Specific concentration of interest
u_j = Velocity
t = time
x_j = direction
D_i = diffusion Constant
R_j = Rate of c_i generalized reaction
S_i = Rate of c_i generalized by source

Eddy diffusivity

$$\langle u_j', c' \rangle = -K_{ij} \frac{\partial(C)}{\partial x_j}$$



#Factor Governing Air–

There are different Factors affecting the movement of the air.

(1) Temperature (T)

Low Pressure ⟶ High Pressure

High Temperature ⟶ Low Temperature

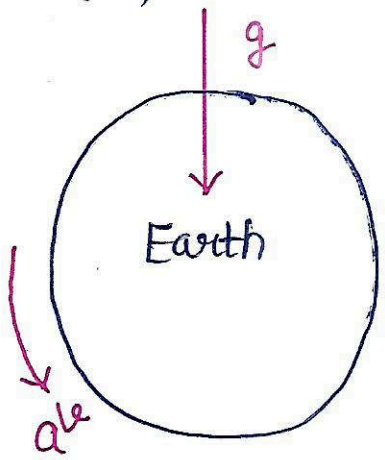
Air Flow

(2) Pressure (P)

↓ Air Pressure decrease

↑ Move Upper to Sea Level

(3) Coriolis Movement (Cm)



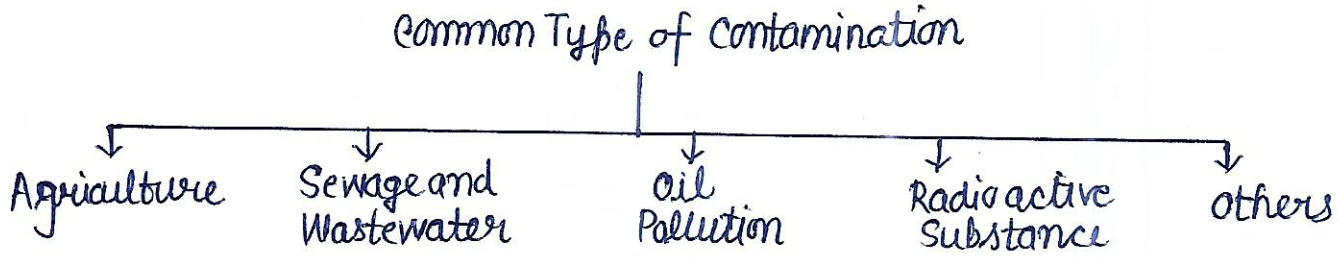
(4) Friction (Ff)

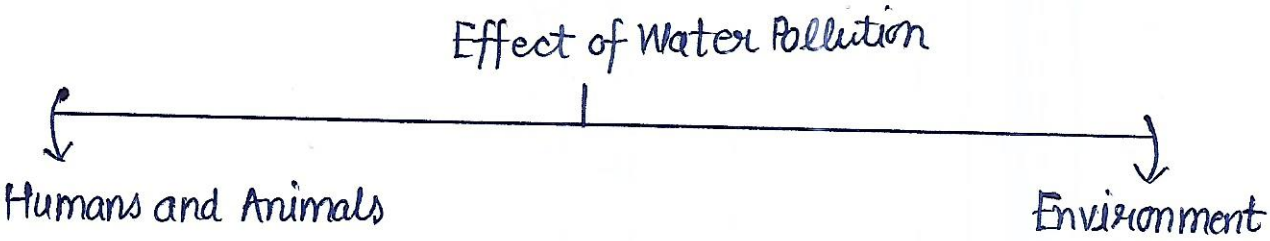
↑ More Friction

↓ Slow Air Movement

Water Pollution-

Water pollution occurs when harmful substances- often chemicals or microorganisms- contaminate a stream, river, lake, ocean, aquifer or other body of water, degrading water quality and rendering its toxic to humans or the environment.

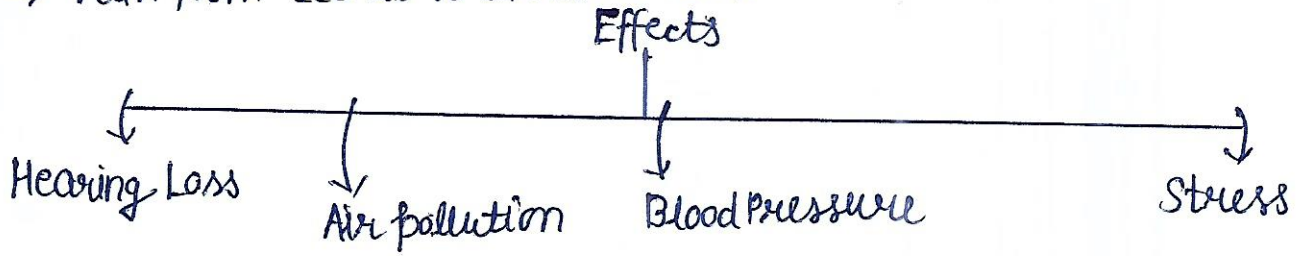




Noise Pollution-

Noise Pollution, unwanted or excessive sound that can have deletrious effect on human health and envimomental quality. Noise Pollution is commonly generated inside many industrial facilities and some other workplaces, but it also comes from highway, railway and airoplane traffic and From outdoor contruction activities.

→ Viberations of air molecules
→ Described in terms of Amplitude, frequency and wave
→ Pain from 120 dB to 140 dB to more.



#Air Quality Standards-

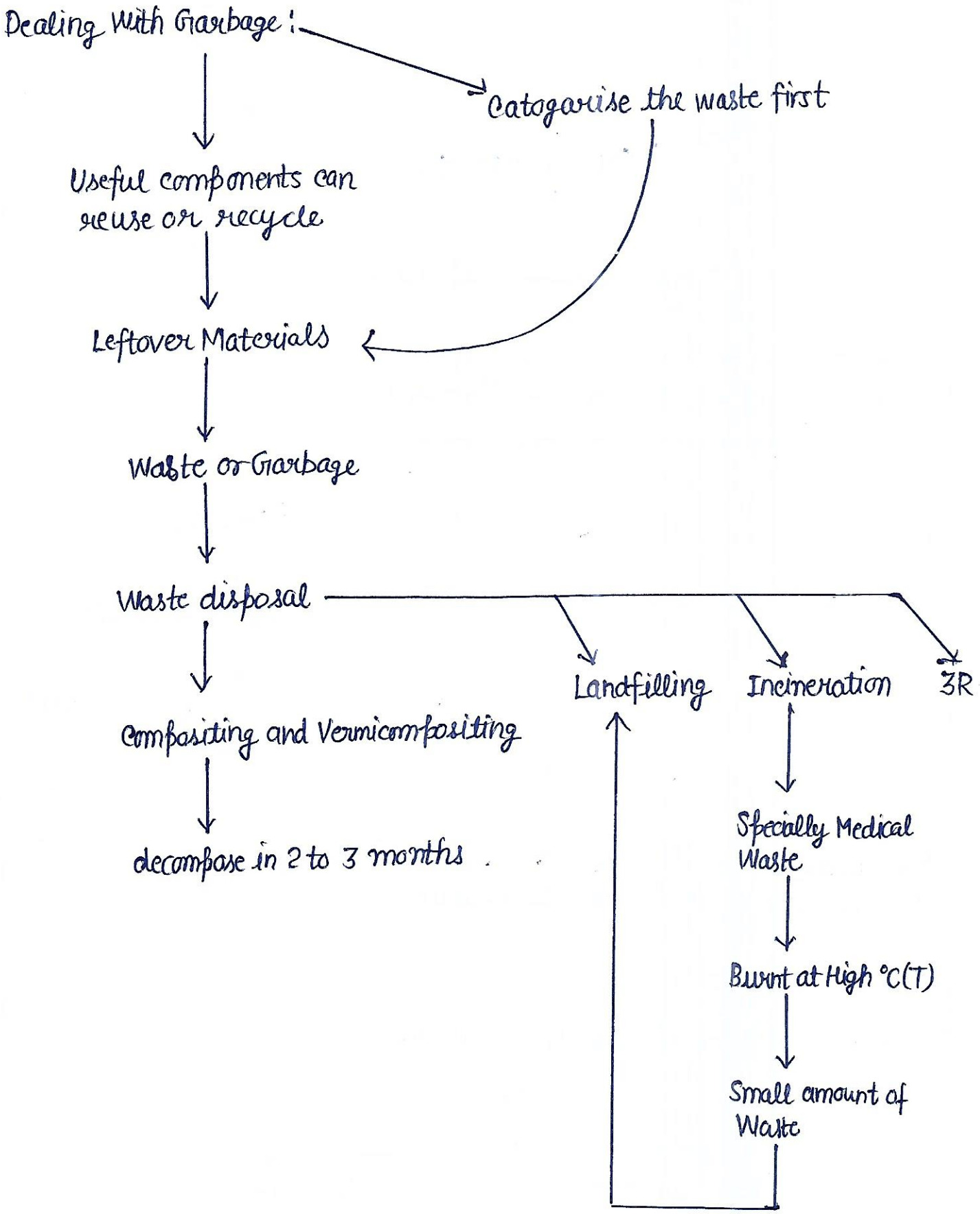
Pollutant	Time Weighted Avarage	Concentration in Ambient Air	
		Industrial and Res.	Ecologically Sens.
SO_2	Annual* 24 Hours*†	50-80 ug/m3	30-80
NO_2		40-80 ug/m3	20-80
PM_{10}		100 ug/m3	100
$PM_{2.5}$		60 ug/m3	60
O_3		180 ug/m3	180
Pb		1.0 ug/m3	1
CO		4 mg/m3	4
NH_3		400 ug/m3	400
C_6H_6		5 ug/m3	5
BaP		1 ng/m3	1
As		6 ng/m3	60
Ni		20 ng/m3	20

#Water Quality Standards-

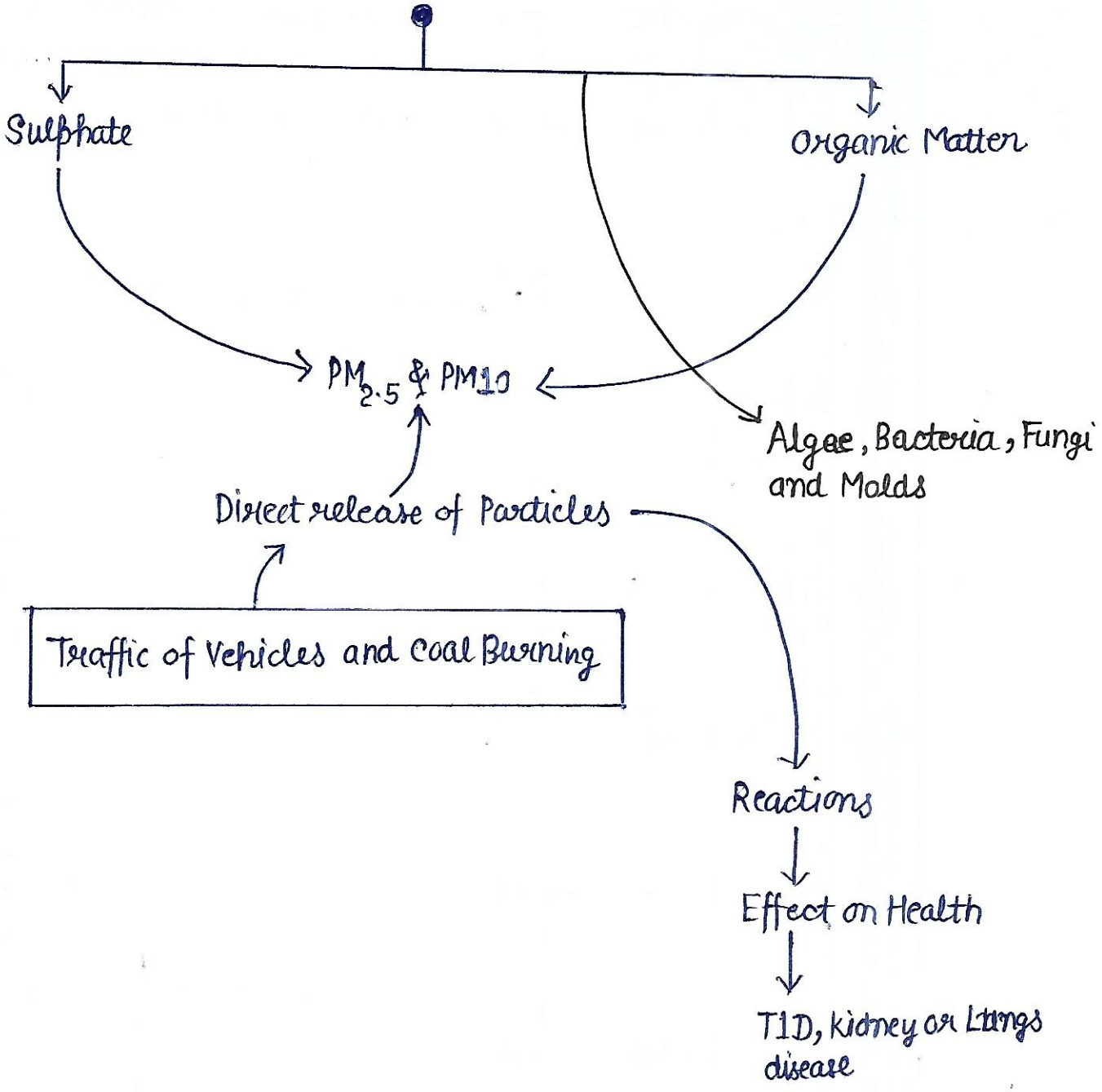
Criteria	Standards	Reasons
Coliform Bacteria	0 C/ml	Health
pH	8.5	Aesthetic
Barium	2 mg/L	Health
Nitrate	10 mg/L	Health
Total dissolved S	500 mg/L	Taste

Waste Disposal –

All the waste we generate we generate in the form of fruits and vegetables peels, leftover food, Packing material, old and unwanted plastic objects, old clothes etc. are all waste or garbage.



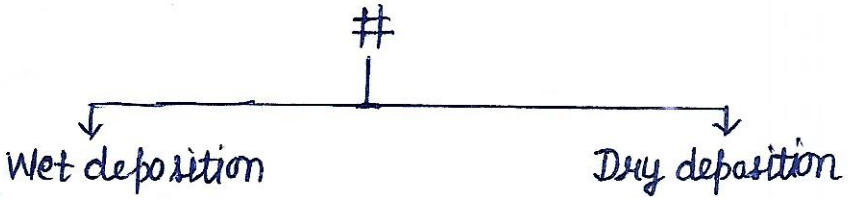
Gaseous and Particulate Matter-



Particulate Matter is the sum of all solids and liquid particles suspended in air, many of which are hazardeous.

Wet and dry deposition-

There are two types of deposition mechanisms.

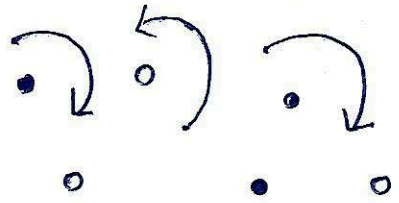


The dry deposition i.e., the uptake at the earth surface (Soil, water, or vegetations) and wet deposition i.e. absorption into droplets following by droplet precipitations or impactation on the earth surface.

