Quantum Physics Easy Notes

Notes

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QUANTUM PHYSICS

- What do we need to know for thermodynamics?
  - How atoms store energy.
  - Details of structure.
  - Statistics.

- Structure → Allowed configuration of Atoms

- Atoms and molecules generally exist in fixed states, known as quantum states.

Quantum Mechanics
↓
Mathematics
↓
Atomic and molecular structure
↓
To determine allowed Quantum states and their energies.

- Short History of Quantum Mechanics
  - dual nature of matter

Democritus 460 BC
Basic principle of matter
↓
Thompson 1897
e^-
↓
Rutherford 1911
Scattering
↓
Bohr 1913
Rules for Atom
↓

COURSERA UC

Waves act like particles

1. Plank Theory of Black Body Radiation — $E_n = nh\nu$
2. Photoelectric effect — $E = h\nu$
3. Compton effect — $p = \frac{h}{\lambda}$

↓

Particles acts like waves.

1. De Broglie Postulates — $\lambda = \frac{h}{p}$
2. Davison-Germer Exp.

↓

Uncertainty Principle

$\Psi \rightarrow$ Oscillating Terms

$$\Delta x \, \Delta p_x \geq \frac{\hbar}{2}$$

↓

Erwin Schrodinger

- Postulates
  1. $\Psi(\vec{r},t)$
     It contains all the information that is known about the system.
  2. $p(\vec{r},t)dV = \psi^*(\vec{r},t)\,\psi(\vec{r},t)\,dV = |\psi(\vec{r},t)|^2 dV$

$$\int \psi^*(\vec{r},t)\,\psi(\vec{r},t)\,dV = 1$$

↓

Wave function is normalized

3.

$\vec{r}$ $\qquad\qquad$ $\vec{r}_{op} = \vec{r}$

$\vec{p}$ $\qquad\qquad$ $\vec{p}_{op} = -i\hbar\vec{\nabla}$

$\qquad\longrightarrow$

E $\qquad\qquad$ $E_{op} = i\hbar\frac{\partial}{\partial t}$

$B(\vec{r},t)$ $\qquad\qquad$ $B_{op} = B(\vec{r}, -i\hbar\vec{\nabla})$

$\uparrow$

function of space-time

4.

$$\langle B \rangle = \int \psi^*(\vec{r},t)\, B_{op}\, \psi(\vec{r},t)\, dV$$

- Energy of a particle

$$K.E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$H = \frac{p^2}{2m} + V(\vec{r},t) = E$$

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi$$

- The Stationary Wave Equation

$V(\vec{r},t) = V(\vec{r})$

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi$$

$$\psi(\vec{r},t) = \phi(t)\,\psi(\vec{r})$$

$$\frac{i\hbar}{\phi}\frac{\partial\phi}{\partial t} = \frac{1}{\psi}\left[-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi\right]$$

$$\frac{i\hbar}{\phi}\frac{\partial \phi}{\partial t} = c$$

$$\frac{1}{\psi}\left[-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi\right] = c$$

$$i\frac{\partial \phi}{\partial t} = \frac{c}{\phi\hbar}\phi$$

$$\phi(t) = \exp\left(-i\frac{c}{\hbar}t\right)$$

$$\psi(\vec{x},t) = \psi(\vec{x})\exp\left(-i\frac{c}{\hbar}t\right)$$

$$\langle E\rangle = \int_V \psi^*\left(i\hbar\frac{\partial}{\partial t}\right)\psi\, dv = \int_V \psi^* e^{i\frac{c}{\hbar}t} c\psi e^{-i\frac{c}{\hbar}t} dv = c$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V(\vec{x})\psi = E\psi$$

- Two Particle system

$$E = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + V(\vec{x})$$

$$\frac{\hbar^2}{2m_t}\nabla^2\psi_e + E_e\psi_e = 0 \qquad \text{Internal}$$

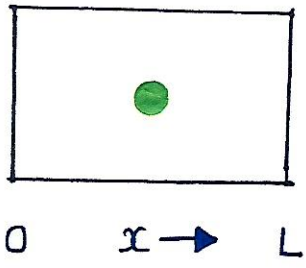
$$\frac{\hbar^2}{2\mu}\nabla^2\psi_{int} + (E_{int} - V(\vec{x}))\psi_{int} = 0 \qquad \text{External}$$

$$E = E_e + E_{int}$$

$$m_t = m_1 + m_2$$

$\mu = \dfrac{m_1 m_2}{m_1 + m_2}$ is called reduced mass.