Dry Deposition

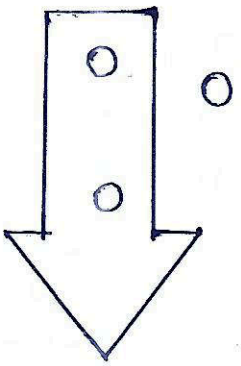
turbulant diffusion

Gases and small particles

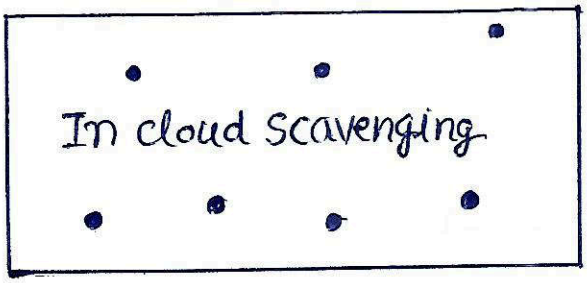


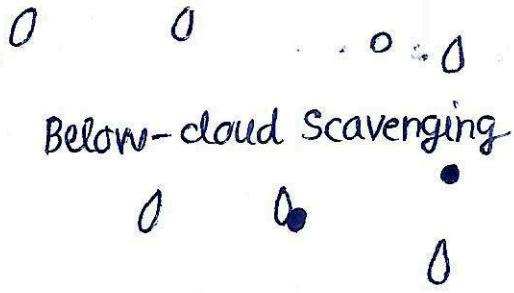
Sedimentation

Large particles



Wet deposition

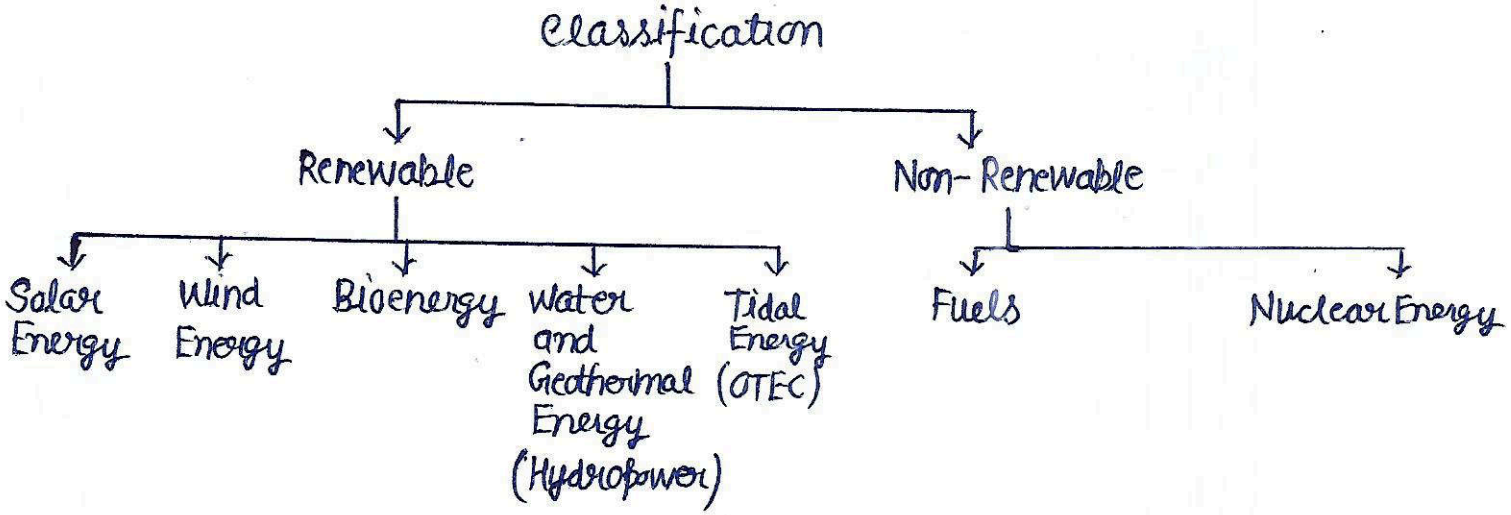




Environmental Change and Remote Sensing

#Energy Source and Combustion Process –

The Energy is the capacity of a physical system to perform work. Energy Exists in several forms such as heat, kinetic or mechanical energy, light, potential energy, electrical or other forms.



given
The Heat off when a specific amount (1 mole) of substance (reactant) burns in Oxygen gas.

$$CH_4(g) + 2O_2(g) \longrightarrow CO_2(g) + 2H_2O(l) + Energy$$

This is combustion process Example.

Types

- Complete combustion
- Incomplete combustion
- Smoldering
- Rapid Combustion
- Spontenous combustion
- Turbulant Combustion
- Micro combution

#Renewable Source of Energy-

Renewable Energy is derived from Natural Processes that are replenished constantly such as Solar, wind, ocean, hydropower, bioenergy, geothermal, biofuels and hydrogen.

Solar Energy-

Sun is primary source of energy. Sunlight is a clean, renewable source of Energy. It is a sustainable resource means it doesn't run out, but can be maintained because the sun shines almost every day. Coal or gas are not sustainable or renewable: once they are gone, there is nothing left. It is called Green Power too.

Advantages:

- Natural Source.
- Free.
- Available in Plenty.
- Non-Polluting.
- Doesn't emit any greenhouse gas.
- Ecofriendly.

Disadvantages:

- Depend on Seasons.
- High Investment.
- Doesn't work at night.
- Bulky.

Technologies:

- Electricity generation.
- DC Voltage.
- Solar Photovoltaic cells.

Wind Energy -

Wind is the natural movement of air across the land or sea. The wind when used to turn the blades of a wind mill turns the shaft to which they are attached. This movement of shaft through a pump or generator produces electricity.
Private agencies own 95% of the wind farms in India.

Advantages -

- Ecofriendly.
- Freely Available

Disadvantages -

- High Investment.
- Speed is not uniform.

Bioenergy -

The Plants fix solar energy through the process of Photosynthesis to produce biomass. The biomass passes through various cycles producing different form of energy source.

Uses -

- 1/3 of total energy of our country.

Advantages -

- Available locally.
- Clean fuel compared to Fossil Fuels.

Disadvantages -

- Air pollution.
- Health Hazardeous.
- Unsustainable.
- Ineffecient use.

#Hydropower-

Hydropower or hydroelectricity refers to the conversion of energy from the flowing water into electricity. It is considered a renewable source of energy because the water cycle is constantly renewed by the Sun.

Advantages-

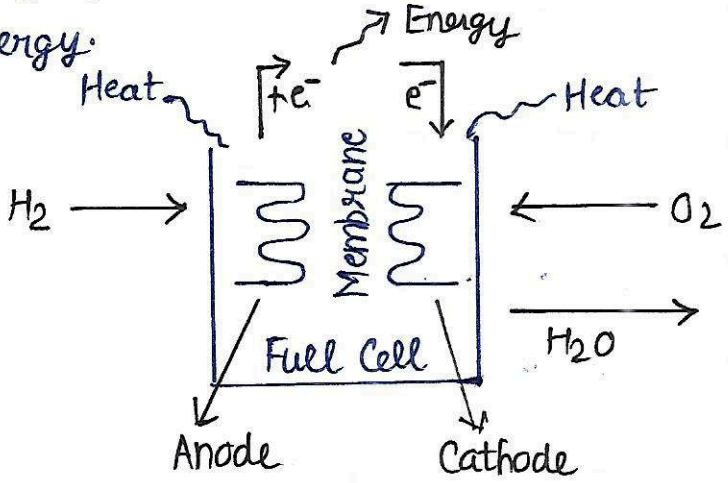
- Reliable.
- Ecofriendly.
- Mature and Proven Technology.
- Non-Palluting.
- Small Capital Investment.
- Minium Transmission Losses.

Other-

- Ocean Energy.
- Wave Energy.
- Current Energy.
- Ocean Thermal Energy Conversion (OTEC).

#Fuel Cells-

A Fuel cell is a device that converts chemical potential energy into electrical energy.



#Nuclear Energy-

Nuclear power is the use of nuclear reactions that release nuclear energy to generate heat.

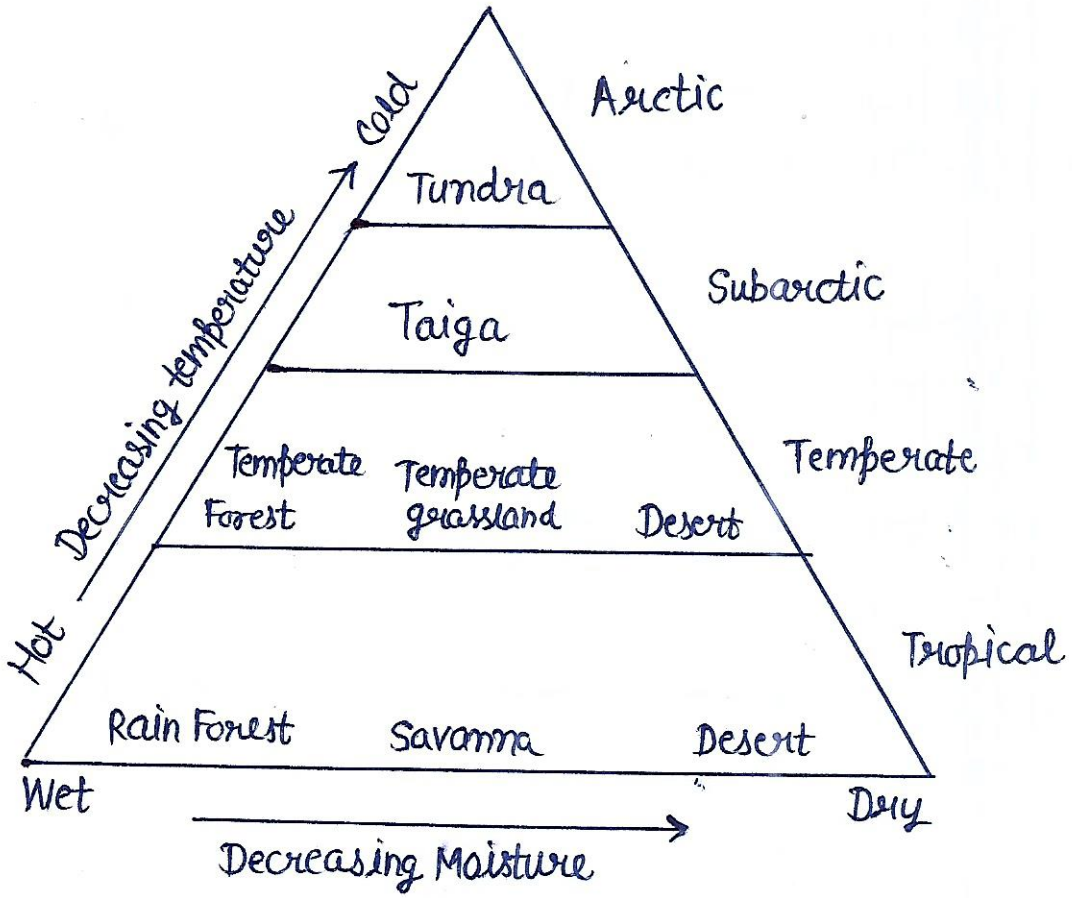
$$^{6}_{3}Li + ^{2}_{1}H \longrightarrow 2\,^{4}_{2}He + 22.2\ MeV$$

UNIT- 5

Global and Regional Climate

Elements of Whether and Climate

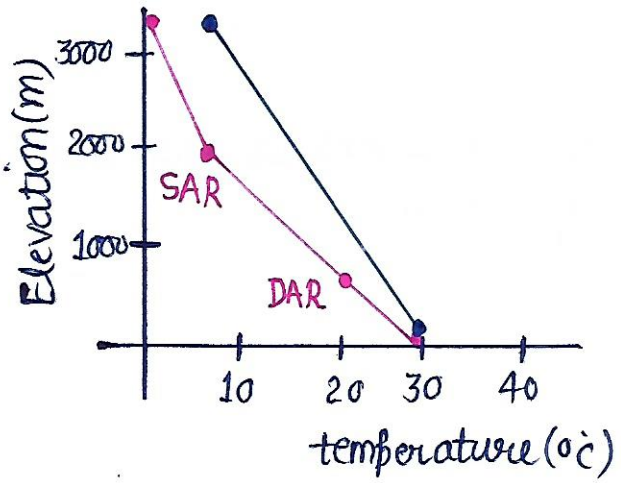
There are many elements that make up both the wheather and the climate of a geographical location. The most significant of these elements are temperature, atmospheric pressure, wind, solar irradiance, humidity, precipitation, and topology.



- Climatology, studies climate change.

Stability and Verticle motion of Air -

Adiabatic temperature change is an important factor in determining the stability of the air. We can think of air stability as the tendancy for air to rise or fall through the atmosphere under its own power. Stable air has a tendancy to resist movement. On the other hand, unstable air will easily rise. The tendency for air to rise or fall depends on the adiabatic and environmental lapse rates.

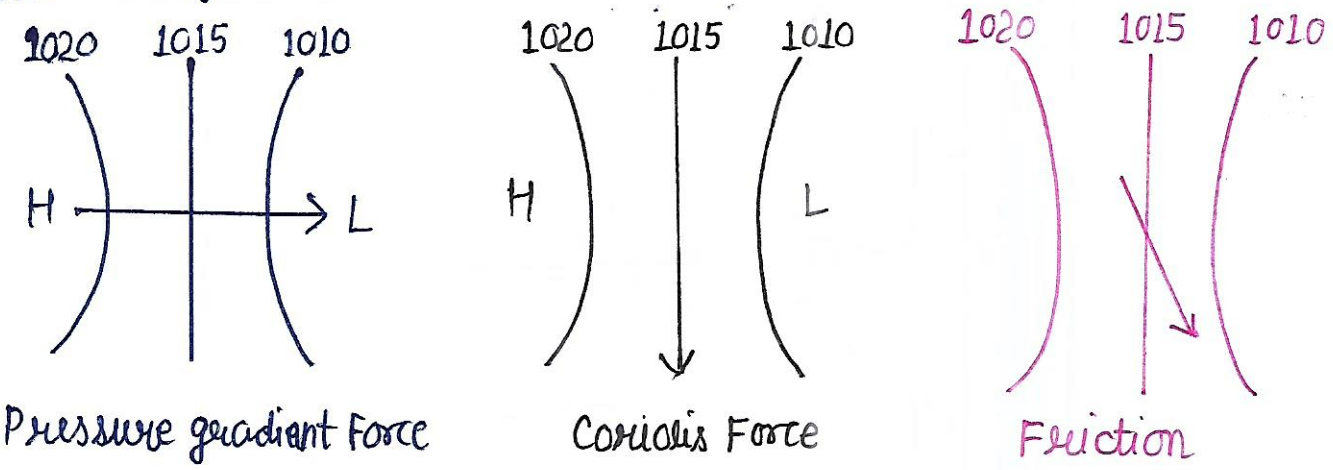


Verticle Air motion –

The important aspect of the airflow around High to Low pressure centers is the verticle movement of the air. It is usually at much lower than horizontal wind. At low pressure centers, air rises. At high pressure centers, air sinks towards the surface.

Horizontal motion of Air–

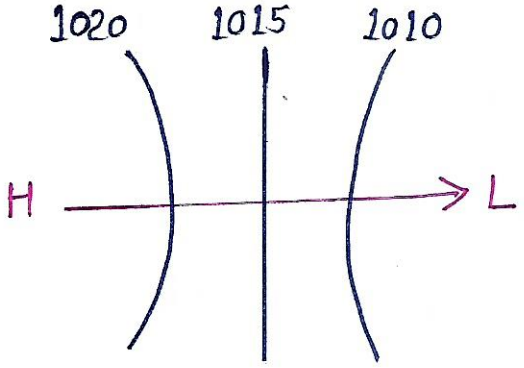
The Horizontal motion of Air is called wind. Wind is initiated because of Horizontal pressure difference. Air flows from areas of higher pressure to lower pressure due to pressure gradient force (PGF). The direction of the wind is influenced by the coriolis force (COF) and friction between the Earth's surface and air. The velocity of the wind is controlled by the difference in pressure between the two regions and the distance separates them.



Wind = Pressure gradient Force + Coriolis Force + Friction

Pressure gradient Force-

The Pressure gradient Force is the Force that results when there is a difference in pressure across a surface.



Pressure gradient Force

Consider a cubic parcel of Fluid with density P, height dz and surface area dA.

$$\text{mass } m = P \cdot dA\, dz \qquad [\because \text{Use} \downarrow\ ma = F]$$

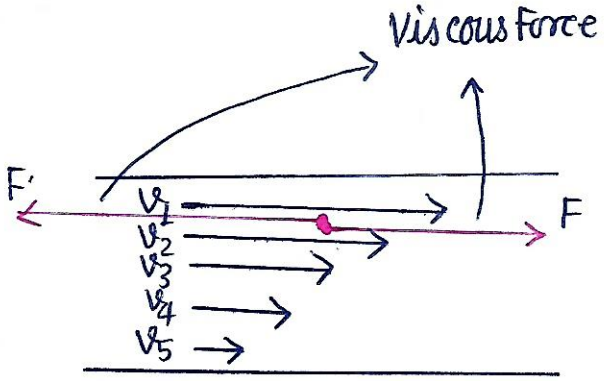
$$F = -dP \cdot dA$$

$$F = PdAdz \cdot a$$

$$a = \frac{-1}{P}\frac{dP}{dz}$$

$$\boxed{\vec{a} = \frac{-1}{P}\vec{\nabla} P}$$

Viscous Force-

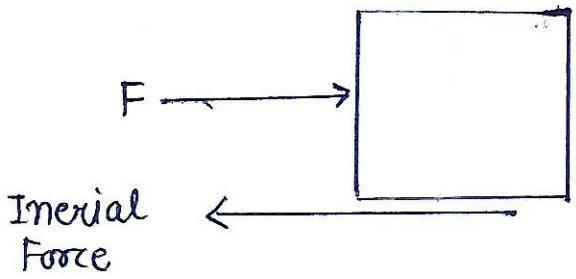


The viscous force is the force between a body and a fluid moving past it, in a direction so as to oppose the flow of the fluid past the object. In particular, the force acts on the object in the direction in which the fluid is moving relative to it. Viscous force is an analogue in Fluids of the force of friction.

Inertial Force -

Inertial Force, also called Friction Force, any force invoked by the observer to maintain the validity of newton's second law in a reference frame.

Inertia is the resistance of any physical object to any change in its velocity.



Raynolds Number -

The Raynolds Number (Re) is an important dimensionless quantity in Fluid mechanics used to help predict flow pattern in different fluid flow situations.

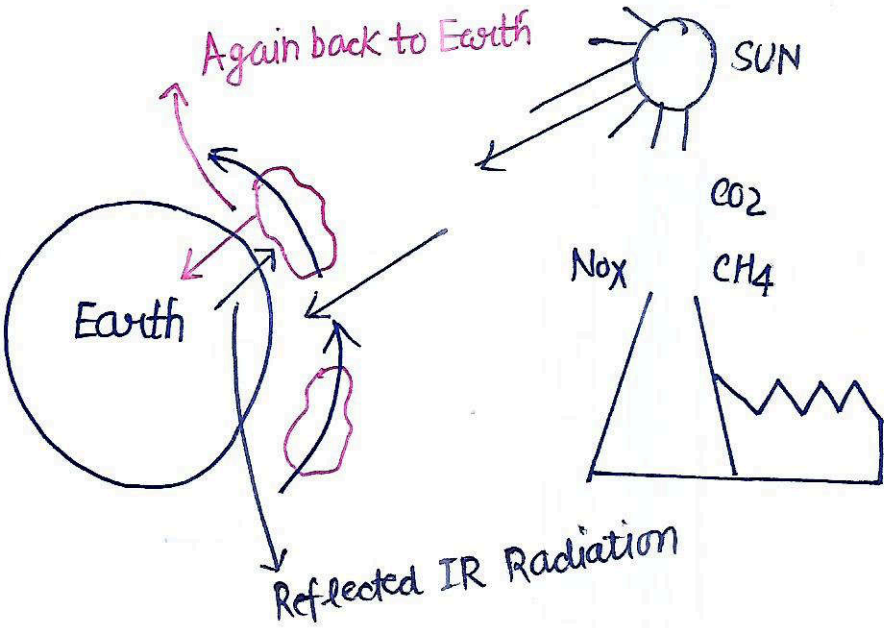
The Raynolds number is the Ratio of inertial force to viscous force within a fluid which is subjected to relative internal movement due to different Fluid Velocities.

$$Re = \frac{\rho u L}{\mu} = \frac{uL}{\nu}$$

- ρ → density
- u → speed
- L → Linear dimension
- μ → dynamic velocity
- ν → Kinematic Viscosity

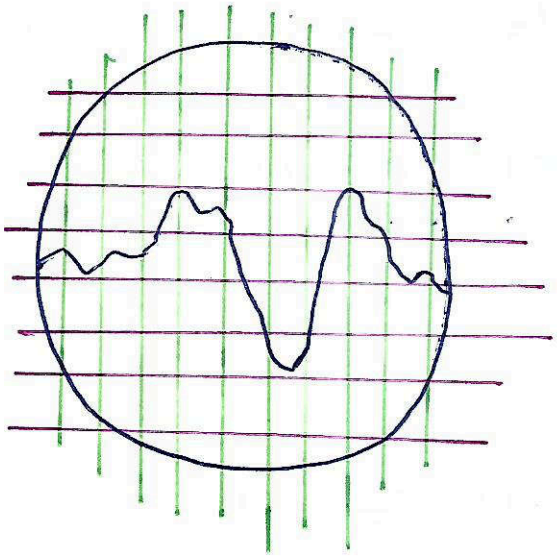
Enhanced Greenhouse Effect –

The disruption to Earth's climate equilibrium caused by the increased concentration of greenhouse gases has led to an increase in the global avarage surface temperature. This process is called Enhanced Greenhouse Effect.



Global Climate Model –

A global climate model (GCM) is a complex mathematical representation of the major climate system components (Atmosphere, land surface, ocean, and sea ice) and their interactions.



- Algorithms for these boxes.

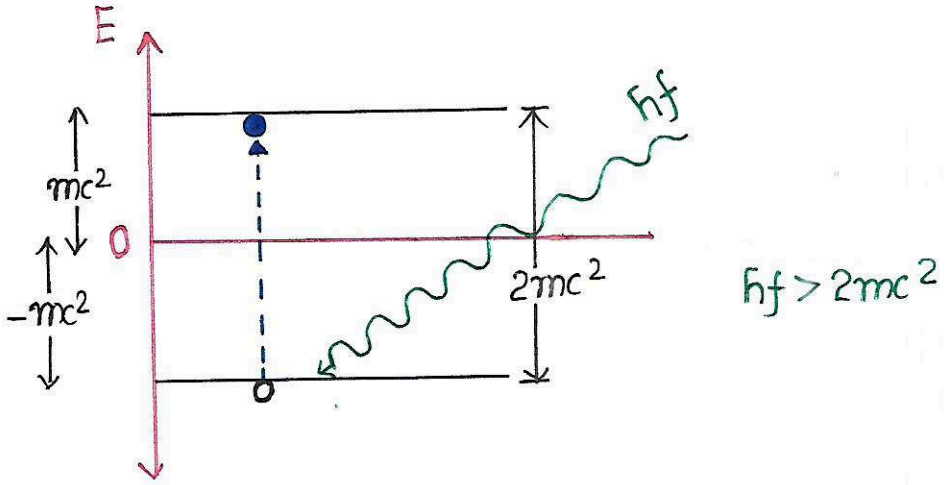
PRESENTATIONS

→ In 1928, Dirac coupled quantum Mechanics and relativity.

$$E^2 = p^2c^2 + m^2c^4$$

$$E = \pm\sqrt{p^2c^2 + m^2c^4}$$

↑ Positive and Negative Energy



$e^+ + e^- \longrightarrow \gamma$- Radiation

What happens if a photon of light with just the right energy ($hf > 2mc^2$) stricks an e^- with a negative energy state? This e^- can now transition to a higher energy (+ve) state leaving an empty spot. We call it empty spot the antiparticle of the e^-, also known as positron. This positron can now move around space. Although the positron is stable when it is by(is) itself.

→ Antimatter–
Since, Antiparticles exist, we can presume that the Antiphoton, Antineutron and positron can combine to form antiatoms, material composed of Antiatoms is known as Antimatter.

Nature ← Conservation of momentum
Conservation of energy
Conservation of massess.

→ e^-, p^+ conserved

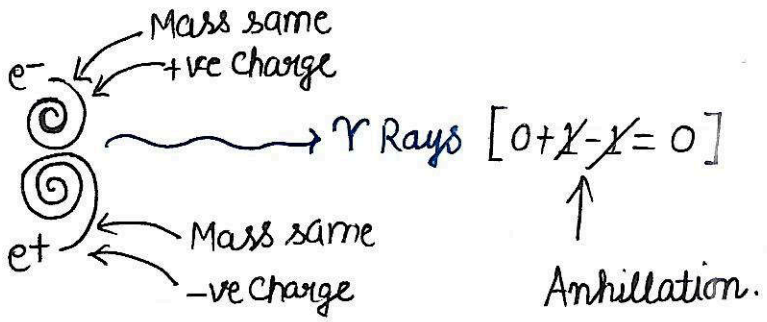
Net Charge of Universe = 0

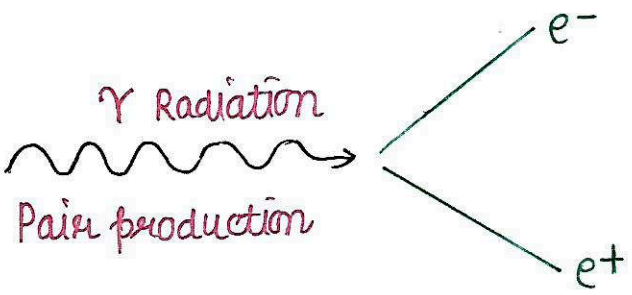
- Charge can only be transferred.

$$E = mc^2$$



Universe's e^- negative energy destroyed. Positive energy became higher. Assymmetry developed.





The Electron:

e^-

β^-

mass = 9.1×10^{-31} kg

C = -1.6×10^{-19} C

Lepton number = 1

The Positron:

e^+

β^+

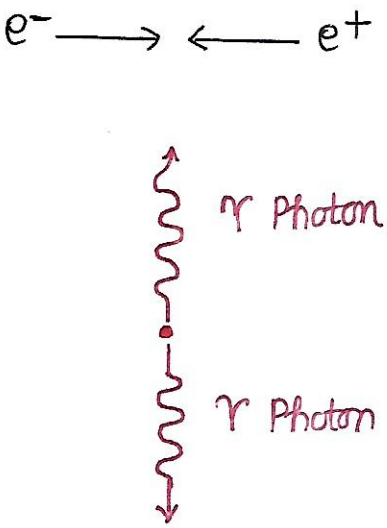
mass = 9.1×10^{-31} kg

C = $+1.6 \times 10^{-19}$ C

L = −1

The Positron is the Antiparticle of e^-.

Annihilation :

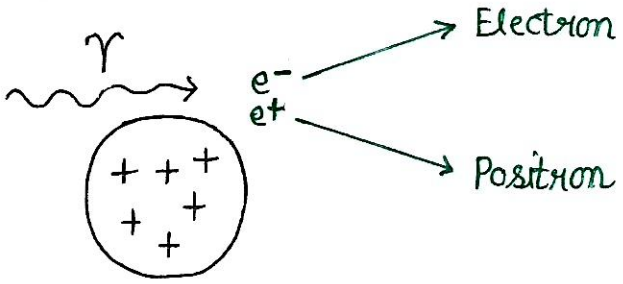


- The e^- and e^+ Annihilation.
- They distroy each other ; their mass has teurned into energy

$$e^- + e^+ \longrightarrow 2\gamma$$

Particle + Antiparticle $\longrightarrow$ Energy

Pair Production :



Energy $\longrightarrow$ Particle + Antiparticle

$\gamma \longrightarrow e^- + e^+$

A gamma photon turns into a particle-Antiparticle pair. The Photon must have a minimum energy of $2m_e c^2$.

Purpose of the nucleus? → To conserve momentum

Why?

The Proton:

p^+

mass = 1.67×10^{-27} kg

C = 1.6×10^{-19} C

Baryon number = 1

The Antiproton:

$\bar{p}$

mass = 1.67×10^{-27} kg

C = $-1.67 \times 10^{-27/19}$ C

Baryon number = -1

The Neutron:

n

mass = 1.67×10^{-27} kg

charge = 0

Baryon number = 1

The Antineutron:

$\bar{n}$

mass = 1.67×10^{-27} kg

Charge = 0

Baryon number = -1

A Particle and its Antiparticle:

- have same mass.
- have opposite charges (If charged).
- have opposite charge family number.
- Annihilation to produce 2 Photons.
- Can be created as a pair (Pair Production).

For every particle, there is a corresponding Antiparticle.

Special Cases:
However the photon, graviton, Z and neutral pion (π^0) are their own Antiparticle

→ If the symbol for a particle include the charge-sign, the Antiparticle has the sign reversed, e.g.

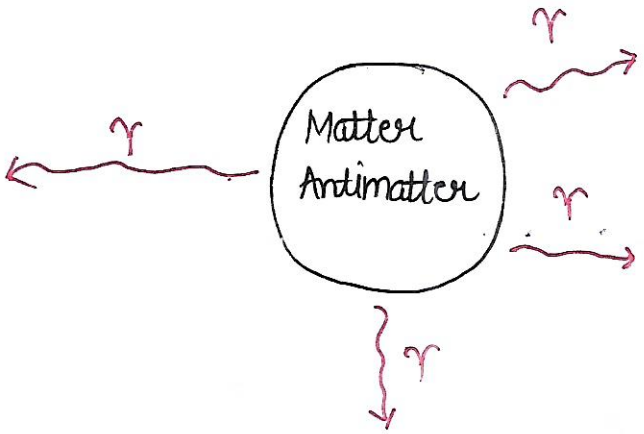
e^- and e^+

μ^- and μ^+

→ Normal matter is material made of normal particles.
e.g. p^+, e^-, n^0

→ Antimatter is made of Antiparticles.
e.g. $\bar{p}, e^+, \bar{n}$

The universe appears to be made from normal matter with very little Antimatter
Why?



We can make Antiparticles in Particle Accelerators.
e.g.

$$P + P \longrightarrow P + P + P + \bar{P}$$

Positronium is an e^- and e^+ in orbit around their center of mass.

- Antihydrogen is a positron is orbit around an antiphoton. Tiny amount has been made.

$$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi$$

↓

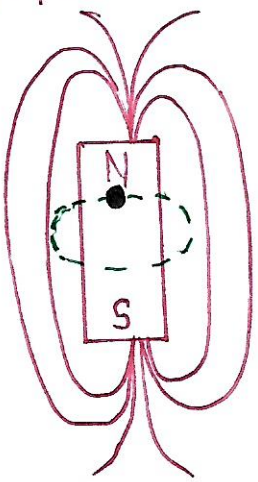
Incompatible with Einstein theory of Relativity

- Only works for slow moving objects.
- Spin ↑ ↓ (Wolfgang Pauli).
- Degree of freedom. ↑ ↓.
- Spinors
- Dirac rewrite Quantum theory + Relativity.

$$\Psi = \begin{bmatrix} \uparrow \\ \downarrow \\ \uparrow \\ \downarrow \end{bmatrix}$$

$$i\hbar\gamma^\mu \partial_\mu \Psi - mc\Psi = 0$$

Motion of e^- at any speed



dirac sea

2. Biological Hazards or Bio-nuclear Weapons

#Hazards-

A Hazard is an agent which has potential to cause harm to a target. Hazards can be both natural or human inducted.

Natural
- Cyclone
- Tsunami
- Earthquakes

Sometimes Human is responsible for them.

Human Inducted
- Flu
- SARS virus
- Ebola
- Dengue
- SARS COV-2

Always we humans are responsible.

Types

- **Chemical**
  - Gases
  - dusts
  - Fumes
  - Vapours
  - Liquids
- **Physical**
  - Noise
  - Viberation
  - Electrical
  - Fire
  - Explosion
  - Machinery
- **Ergonomic**
  - Poor design
  - Workstation
  - Manual Handling
  - Overuse Injuries
- **Radiation**
  - X-Rays
  - Gamma Rays
  - Microwave
  - UV
  - IR
  - LASER
  - Antiparticles
- **Psylogical**
  - Stress
  - Shift work
  - Harassment
  - Discrimination
- **Biological**
  - Infections
  - Becterial,
  - Viruses
  - Parasitic

#Biological Hazards-

- Human & Blood Products
- Animals Waste and Sharp Waste

- Introduction
- Types
- Entrance for Body
- How these spread
- Levels
- Risk Assessment
- Control and Strategies for Preventation

- A threat to the health of human and animals also.
- Could Include simple microorganism, virus, toxin.

Conclusion :-

These things are good as well as bad. Bectrias like curd or yogart, you eat. We treat various infections using virus vaccine which keep balanced amount of virus.

3. Study of Computational Software

MathWorks® MATLAB

#Alternative Softwares !

- GNU Octave
- SciLab
- Maxima
- Julia
- Mathematica

#System Requirements !

- Window 10
- 29GB Harddrive space for software
- 8GB
- 1GB GPU
- Minimum i3

#What is MATLAB ?

- Getting started with MATLAB Videos.

- MATLAB Basics
- Arrays : Unlocking potential
- Loop and Execution
- Script and Function
- Program Output

→ Command Windows
→ Workspace → Variables

edit myfirstfile.m

Indian captain, Mahendra Singh Dhoni, hits a ball with initial velocity of 35 m/s and angle of 45°.

Prerequirement -
Linear Algebra
Numerical Methods

$v_{net} = 35$

$u_0 = v_{net} \cos(\pi/4)$

$v_0 = v_{net} \sin(\pi/4)$

$u' = -ku$

$v' = -g$

$x' = \frac{dx}{dt} = u'$

$y' = \frac{dy}{dt} = v$

Input Block

Computational

Output Blocks

%% Comments ——→ Just for our understanding

Param.g = 9.81; ——→ Assignment

u₀ = 35*Cos(Pi/4) ——→ Math (Expression)

[tout, Xout] ——→ Calling a function

Plot ——→ Calling a function

Loops:

Iteration

↓

Iteration is the repition of a process in order to generate a sequence of outcomes.

$$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 0.1 \end{bmatrix} = \begin{bmatrix} 2a & 0.1b \\ 2c & 0.1d \\ 2e & 0.1f \end{bmatrix}$$

Scale

- Show them programmes.

Learning Physics through Simple Experiments

- Optics Experiments
- Mechanics Experiments
- Electricity / Magnetism Experiments
- Oscillations and Waves Experiments

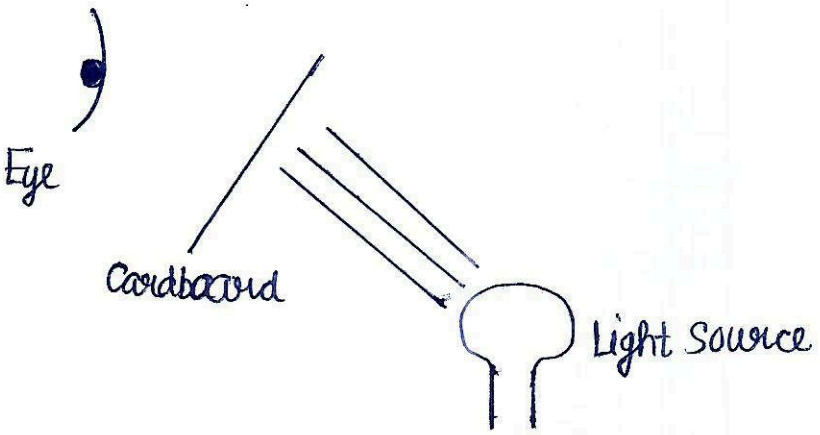
By Prof. H. C. Verma
Sharp Programme
CCE, IIT Kanpur

From Shadow of Differection !