- The Particle in a Box



$$\psi(0) = \psi(L) = 0$$

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = -E_x\psi$$

Therefore

$$\psi(x) = A\sin\sqrt{\frac{2mE_x}{\hbar^2}}\,x + B\cos\sqrt{\frac{2mE_x}{\hbar^2}}\,x$$

$$B = 0$$

$$0 = A\sin\sqrt{\frac{2mE_x}{\hbar^2}}\,L$$

$$E_x = \frac{\hbar^2\pi^2}{2mL}n_x^2$$

Thus, not all energies are allowed as stationary state of the particle hence QUANTUM!

Postulate 2 $\longrightarrow \int_0^L |\psi|^2 dx = 1 \longrightarrow A = \sqrt{\frac{2}{L}}$

$$\psi(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi n_x x}{L}\right) \qquad n = 0, 1, 2, \ldots.$$

$$E = E_x + E_y + E_z = \frac{h^2}{8mV^{2/3}}(n_x^2 + n_y^2 + n_z^2)$$

$$\psi(x) = \sqrt{\frac{8}{L^3}}\sin\left(\frac{\pi n_x x}{L}\right)\sin\left(\frac{\pi n_y y}{L}\right)\sin\left(\frac{\pi n_z z}{L}\right)$$

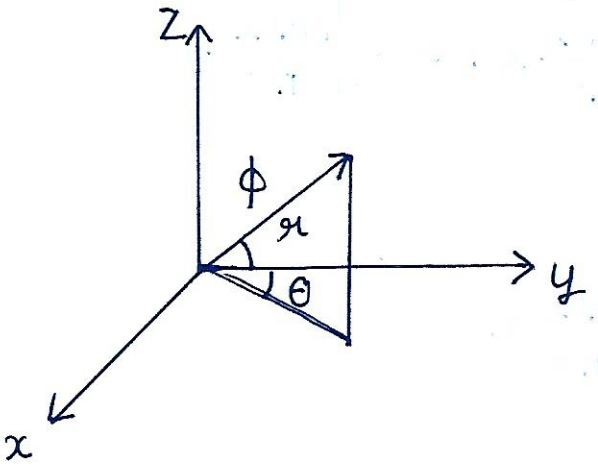
- When you have more that one quantum state that contribute to the same energy that is called degeneracy.

↓

Symbol g

$$n_{typical} \sim \sqrt{n_x^2+n_y^2+n_z^2} = \left(\frac{8mV^{2/3}}{h^2}\langle E\rangle\right)^{1/2} = 1.38\times10^{8}$$

- Internal Motion: Radial and Angular Motion-



$$\psi_{int}(r,\theta,\phi) = R(r)\,Y(\theta,\phi)$$

$$\frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\frac{2\mu r^2}{\hbar^2}\left[E_{int}-V_{int}(r)\right] = -\frac{1}{Y}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial Y}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2 Y}{\partial\phi^2}\right]$$

$$Y(\theta,\phi) = \Theta(\theta)\,\Phi(\phi)$$

$$\frac{d^2\Phi}{d\phi^2}+\beta(\Phi)=0$$

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right)+\left(\alpha-\frac{m_\ell^2}{\sin^2\theta}\right)\Theta=0$$

The is Legendre Equation.

$$\langle B \rangle = \int \psi^*(\vec{r},t)\, B_{op}\, \psi(\vec{r},t)\, dV$$

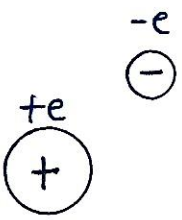
$$\langle L^2 \rangle = l(l+1)\hbar^2$$

↑ Expectation value of Angular momentum

$$\langle L_z \rangle = m_l \hbar$$

↑ Magnetic quantum number

- The Hydrogenic Atom-



$$V(r) = \int_{\infty}^{r} \frac{e^2}{4\pi\epsilon_0} \frac{dr}{r^2} = -\frac{e^2}{4\pi\epsilon_0 r}$$

$$\frac{d}{dr}\left(r^2 \frac{dR}{dr}\right) + \left\{\frac{2\mu r^2}{\hbar^2}[E_{int} - V_{int}(r)] - \alpha\right\} R = 0$$