OPTICS

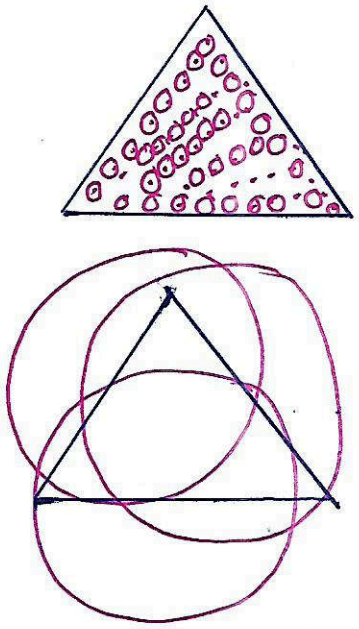
Light → Learning → Theory Developed.

- Light travels in straight lines (Approximation in Homogenous Medium).



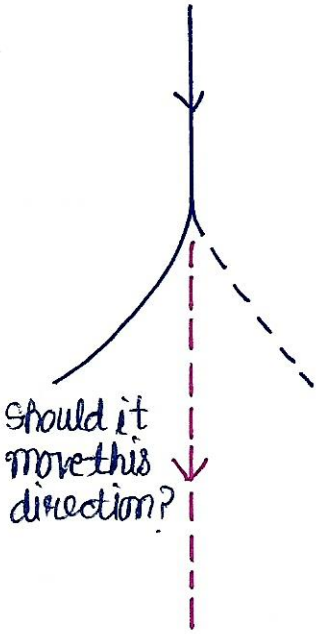
- Bulb shadow Experiment.
- Sunlight, cardboard shapes +, ○, □.

Wave Nature of light.

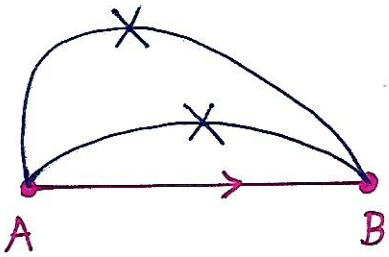


- LASER Light and thin wire, Pen Experiment.

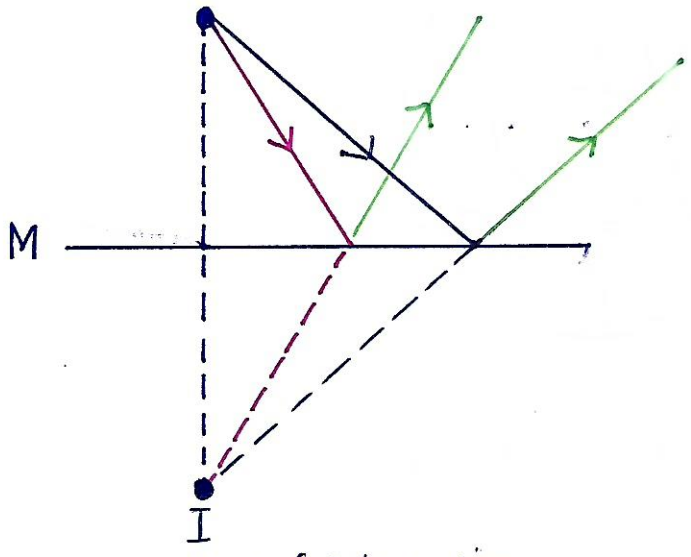
From Fermat Principle to Reflection



Reality
(Explained by Symmetry)



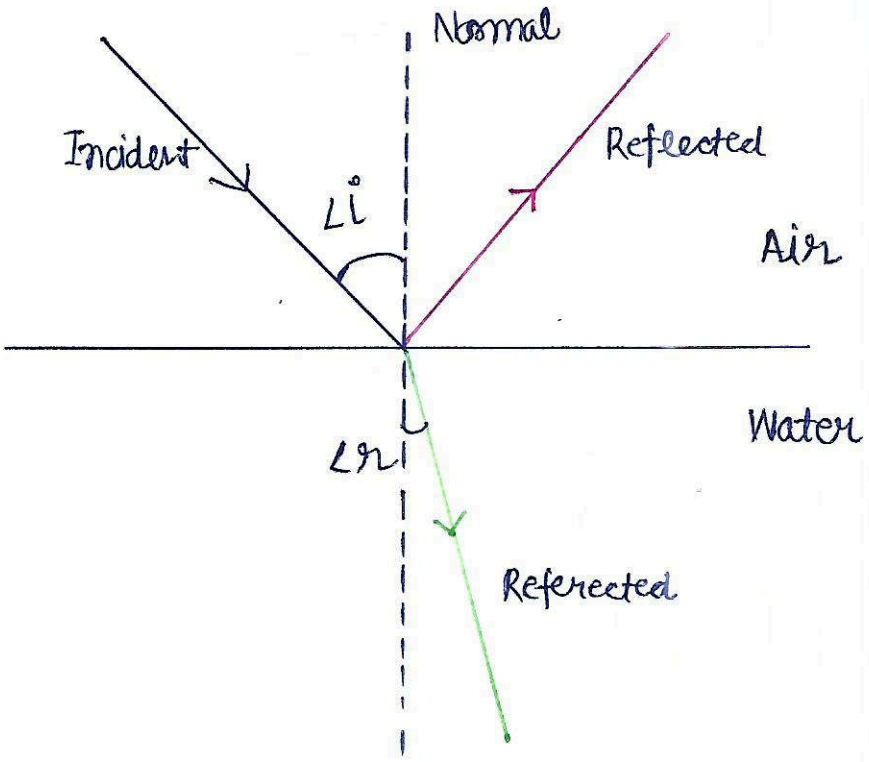
Fermat's principle in a given situation (optical elements) Light chooses a path for going from one given point to the other given point, such that it takes least time



Point of Intersection

- Inversion depends on viewer.

#From One Medium to Another | Refraction-



$$\frac{\sin i}{\sin r} = \frac{\text{Speed of Light in Air}}{\text{Speed of Light in Water}}$$

Total Internal Refeection = total incident Reflected due to critical Angle.

- Exp. using Magnify Lens, screen and Bulb which shows wavelengths.

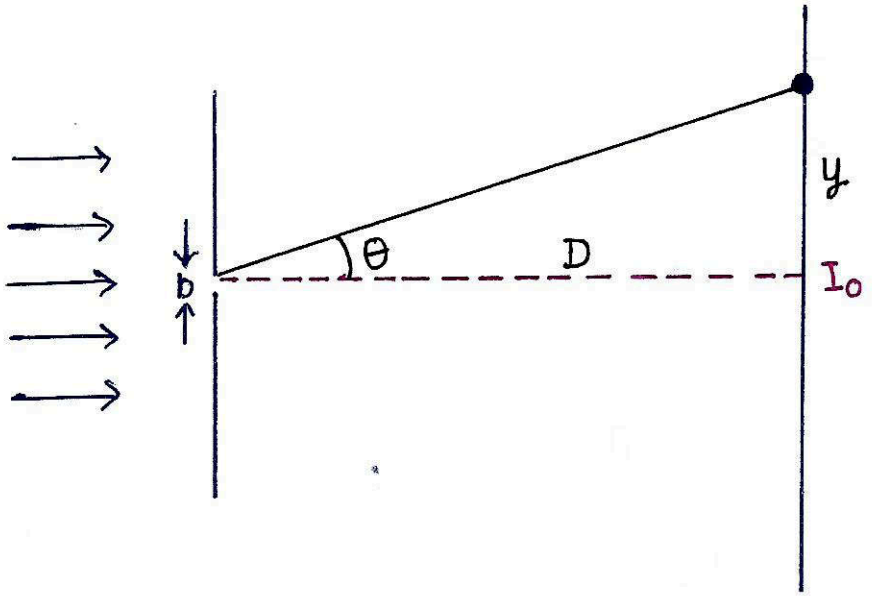
#From Differection to Interference -

- Light is EM Wave.

$$\vec{E} = \vec{E}_0 \sin(\vec{k}\cdot\vec{r} - \omega t)$$

$$|\vec{k}| = \frac{2\pi}{\lambda}$$

$$\omega = \frac{2\pi}{T} = 2\pi\nu$$

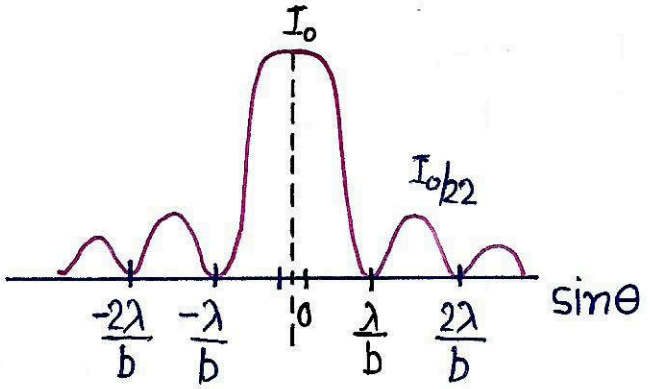


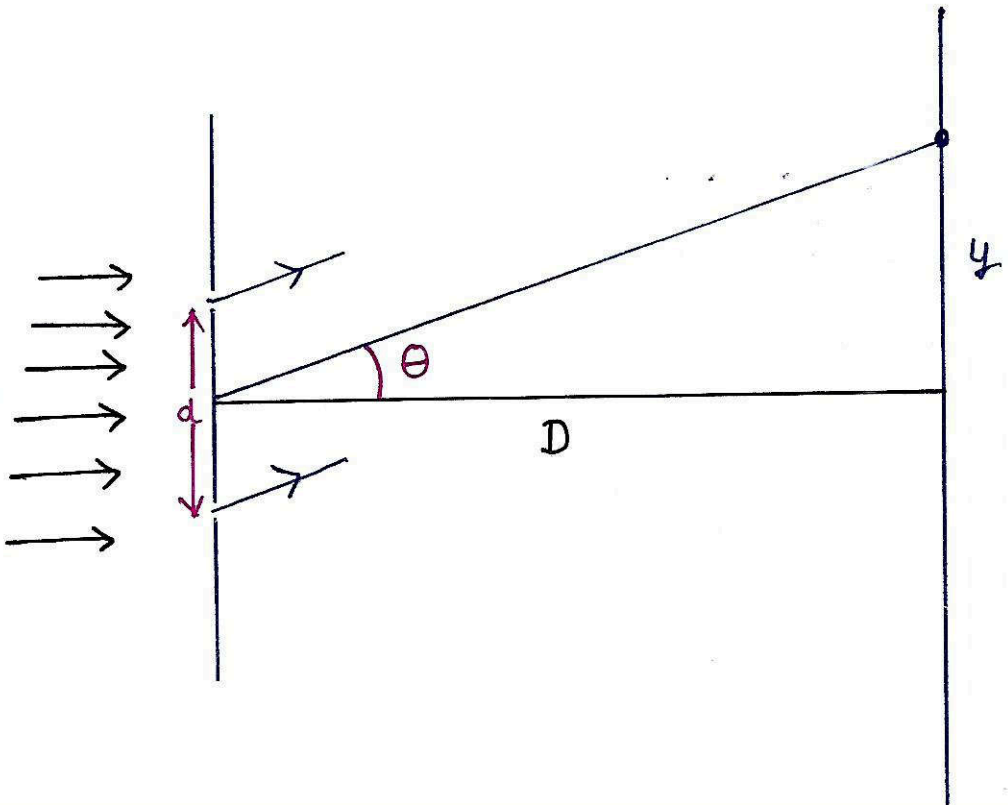
$$\tan\theta = \frac{y}{D}$$

$$\approx \sin\theta$$

$$I = I_0 \frac{\sin^2\beta}{\beta^2}$$

$$\beta = \frac{\pi}{\lambda} b \sin\theta$$

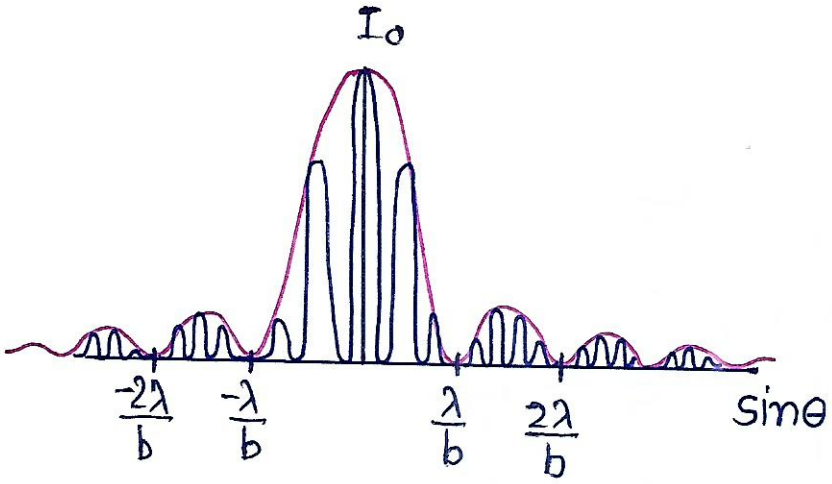




$$I = I_0 \frac{\sin^2\beta}{\beta^2} \cos^2\gamma$$

$$\beta = \frac{\pi}{\lambda} b\sin\theta$$

$$\gamma = \frac{\pi}{\lambda} d\sin\theta$$



Unpolarized to Polarized Light-

- Light is EM Wave, consisting of $\vec{B}$, $\vec{E}$.
- $\vec{E}$ oscillating in the plane perpendicular to the direction of propagation and $\vec{B}$ also in the plane perpendicular to the direction of propagation, and $\vec{E}$.
- $\vec{E}$ oscillating in the plane parallel to the direction when passes through Polarizer.
- Unpolarized (Sheet) light has no change in Intensity.

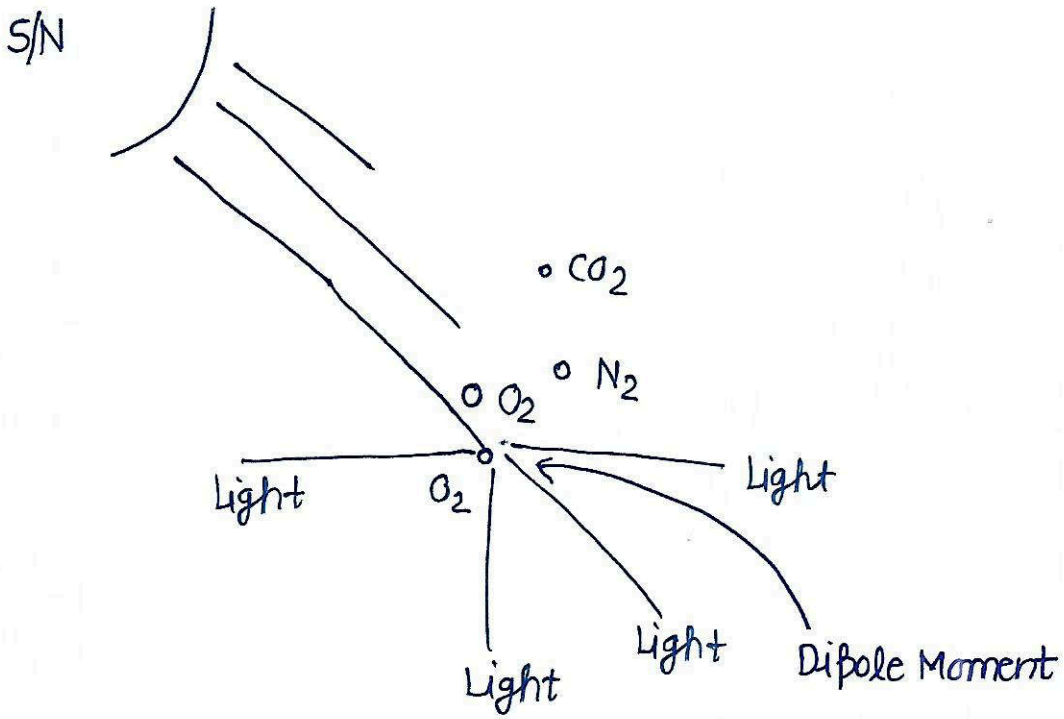
→ Experiments using

- Polarizers
- LED
- SUNLIGHT
- Laser
- Blue LED

#From Blue Sky to Coloured Cello Tape-

- Scattering of Light
- Optical Activity
- Birefrengence

In Morning, you see everything is lightning up but the light is coming from East but why is everything lightning up in every direction.

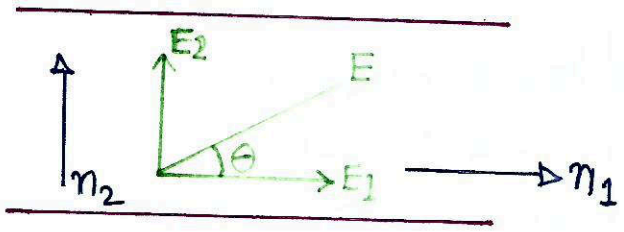


Rayleigh Scattering $I \propto \frac{1}{\lambda^4}$

→ Kiral Molecules
→ Sugar Activity using optical Polarizer
→ Birefrengence

Tape Layer with two polarizers, Tape Layers Increase

B
Y
G



$$n_1 t - n_2 t$$

$$\Delta\phi = \frac{2\pi}{\lambda}(n_1 - n_2)t$$

- Tape is Birefrenge material.

MECHANICS

Let us be Non-Inertial-

- Inertial Frames are those which are at rest or constant Velocity.
- You can use Inertial frame for Newton's Mechanics Law $F=ma$.
- For Non-Inertial Frame, $F=ma$ doesn't work.

$$\vec{F} \neq m\vec{a}$$

$$m\vec{a} = \vec{F} + \vec{F}_p \leftarrow \text{Pseudo Force}$$

$\vec{F}$ ↑ Resultant of all the forces applied to the particle

$$\vec{F}_p = m\vec{a} - \vec{F}$$

- Psudo Force balance acceleration and works in opposite direction.

a_o = acceleration of the non-inertial frame with respect to the lab

$$\vec{F}_p = -m\,\vec{a}_o$$

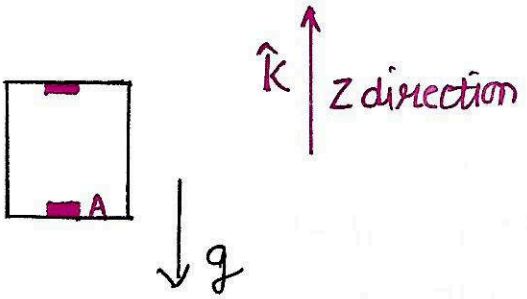
m ↑ mass of the particle

$$-m\vec{a}_o = m\vec{a} - \vec{F}$$

$$+m\vec{a}_o + m\vec{a} = F$$

$$\boxed{\vec{F} = m(\vec{a} + \vec{a}_o)}$$

- Magnets Experiment
- Train Experiment



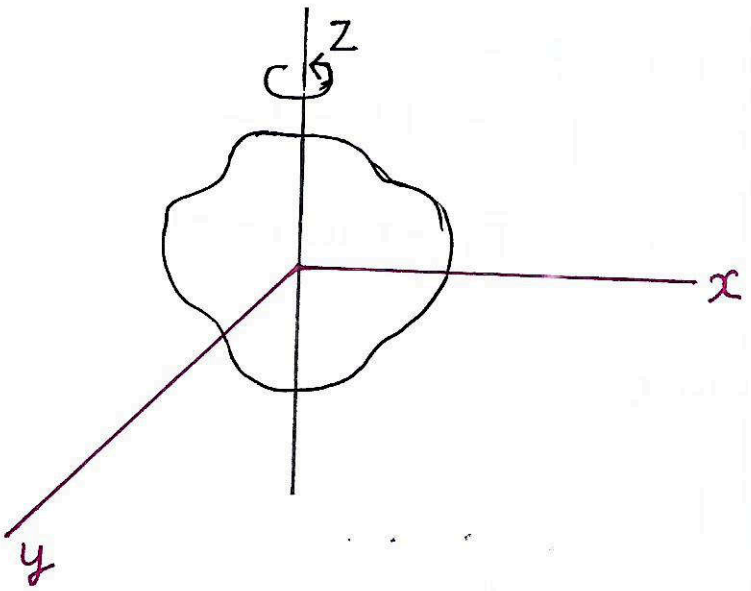
Forces on the magnet A

$$mg(-\hat{k}) + N\hat{k} + F_{mag}\hat{k} + mg\hat{k} = m\vec{a}$$

- $mg(-\hat{k})$: Weight
- $N\hat{k}$: Normal Force due to the bottom
- $F_{mag}\hat{k}$: Magnetic attraction due to the upper weight
- $mg\hat{k}$: Psudo Force
- $N\hat{k} + F_{mag}\hat{k}$: dominating

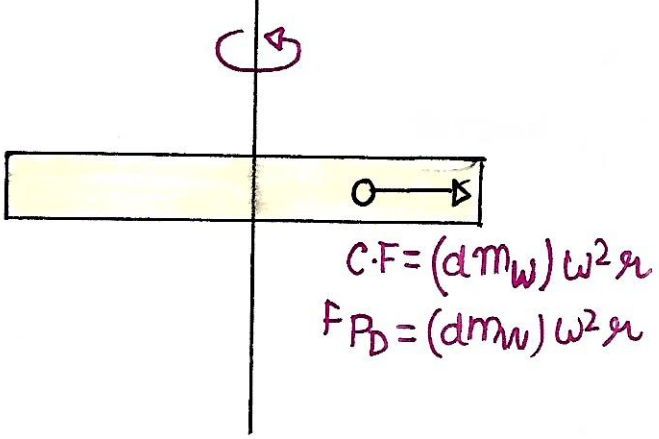
Sometimes it can called Weightlessness.

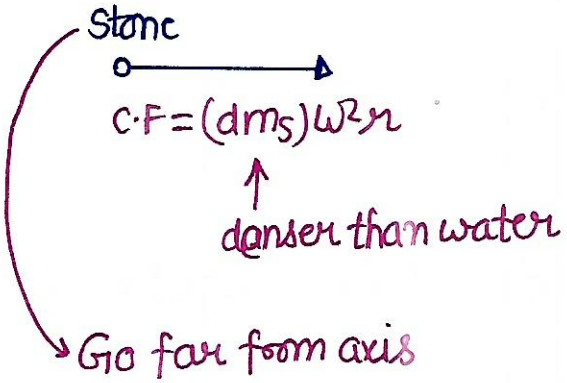
Take things away : Centrifugal Force -

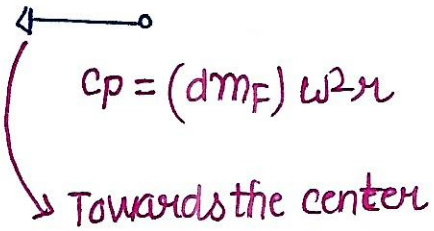


$$F_{CF} = m\omega^2 r$$

- m: mass of the particle
- ω: Angular speed of the frame
- r: Distance of the particle from the axis of rotation of the frame

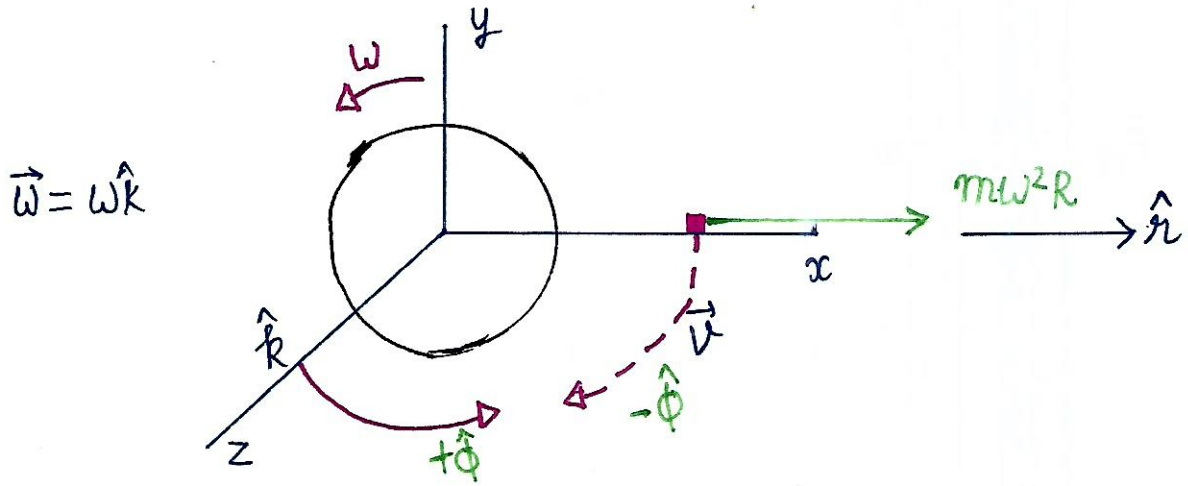






- density plays role here.

#Deflects things sideways ! Coriolis Force–



$\vec{\omega} \times \vec{v} = \omega \hat{k} \times \omega R(-\hat{\phi})$

$\vec{\omega} \times \vec{v} = \omega^2 R \hat{r}$

$m\vec{a} = \vec{F} = \vec{F}_{real} + \vec{F}_{pseudo}$

$m\vec{a} = m\omega^2 R \hat{r}$

$m\omega^2 R(-\hat{r}) = m\omega^2 R \hat{r}$

There is another force.

$m\omega^2 R(-\hat{r}) = m\omega^2 R\hat{r} + (-2m\omega^2 R\hat{r})$

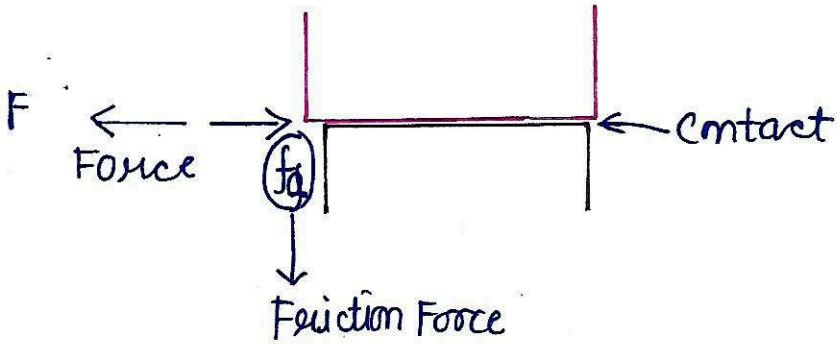
$\Downarrow$

$-2m(\vec{\omega} \times \vec{v})$

$\swarrow$

Coriolis Force

Be with me ! Friction-



* Liquids, gases also exerts Friction force.

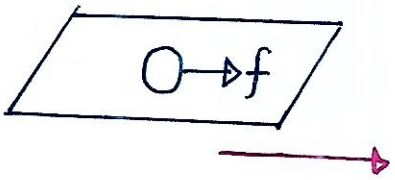
⇒ Card has length L
Mass of the card M
Mass of the coin m
Friction coefficient b/w card and coin μ
Force during flicking F
Duration of Flicking Δt

→ Coin Card

During the flicking

$$(\vec{F} - f)\Delta t = Mv$$

Neglected ↑



$$v = \frac{F\Delta t}{M}$$

$$T = \frac{L}{v} = \frac{LM}{F\Delta t}$$

→ card

$$f = \mu N = \mu mg$$

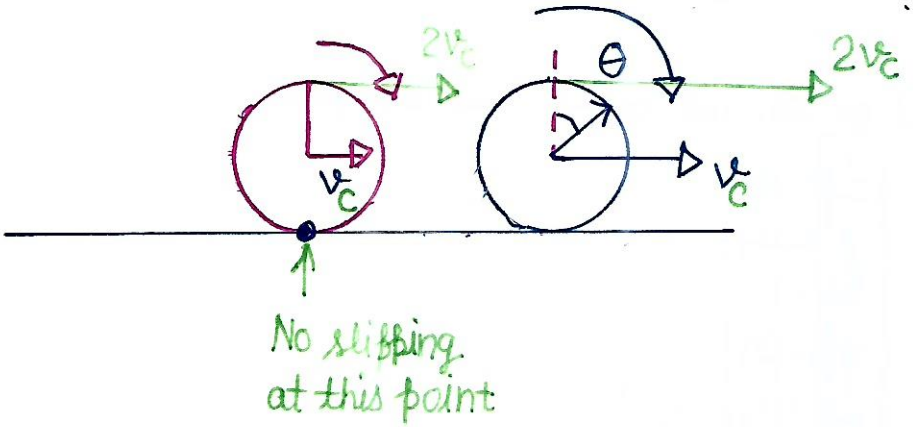
$$a = \frac{f}{m} = \mu g$$

$$\Delta x = \frac{1}{2}aT^2 = \frac{1}{2}\mu g \frac{L^2M^2}{F^2(\Delta t)^2} < R$$

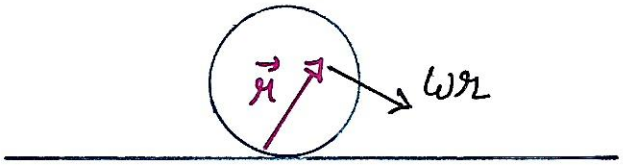
For coin to fall in the tumbler $\Delta x < R$

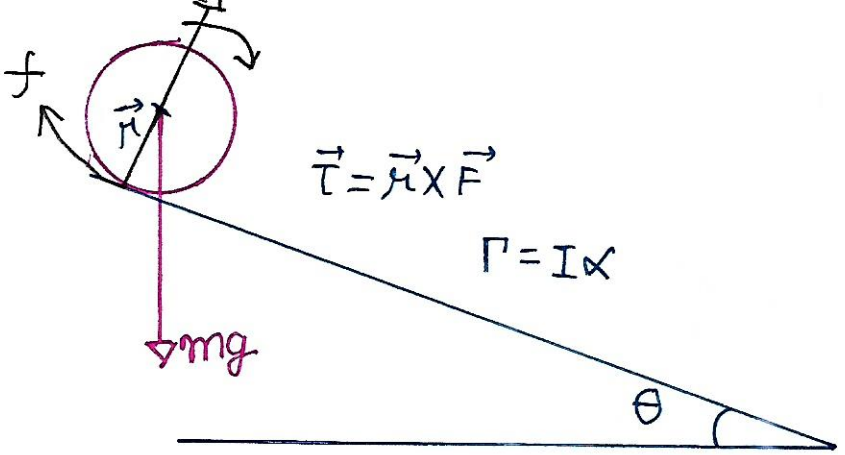
Set the ball rolling-

$$\omega = \frac{\Delta\theta}{\Delta t}$$



$$v_{top} = 2v_c$$





$\alpha = \frac{a_c}{R}$ (condition for No slipping)

$Mg\sin\theta - f = Ma_c$

$f R = \left(\frac{2}{5}MR^2\right)\alpha$

$f = \frac{2}{5}MR\alpha$

$f = \frac{2}{5}MR\frac{a_c}{R}$

$f = \frac{2}{5}Ma_c$

$Mg\sin\theta - \frac{2}{5}Ma_c = Ma_c$

$Mg\sin\theta = \frac{7}{5}Ma_c$

$$\boxed{a_c = \frac{5}{7}g\sin\theta}$$

Make things rotate-

- Rigid Bodies
- Rotational motion

→ Angular velocity $\vec{\omega}$
→ Angular momentum $\vec{L}$
→ Moment of Inertia I
→ torque τ

$$\vec{v}_i = \vec{\omega} \times \vec{r}_c$$

$$\omega = \frac{d\theta}{dt}$$

Angular acceleration $\vec{\alpha} = \frac{d\vec{\omega}}{dt}$

$$\vec{L} = \sum \vec{r}_{ic} \times \vec{p}_i = \sum \vec{r}_c \times m_i \vec{v}_i$$

$$\tau = \vec{r}_{ic} \times \vec{F}_i$$

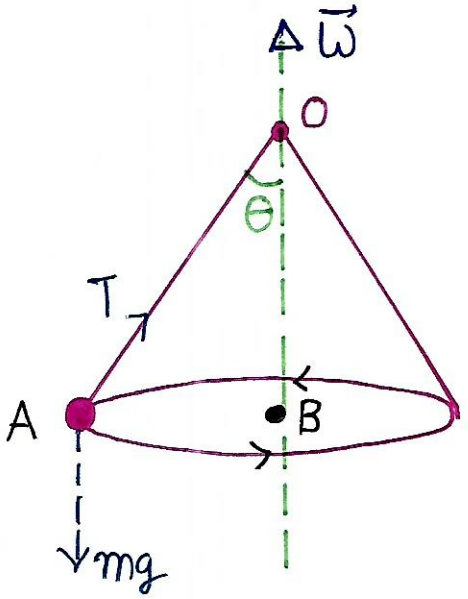
$$I = \sum_i m_i r_i^2$$

$$\left[\begin{array}{l} \vec{\tau} = I\vec{\alpha} \\ \vec{L} = I\vec{\omega} \end{array} \right] \rightarrow \text{(Valid for certain situation)}$$

$\omega = \frac{\Delta\theta}{\Delta t}$ | Vertically Upward $\quad \vec{\alpha} = \frac{d\vec{\omega}}{dt} = 0$