Put the values of V and α

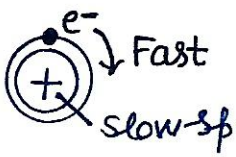
$$\frac{1}{\rho^2}\frac{d}{d\rho}\left(\rho^2 \frac{dR}{d\rho}\right) - \left[\frac{l(l+1)}{\rho^2} - \frac{n}{\rho} + \frac{1}{4}\right] R = 0$$

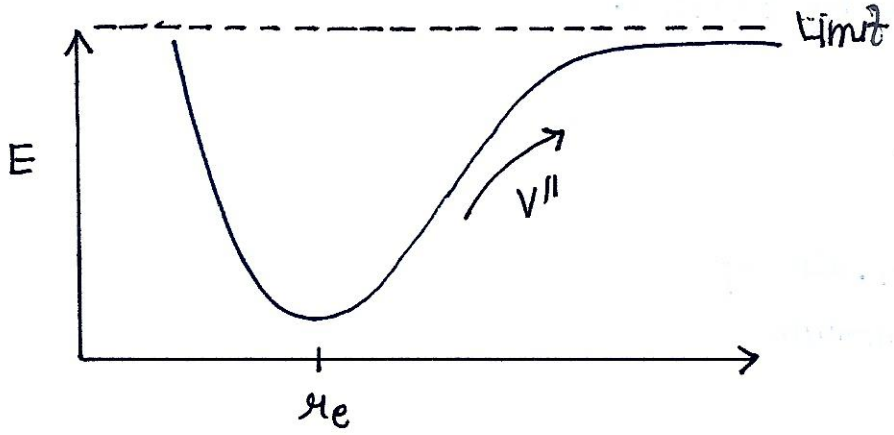
This is Laguerre Equation.

- The Diatomic Molecule-

↓

Born-Oppenheimer Approximation





- Real Atoms –

  – Higher order Effect

    - Spin Orbit Coupling
    - Relativistic
    - Nuclear Spin
    - Zeeman Effect

$$\vec{J} = \vec{L} + \vec{S}$$

$$J = \sqrt{j(j+1)}\hbar$$

$$J_z = m_j \hbar$$

  – Good quantum numbers

$$n, l, j, m_j$$

- Polyatomic Molecules

  – NIST database.

  – n atoms, there are 3n degree of freedom

Types of Bonds

Ionic Covalent

- Classification of compounds
  1. Hydrides
  2. Oxides
  3. Halides
  4. Organic VS Inorganic
  5. Ionic
  6. Organometallic
- Types of Organic compounds
  1. Alkanes –
  2. Alkenes =
  3. Alkynes ≡
  4. Arines O

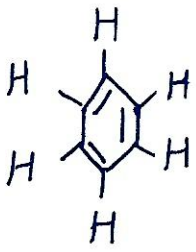
C–C

C=C

C≡C







Benzene

- Functional Groups
  - –OH
  - – Ketone
  - –Aldehyde
  - – COOH
  - – Easter
  - – Amide
  - – Nitrate

- Numerical Methods for solving the wave equation

$$H\Psi = W\Psi$$

- Ab Initio — Most accurate, Slowest
- Density Functional — Less accurate, faster
- Semi-Empirical — Least accurate, fastest