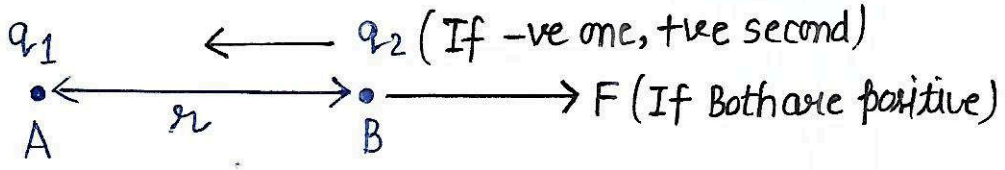
$$\vec{\tau} = \sum \vec{r}_i \times \vec{F}_i = \vec{OA} \times m\vec{g} = (\vec{OB} + \vec{BA}) \times m\vec{g}$$

$$\vec{\tau} = \vec{BA} \times m\vec{g} = Rmg$$



ELECTRICITY AND MAGNETISM

Field, and not charge, exerts force-



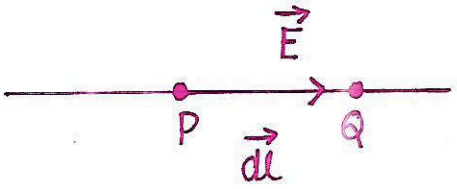
Force on B due to A $F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$

→ charge q_1 produces Electric Field

$$\vec{E} = \frac{q_1}{4\pi\epsilon_0 r^2}$$

→ Force exerted by E-field on q_2

$$\vec{F} = q_2\vec{E}$$

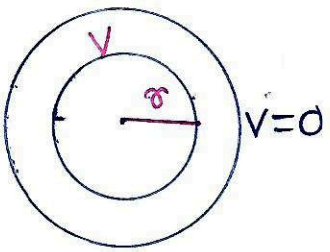


$$V_Q - V_P = -\int E\,dl$$

→ Potential difference = Voltage at terminals

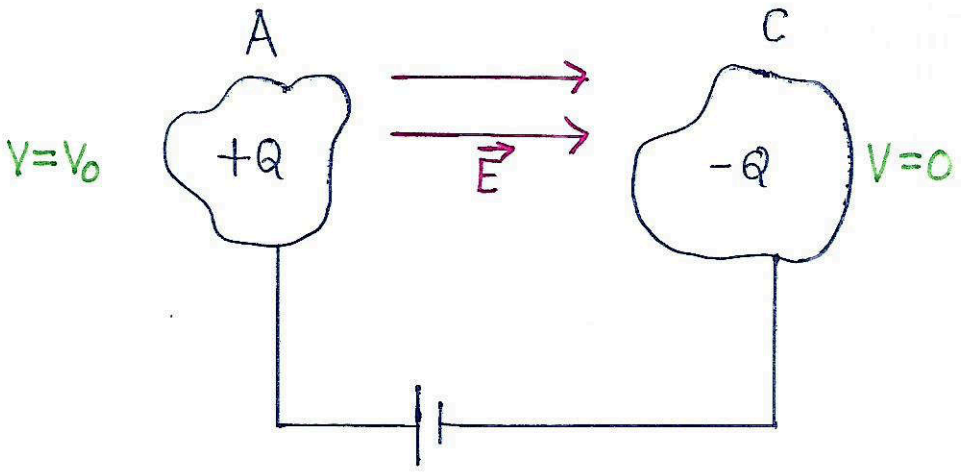
* Experiments can be performed anywhere

* Experiment-



r(cm)	V(Volts)
2.5	4
5	2.4
7.0	1.3

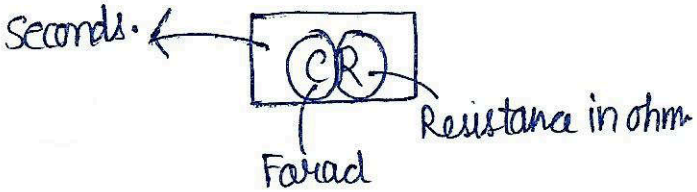
Building up capacity to carry charge-



→ Capacitor
→ Charge on the capacitor = Q
• Voltage on the capacitor = V

$$C = \frac{Q}{V} \quad C/V \text{ or } F \text{ (farad)}$$

Capacitance



A force that never changes speed-

• Magnetic Field is produced by moving charge particles or currents.

$$\vec{F} = q\vec{v} \times \vec{B}$$

• Force due to a current carrying wire:

$$d\vec{F} = I\, d\vec{l} \times \vec{B}$$

Swings to Motors-

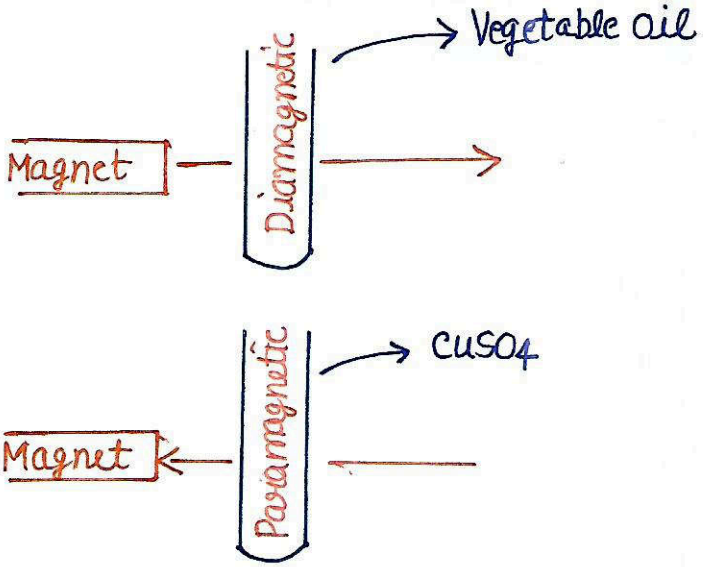
$$d\vec{F} = I\,d\vec{l} \times \vec{B}$$

↓

Inside the magnet

Whom does a magnet attract-

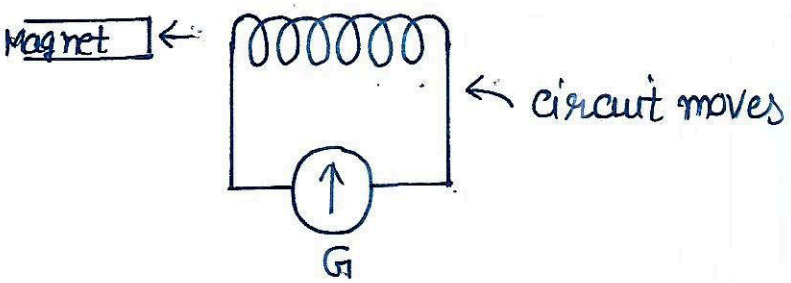
→ Inhomogenous $\vec{B}$ create exert force.



→ When Thermal property dominates in a Feromagnetic material, becomes paramagnetic.

Let us create Electric current-

→ Faraday Law of Electromagnetic Induction.

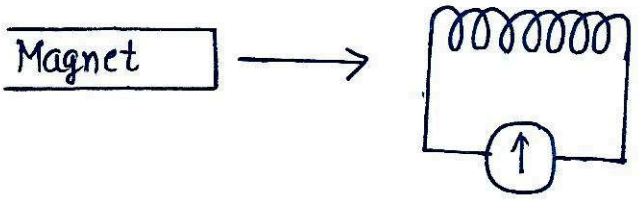


$$\vec{F} = q\vec{v} \times \vec{B}$$

EMF is generated

Φ_B flux of B field $= \int \vec{B} \cdot d\vec{a}$

$$E = \frac{-d\Phi_B}{dt}$$



B-Field changes with time
E-field is created
→ Induced E-field

$$\vec{F} = q\vec{E}$$

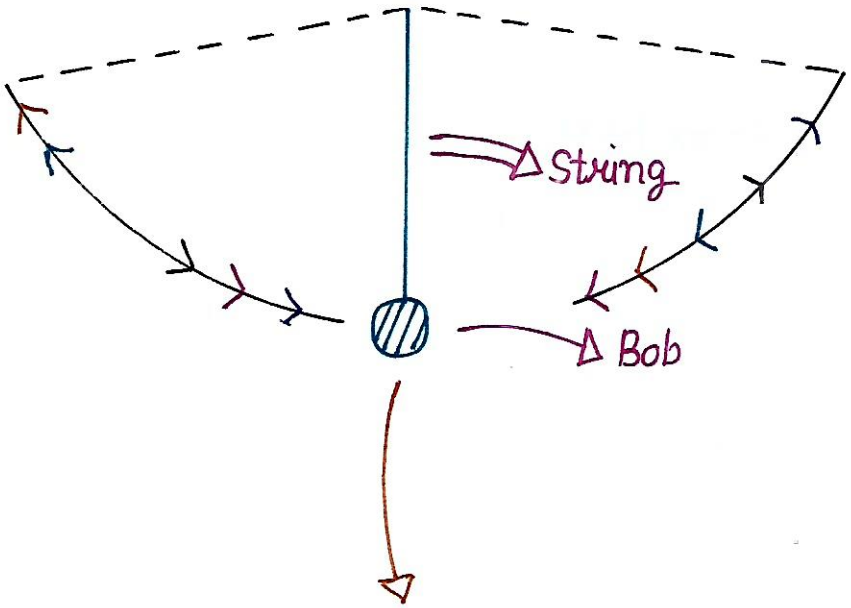
$$\vec{E} = \frac{-d\Phi_B}{dt}$$

- Changing E field
  ↓
  Changing B-Field
  ↓
  Create E field
  ↓
  Current drive → Bulb glow

} Tesla Coil

Oscillations and Wave Experiments

#Let us Oscillate-



$$ma = -kx$$

$$m\frac{dv}{dt} = -kx$$

$$v = \frac{dx}{dt}$$

$$m\frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} = -\frac{kx}{m}$$

$$\boxed{\frac{d^2x}{dt^2} = -\omega^2 x}$$

$$\omega = \sqrt{\frac{k}{m}}$$

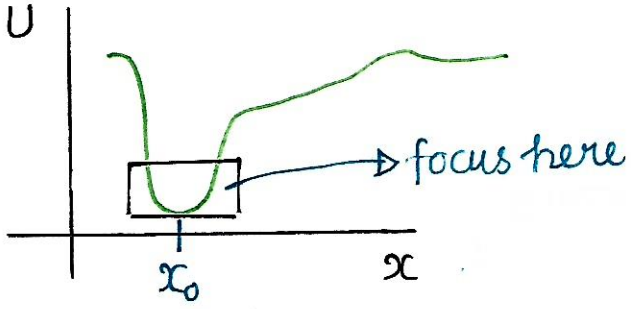
$$x = A\cos\omega t$$

SHM → Simple harmonic motion

$U = \frac{1}{2} K(x - x_0)^2$

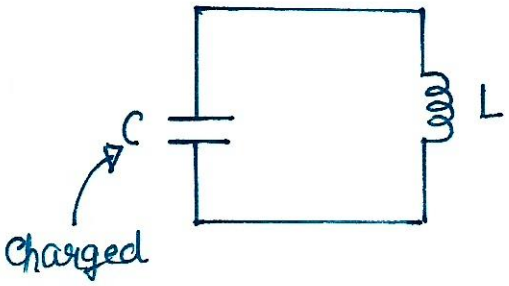
$ma = -kx - bv$

$x = [A\cos\omega t]\, e^{-\beta t}$



$x = A\cos\omega t$

↳ Ideally, Amplitude is constant but in real systems, it decreases with time.



Charges Flow through L

↓

Current built up

↓

$\vec{B}$

↓

$\frac{\partial \vec{B}}{\partial t}$

↓

$\vec{E}$ (Induced)

↓

EMF (OWN)

$$\frac{q}{C} = -L\frac{di}{dt}$$

$$i = \frac{dq}{dt}$$

$$\frac{q}{C} = -L\frac{d^2q}{dt^2}$$

$$L\frac{d^2q}{dt^2} = -\frac{q}{C}$$

$$\frac{d^2q}{dt^2} = -\frac{q}{LC}$$

$$\boxed{\frac{d^2q}{dt^2} = -\omega^2 q} \qquad \left[\because \omega = \frac{1}{\sqrt{LC}}\right]$$

$$q = q_0 \cos\omega t$$
$$i = i_0 \sin\omega t$$

$$Ri + \frac{q}{C} = -L\frac{di}{dt}$$

$$q = [q_0 \cos\omega t]e^{-\beta t}$$

#Let us Resonate-

$$m\frac{dv}{dt} = -Kx$$

$$\omega = \sqrt{\frac{k}{m}}$$ ← frequency

Add some external Forces

$$m\frac{dv}{dt} = -kx - bv + F_0\cos\omega t$$

$$\frac{d^2x}{dt^2} = -\omega_0^2 x$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

$x = A\cos\omega t$

$x = B\cos\omega_0 t$

ω too close to ω_0
A is large

Basic Oscillation

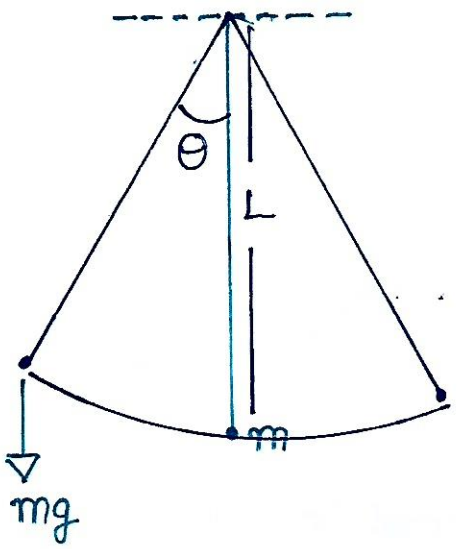
$$L\frac{di}{dt} = -\frac{q}{C} - Ri + \mathcal{E}_0\cos\omega t$$

$\mathcal{E}_0\cos\omega t$ → frequency of External power source

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

- Used in tuned circuits

Simple Pendulam isn't that simple -



$$-mgL\sin\theta = mL^2\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$$

$$\omega = \sqrt{\frac{g}{L}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

$$\left(\because \sin\theta = \theta - \frac{\theta^3}{3!} + \dots \right)$$

↓

Approximation

Small Amplitude

10 Oscillations ⇒ 12.6 Seconds

Large Amplitude

10 Oscillations ⇒ 13.0 Seconds

$$T = T_0\left(1 + \frac{\alpha^2}{16} + \dots\right)$$

$$180^\circ = 3.14^c$$

$$1^c = \frac{180}{3.14} = 57^\circ$$

Mechanics
Synge & Griffiths

From one to many-

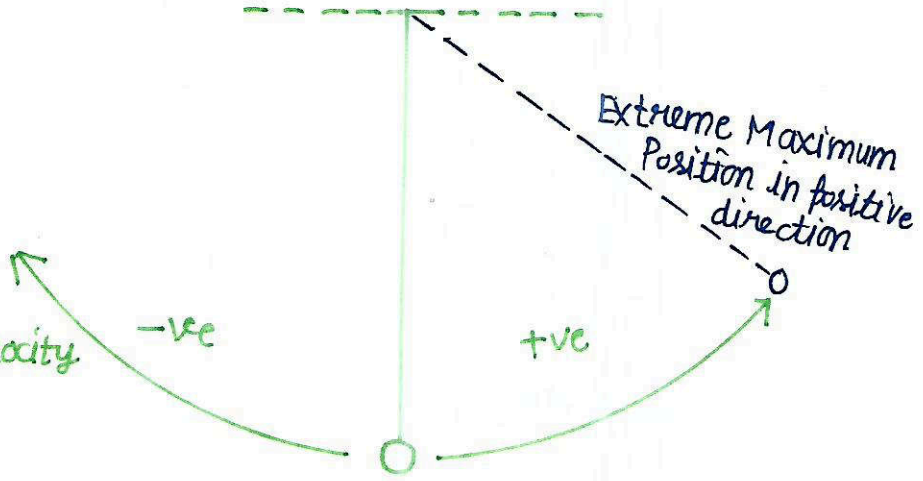
Phase → Phase repeat
State
Phase

Position $\theta = \theta_0 \cos\omega t$

θ_0 → Amplitude

$\omega t = 0$

$\theta = \theta_0$



ωt	θ (Position)	$\frac{d\theta}{dt}$ (Angular Velocity)
0	θ_0	0
$\pi/2$	0	$-\theta_0\omega$

$\frac{d\theta}{dt} = -\theta_0 \omega \sin\omega t$

Angular velocity

Phase is defined.

If you have two pandulam, phase difference occur.

$\theta_1 = \theta_{01}\cos\omega t$

$\theta_2 = \theta_{02}\cos\omega t$

zero phase difference.

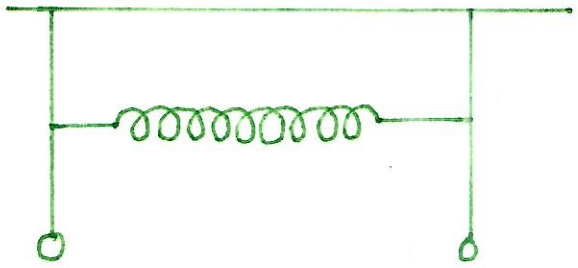
$\theta_1 = \theta_{01}\cos\omega t$

$\theta_2 = \theta_{01}\cos(\omega t + \pi)$

$\frac{d\theta_1}{dt} = -\theta_{01}\omega \sin\omega t$ → Phase

$\frac{d\theta_2}{dt} = -\theta_{02}\omega \sin(\omega t + \pi)$

Phase difference = π



More than one frequencies and time period.

$x_1 = A\cos\omega_1 t$

$x_2 = A\cos\omega_2 t$

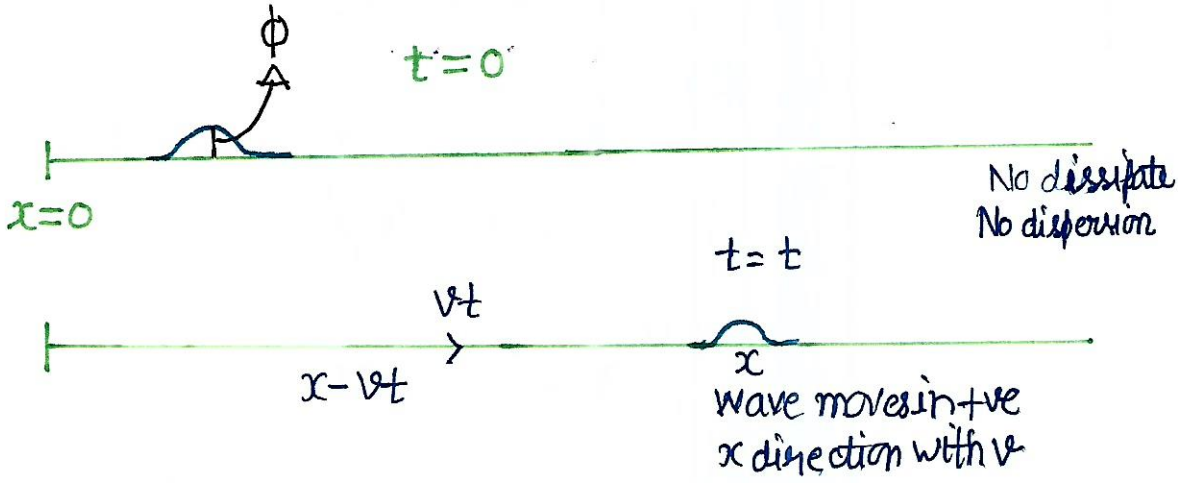
$x = x_1 + x_2 = A\cos\omega_1 t + A\cos\omega_2 t$

$= 2A\cos\left(\frac{\omega_1+\omega_2}{2}t\right)\cos\left(\frac{\omega_1-\omega_2}{2}t\right)$

$= \left[2A\cos\frac{\omega_1-\omega_2}{2}t\right]\cos\frac{\omega_1+\omega_2}{2}t$

$= A'\cos\frac{\omega_1+\omega_2}{2}t$

Let us create wave -



$\phi(x,t)$ $\qquad\qquad$ $\phi(x-vt,0)$

$$\phi(x,t) = \phi(x-vt,0)$$

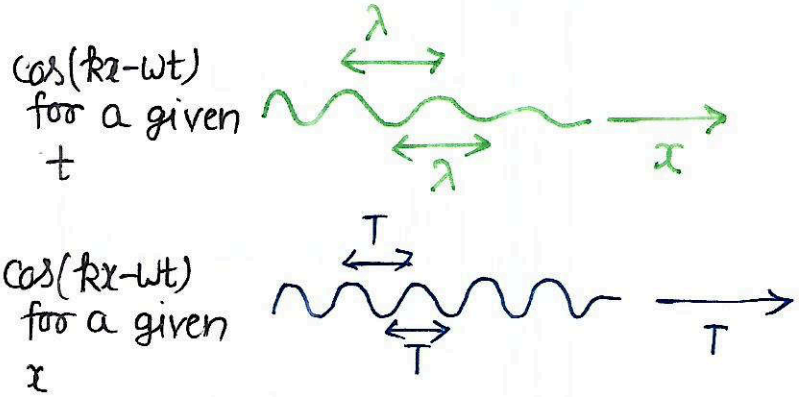
$$\phi(x,t) = A e^{-\left(\frac{x-vt}{a}\right)^2}$$

$$\phi(x,t) = A\left[\frac{x^2}{a^2} + \frac{v^2}{v_0^2} + \frac{t^2}{t_0^2}\right]$$

#Oscillate and Don't oscillate-

$$\xi = f(x-vt)$$

$$= A\cos[k(x-vt)] = A\cos(kx-\omega t) \qquad kv = \omega$$



$$\xi_1 = A\sin(kx-\omega t)$$

$$\xi_2 = A\sin(kx+\omega t)$$

$$\xi_N = \xi_1 + \xi_2 = 2A\sin kx\cos\omega t$$

At places,

$kx = 0, \pi, 2\pi, \ldots$ ⟶ Nodes

$\xi = 0$ Always

At places

$$kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

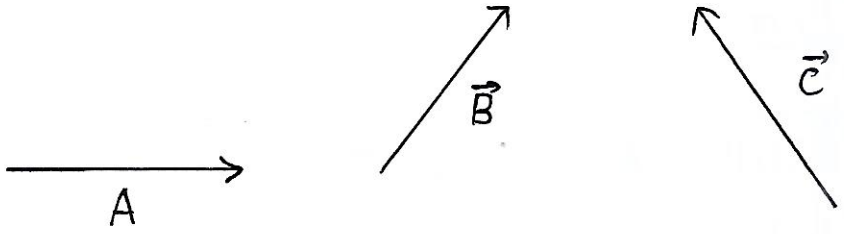
Amplitude = 2A (maximum) ⟶ Anti-Nodes

VECTORS AND SCALARS

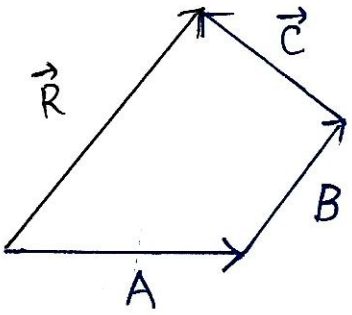
$\vec{A} = |A|\hat{A}$ → Direction

↓ Magnitude

Addition of vector -



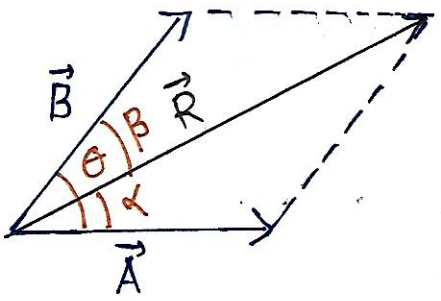
(1) Head-Tail Method or Polygon Method-



$$\vec{R} = \vec{A} + \vec{B} + \vec{C}$$

(2) Parallelogram Law-

$$\vec{A} + \vec{B} = \vec{R}$$

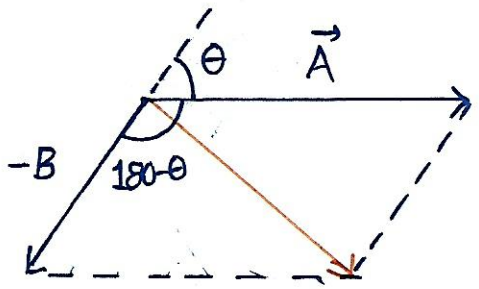


$$|R| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

$$\tan\beta = \frac{A\sin\theta}{B + A\cos\theta}$$

Subtraction $= \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$



$$\vec{R} = \vec{A} + (-\vec{B})$$

$$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos(180^\circ - \theta)} = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

Problem:

$$|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$$

$$\sqrt{A^2 + B^2 + 2AB\cos\theta} = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$$4AB\cos\theta = 0$$

$$A \neq 0$$

$$B \neq 0$$

$$\cos\theta = 0$$

$$\cos\theta = \cos\frac{\pi}{2}$$

$$\theta = 90^\circ$$

Problem:

$$\vec{A} + \vec{B} = \vec{R}$$

$$|R| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$1 = \sqrt{1 + 1 + 2\cos\theta}$$

$$1 = \sqrt{2 + 2\cos\theta}$$

$$-1 = 2\cos\theta$$

$$\cos\theta = \frac{-1}{2}$$

$$\theta = 120^\circ$$

Problem -

$$\vec{A} - \vec{B} = \vec{R}$$

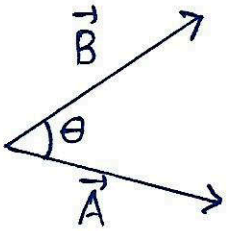
$$|R| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$$|R| = \sqrt{2 - 2\cos\theta}$$

$$|R| = \sqrt{2 - 2 \times 1 \times 1 \times -\frac{1}{2}}$$

$$\boxed{|R| = \sqrt{3}}$$

- **Multiplication -**
  - Dot Product $A \cdot B$ (Scalar Product)
  - Cross Product $\vec{A} \times \vec{B}$ (Vector Product)

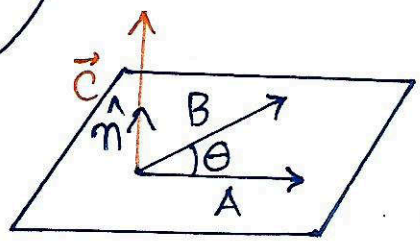


$\boxed{A \perp B \quad A \cdot B = 0}$

$$\vec{A} \cdot \vec{B} = |A||B|\cos\theta$$

$$\vec{A} \times \vec{B} = \vec{C}$$ (It is perpendicular to $\vec{A}$ and $\vec{B}$)

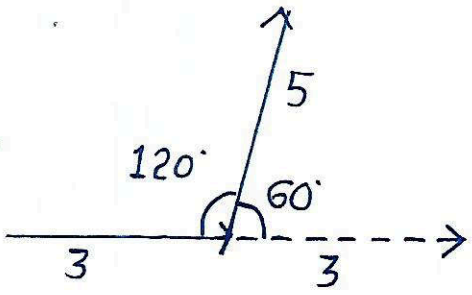
Screw Rule



$$\vec{C} = \vec{A} \times \vec{B} = |A||B|\sin\theta\,\hat{n}$$

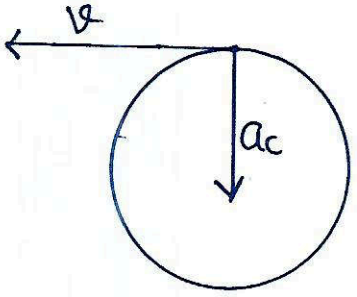
magnitude, direction

$\boxed{A \perp B \quad \vec{A} \times \vec{B} = 0}$



$$\vec{R} = 3 \times 5\cos 60^\circ$$

$v \cdot a_c = 0$



$\boxed{\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}}$

$\boxed{\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}}$ → Values same but direction is opposite

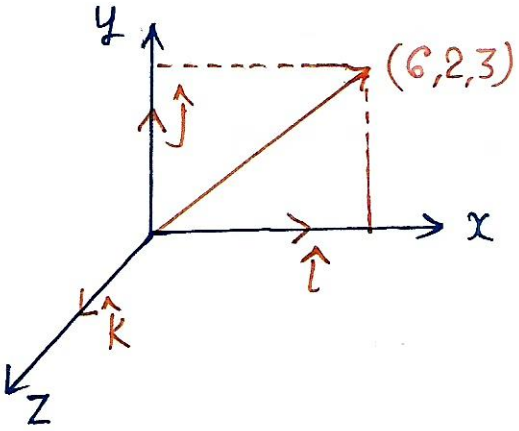
$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

$A\cdot A = A^2$

$|A||A|\cos 0^\circ = A^2 = |A|^2$

$\vec{A}\times\vec{A} = 0$

- Orthogonal Unit Vector-



$\vec{A} = 4\hat{i}$

$\vec{B} = 4\hat{i} + 5\hat{j}$

$\vec{C} = 4\hat{i} + 5\hat{j} + 3\hat{k}$

$\vec{A} = 6\hat{i} + 2\hat{j} + 3\hat{k}$

$|A| = \sqrt{(6)^2 + (2)^2 + (3)^2} = \sqrt{49} = 7$

Problem: Find a unit vector in the direction of $6\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{A} = |A|\,\hat{A}$

$$\boxed{\hat{A} = \frac{\vec{A}}{|A|}}$$

$$\hat{A} = \frac{6\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{(6)^2+(2)^2+(3)^2}}$$

$$\hat{A} = \frac{6\hat{i}+2\hat{j}+3\hat{k}}{7}$$

$$\hat{A} = \frac{6}{7}\hat{i} + \frac{2}{7}\hat{j} + \frac{3}{7}\hat{k}$$

Problem:

$\vec{A} = 3\hat{i}+2\hat{j}+\hat{k}$
$\vec{B} = 2\hat{i}-\hat{j}+3\hat{k}$
} Find the Angle b/w them.

$$\vec{A}+\vec{B} = 5\hat{i}+\hat{j}+4\hat{k}$$

$$\vec{A}-\vec{B} = \hat{i}+3\hat{j}-2\hat{k}$$

$$\vec{A}\cdot\vec{B} = 6+2\times(-1)+3\times1$$

$$\vec{A}\cdot\vec{B} = 6-2+3$$

$$\vec{A}\cdot\vec{B} = 7$$

$$\vec{A}\cdot\vec{B} = |A||B|\cos\theta$$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|A||B|}$$

$$\cos\theta = \frac{7}{\sqrt{14}\cdot\sqrt{14}}$$

$$\cos\theta = \frac{7}{14}$$

$$\cos\theta = \frac{1}{2}$$

$$\cos\theta = \cos 60^\circ$$

$$\theta = 60^\circ$$

$a_c = 2\hat{i} - 3\hat{j}$

$v = 3\hat{i} + x\hat{j}$

$\vec{v} \cdot \vec{a}_c = 0$

$6 - 3x = 0$

$3x = 6$

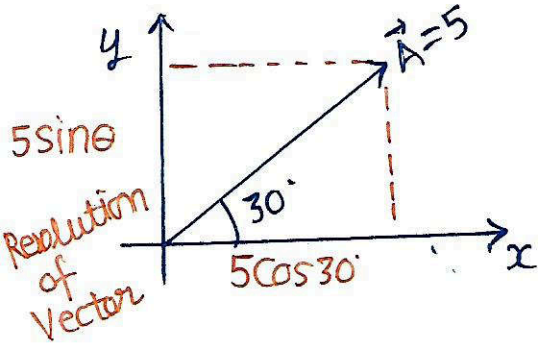
$\boxed{x = 2}$

$\vec{A} \times \vec{B} = ?$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 1 \\ 2 & -1 & 3 \end{vmatrix} = \hat{i}[2\times3 - (1\times1)] + \hat{j}[1\times2 - 3\times3] + \hat{k}[3\times-1 - 2\times2]$$

$$\vec{A} \times \vec{B} = \vec{C} = (6+1)\hat{i} + (2-9)\hat{j} + \hat{k}(-3-4)$$

$$\boxed{\vec{A} \times \vec{B} = \vec{C} = 7\hat{i} - 7\hat{j} - 7\hat{k}}$$



$$|A| = \sqrt{Ax^2 + Ay^2}$$

$$|A| = \sqrt{\left(\frac{5\sqrt{3}}{2}\right)^2 + \left(\frac{5}{2}\right)^2} = \sqrt{\frac{100}{4}} = 5$$

$\vec{R} = \vec{A} + \vec{B} + \vec{C}$

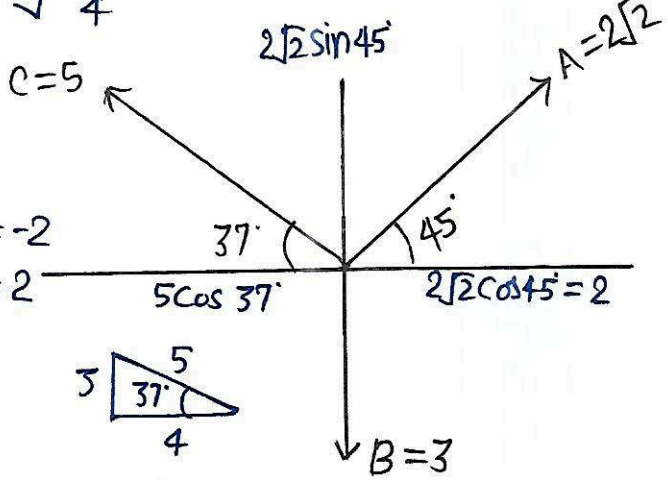
$R = \sqrt{Rx^2 + Ry^2}$

$Rx = Ax + Bx + Cx = 2 + 0 - 4 = -2$

$Ry = Ay + By + Cy = 2 + (-3) + 3 = 2$

$R = \sqrt{4+4} = \sqrt{8}$

$\boxed{R = 2\sqrt{2}}$



Nuclear Physics

- Formulea
- Shell Model
- Nuclear Structure

General Notation-

$${}^{A}_{Z}X$$

$Z \rightarrow$ Atomic Number (No. of Protons inside Nucleus)

$A = Z + n \rightarrow$ No. of Neutrons

$A =$ Atomic mass

- Binding Energy-

$$E_B = [ZM_p + NM_N - M_{nuc}]c^2$$

M_p: mass of Proton; M_N: mass of Neutron; M_{nuc}: mass of nucleus

- Binding Energy per nucleon or Binding Fraction-

$$F_B = \frac{E_B}{A}$$

- Mass Defect-

$$\Delta M' = M(A,Z) - A$$

$A \rightarrow$ Mass Number

$M(A,Z)$: Measured atomic mass

- Packing Fraction-

$$f = \frac{\Delta M'}{A}$$

$\rightarrow$ mass defect per nucleon in the nucleus.

$\rightarrow M(A,Z) = A(1+f)$

- Radius of Nucleus-

$$R = R_0 A^{1/3}$$

$$R_0 = 1.2 \times 10^{-15} m$$

$$R_0 \approx (1.1 - 1.5) \times 10^{-15} m$$

- Charge Density -

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-R)/a}}$$

$r^2 = x^2 + y^2 + z^2$

R = Radius of Nucleus

a = constant

- Quardpole moment -

$$Q = \frac{1}{e}\int (3z^2 - r^2)\,\rho\, dV$$

e = Charge of Proton

If $Q = 0$ Spherically Symmetric Nucleus

$Q > 0$ Prolate Spheroid Nucleus 

$Q < 0$ Oblate Spheriod Nucleus 

Spin of nucleus = $+1/2$

- Energy of nth State of Hydrogen -

$$E_n = \left(\frac{-13.6}{n^2}\right) eV$$

Z_{eff} (2) → Effective nuclear charge

$$E_n = \frac{Z_{eff}^2}{n^2} \times E_1$$

E_1 → Energy of 1st Orbit of $_1H$.

Note - Nuclear forces are spin dependant and have a non-central part, velocity dependant, saturated by surrounding nucleus, stronger known forces in nature, charge independent, short range forces.

- Rydberg Constant for an atom having an infinitly heavy nucleus,

$$R_\infty = \frac{me^4}{8\epsilon_0^2 h^3 c}$$

- For nucleus of mass M,

$$R_M = \frac{\mu e^4}{8\epsilon_0^2 h^3 c}$$

$$\text{Reduced mass } \mu = \frac{m_e M}{m_e + M}$$

m_e → mass of e^-, M → nuclear mass

- Hygenberg Uncertainty Relation-

$$\Delta x \cdot \Delta p = \hbar$$

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{\hbar}{2R}$$

$\Delta x = 2R$ → diameter of Nucleus

$$E_K = \frac{(\Delta p)^2}{2m}$$

- Shell Model-

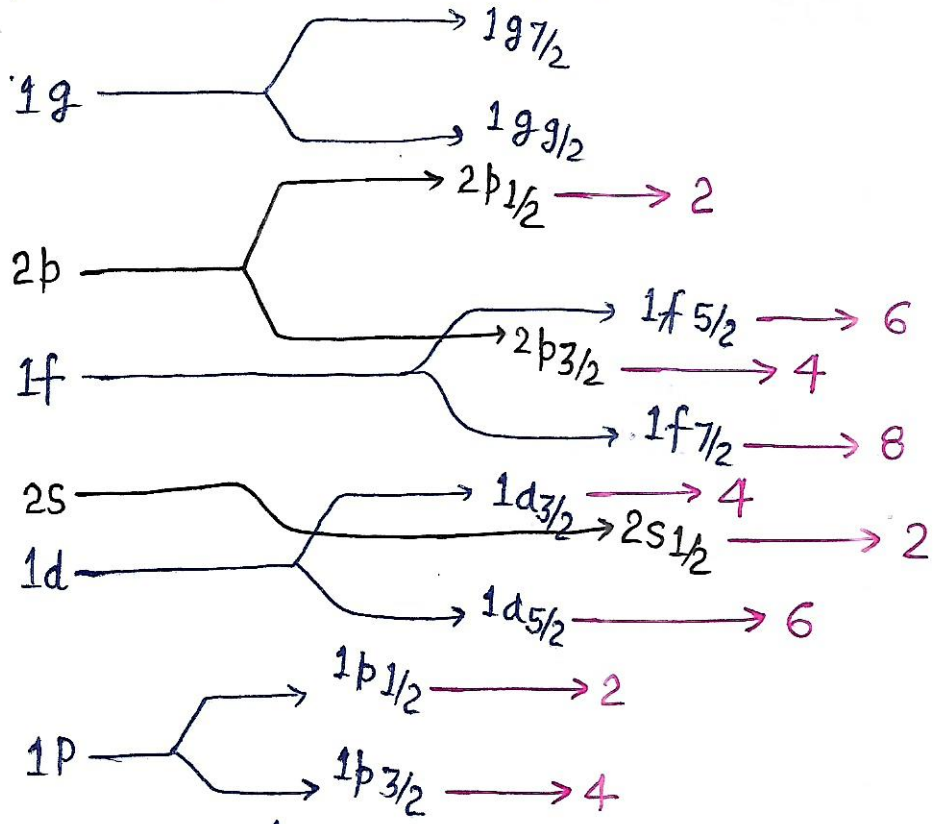
(1) 1. We want to find Angular momentum & Parity of a nucleus having

No. of Proton → Even
No. of Neutrons → Even
} $J = 0$, Parity $= +$ $\quad J^P = 0^+$

2. When any one of these

No. of Proton or Neutron → Even → Won't contribute in Angular momentum.

(2) When one is even and other odd. We will find angular momentum due to that odd no. & angular momentum will be equal to the j value of last unpaired e- Parity $(-1)^l$.



$n \rightarrow 1s \longrightarrow 1s_{1/2} \longrightarrow 2 \longrightarrow 2J+1$ degen

Principal Quantum Number

l	0	1	2	3
	s	p	d	f

Parity = $(-1)^l$

(3) When both no. of Proton and neutron are odd, total angular momentum will be due to total (nuclear) nucleons.

Let's discuss steps-

$$N^1 = (j_p - l_p) + (j_N - l_N)$$ → For Neutron

j_p: Angular moment corresponding to proton with that table

l_p: Orbital quantum number (l)

//

$N' = 0, \pm 1$

(i) $N' = 0$ Then $J = |J_p - J_n|$, Parity = -ve

(ii) $N' = \pm 1$ and $N = Z$

$$J = |J_p + J_N|, \text{ Parity} = \pm 1$$

(iii) $N' = \pm 1$ and $N \neq Z$

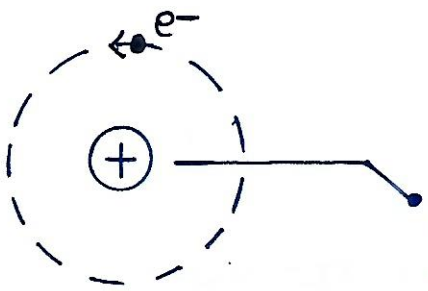
$$J = (J_p - J_N - 1), \text{ Parity} = +ve$$

#Nuclear Structure-

Nuclei ⇒ Plural of Nucleus

↓

Multiple of Nucleus



Entire mass & +ve charge of an atom is concentrated into a very small volume called Nucleus.

Radius = 10^{-14}m to 10^{-15}m

$\text{Radius}_{atom} = 10^{-10}$ m

- Nucleus is made up of neutrons & protons → Nucleons

Proton:

$C = +e = +1.6 \times 10^{-19}\ C$

$m = 1.6726 \times 10^{-27}\ kg = 1\ amu = 1\ u$

Symbol = ${}^{1}_{1}H$ (mass / charge ^{H})

↓

H-nucleus

Neutron:

$C = 0$

$m = 1.6749 \times 10^{-27}\ kg = 1\ amu$

Symbol = ${}^{1}_{0}n$

- **Atomic Number (Z) –**

$Z = p$

- **Mass Number (A) –**

$A = p + n$

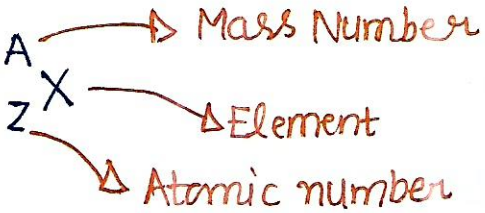
A = no. of nucleons

- **Atomic mass –**

Atom's mass ⟶ Nucleus mass = mass of protons and n.

Atomic mass = (No. of protons + no. of neutrons) × 1 amu

= A × amu



No. of neutron = A−Z

e.g.

$_{8}O^{16}$

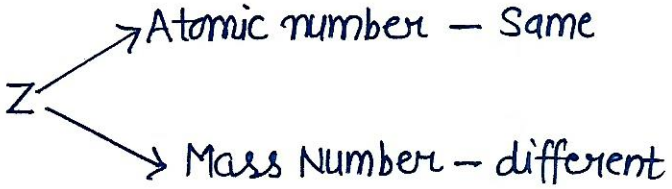
8P, 8n, 8e⁻

$_{6}C^{13}$

6P, 7n, 6e⁻

- Classification of Nuclei-

(1) Isotopes- Atoms of same element whose nuclei have same number of protons and different number of neutrons.



e.g. $_{1}^{1}H$, $_{1}^{2}H$, $_{1}^{3}H$

$_{8}^{16}O$, $_{8}^{17}O$, $_{8}^{18}O$

→ Characteristic property of Atom is Atomic number.

• Fractional Atomic mass-

↓

due to Isotopes

Avarage of Atomic mass = $A_1n_1 + A_2n_2 + \ldots$

A_1 = Atomic mass of 1st Isotope

n_1 = % of ——— o ———

A_2 = Atomic mass of 2nd Isotope

n_2 = % of ——— o ———

Cl → Atomic mass = 35.5 a.m.u.

$^{35}_{17}Cl$ $^{37}_{17}Cl$ ← Isotopes

75% 25%

$\langle A \rangle = 35 \times 75\% + 37 \times 25\%$

$\langle A \rangle = \frac{35 \times 75}{100} + \frac{37 \times 25}{100}$

$\langle A \rangle = \frac{105 + 37}{100 \times 4}$

$\langle A \rangle = \frac{142}{4}$

$\boxed{\langle A \rangle = 35.5 \text{ amu}}$

(2) Isobars- Atomic number – differents

Atomic mass – same

$\boxed{\text{Atomic element – different}}$

Atoms of different elements whose nuclei have same number of nucleons and different number of protons.

e.g.

$_{6}^{14}C$ & $_{7}^{14}N$

6P
8n

7P
7n

$_{18}^{40}Ar$ $_{20}^{40}Ca$

18P
22n

20P
20n

(3) Isotons -

Atoms of different elements whose nuclei have same numbers of neutrons but different number of protons.

Z → different
A → different
A-Z → same

e.g. $_{3}^{7}Li$ $_{4}^{8}Be$

3P
4n

4P
4n

Isotops → no. of protons
Isobars → no. of nucleons
Isotons → no. of neutron

- Nuclear Size, Shape and density -

- Practically Spherical
deviation ≤ 10%

-Size

$_{1}^{1}H$ $_{11}^{23}Na$

1P 11P

on 12n

R

$V = \frac{4}{3}\pi R^3$

Volume of nucleus $\propto$ no. of nucleons in nucleus

$V \propto A$

$\frac{4}{3}\pi R^3 \propto A$

$R^3 \propto A$

$\boxed{R \propto A^{1/3}}$

$R = R_0 A^{1/3}$

R → Radius of Nucleus

R_0 → Constant $R_0 = 1.2 \times 10^{-15} m = 1.2$ fermi

$R = 1.2(A)^{1/3}$

↳ In fermi

Density -

$$\rho = \frac{\text{mass of nucleus}}{\text{V of nucleus}}$$

$$\rho = \frac{A \times amu}{V}$$

$$\rho = \frac{A \times 1\,amu}{\frac{4}{3}\pi R^3}$$

$$\rho = \frac{A \times 1\,amu}{\frac{4}{3}\pi [R_0 (A)^{1/3}]^3} = \frac{A \times 1\,amu}{\frac{4}{3}\pi R_0^3 A}$$

$$\rho = \frac{1.67 \times 10^{-27}\ kg}{\frac{4}{3}\pi (1.2 \times 10^{-15}\ m)^3}$$

$$\boxed{\rho \approx 2 \times 10^{17}\ kg/m^3}$$

Radioactivity -

⇒ Among 1500 known nuclei, less than 260 are stable.
⇒ The unstable nuclei gains stability by emitting α, β, γ.
⇒ α- particle or β- particle and γ EM waves.

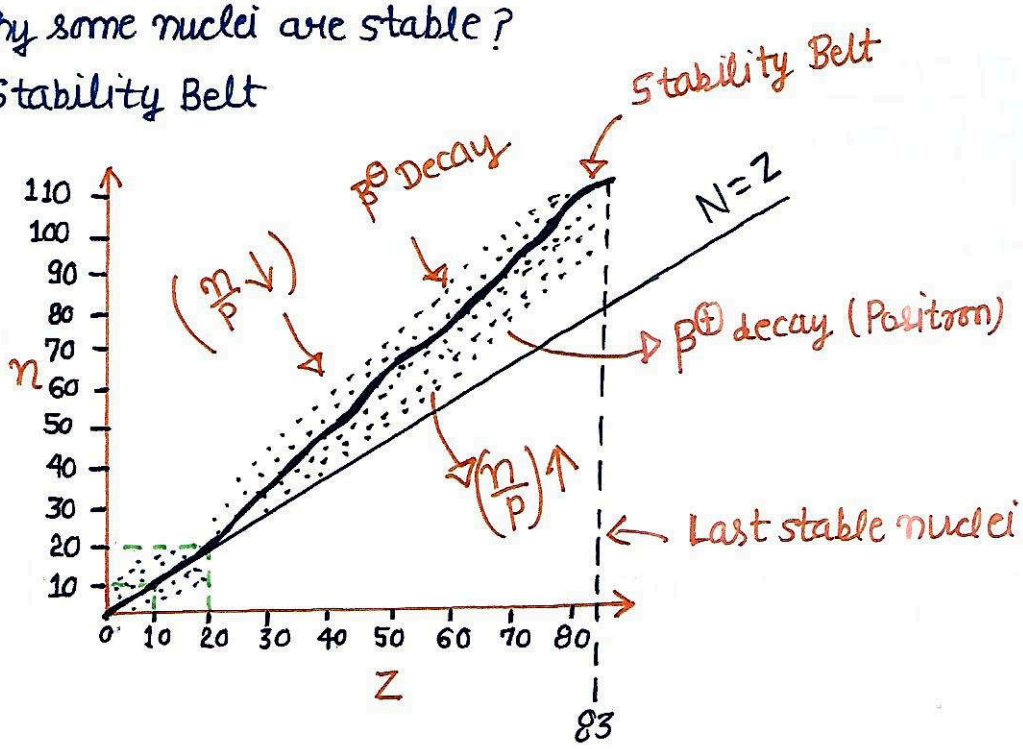
This phenomena is called radioactivity.

→ Radioactivity is a nuclear phenomena. The rate of emission of α, β, γ depends only on constentration of nuclei. These reactions are unaffected by temperature, pressure and other physical conditions.

→ Radioactivity was first discovered by Henry Becqural in 1896, accidently.

Why some nuclei are stable?

-Stability Belt



For atoms with $Z \leq 20$

stable nuclei have $\frac{n}{p} \approx 1$

$N \approx Z$

e.g $^{4}_{2}He$ $^{12}_{6}C$ $^{14}_{7}N$

2P 6P 7P

2n 6n 7n

$^{23}_{11}Na$ $^{24}_{12}Mg$ $^{40}_{20}Ca$

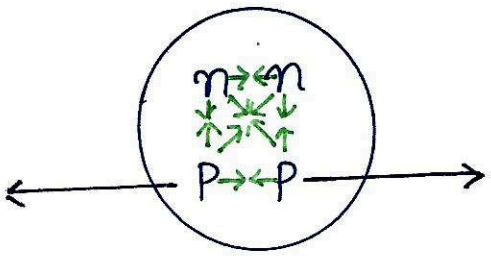
11P 12P 20P

12n 12n 20n

$^{14}_{6}C \longrightarrow$ Radioactive

6P

8n



- dominating nuclear force.
  ↓
  charge independent.

For atoms with Z 720
stability nuclei have $1 \leq \frac{n}{P} \leq 1.6$

e.g. $^{56}_{26}Fe$ $^{108}_{47}Ag$

26P 47P
30n 61n

- Last stable nuclei → $^{209}_{83}Bi$

Notes