• Normalization of Wave Function –

– Schrodinger's equation

$$\frac{-\hbar^2}{2m}\frac{d^2\psi}{dt^2} + V(x)\psi = E\psi$$

$\psi(x) \equiv$ Wave Function solution

$$\frac{d^2y}{dx^2} + y = 0$$

$y = \sin(x)$

$y = 2\sin(x) \Rightarrow \frac{dy}{dx} = 2\cos(x)$

$$\frac{d^2y}{dx^2} = -2\sin(x)$$

$$\frac{d^2y}{dx^2} + y = 0$$

$y = 3\sin(x)$

$\vdots$

$y = n\sin(x)$

– Infinite solutions

$c_1\psi(x)$ $\quad c_1 =$ constant

↓

Also solution of schrodinger eqn

$c_2\psi(x)$

$c_3\psi(x)$

$c_4\psi(x)$

$\vdots$

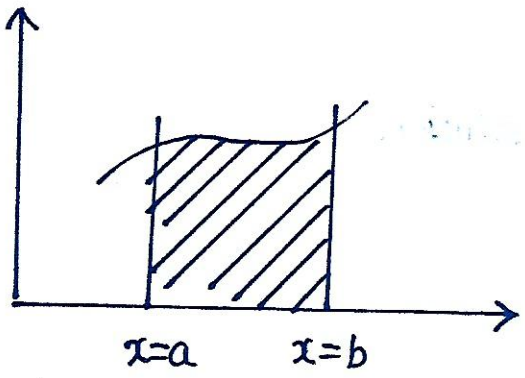
$c_n\psi(x)$

– These are mathematical only. Only one can use as actual one.

YOUTUBE

- Born statistical Interpretation

$$\int_{x=a}^{b} |\psi(x)|^2 dx = P_{ab}$$



$$F = ma$$

$$= m\frac{d^2r}{dt^2}$$

↓

All Information of particle

- Probability of finding the particle.

$$\int_{x=-\infty}^{x=\infty} |\psi(x)|^2 dx = 1$$

$\psi(x)$ ⟶ NORMALIZE It ⟶ ? HOW?

↓

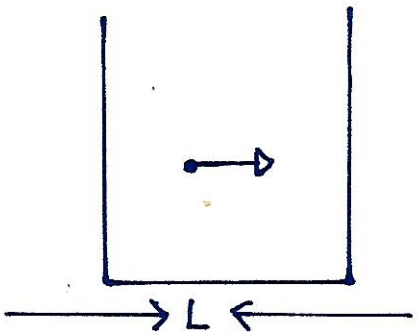
NOT NORMALIZE

↓

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx \neq 1$$

$$\int_{-\infty}^{\infty} |N\psi(x)|^2 \, dx = 1$$

$$\boxed{N^2 = \frac{1}{\int_{-\infty}^{\infty} |\psi(x)|^2 \, dx}}$$



$$\rightarrow \psi(x) = \sin\left(\frac{n\pi}{L} x\right)$$

$$\int_{-\infty}^{\infty} |N\psi(x)|^2 \, dx = 1$$

$$\rightarrow N^2 \underbrace{\int_{-\infty}^{\infty} \sin^2\left(\frac{n\pi}{L} x\right) dx}_{\frac{L}{2}} = 1$$

$$\rightarrow N^2 \times \frac{L}{2} = 1$$

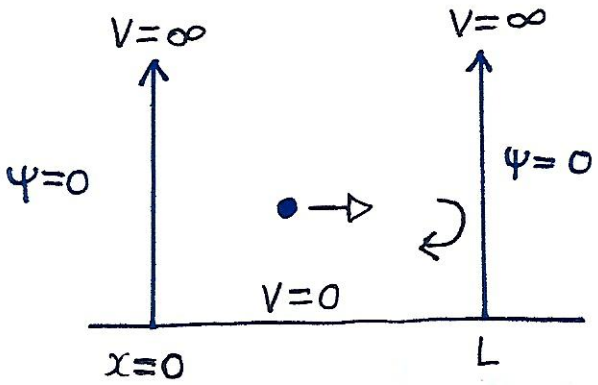
$$\rightarrow \rightarrow N^2 = \frac{2}{L}$$

$$\boxed{N = \sqrt{\frac{2}{L}}}$$

$$\boxed{N\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} x\right)}$$

- Particle in an Infinite Potential Well-

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$



$\Downarrow$ $V(x)$ → Potential is independent of time

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi$$

→ $\psi(x) \equiv$ Eigen function soln

$$\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$$

$$V(x) = \begin{cases} \infty & \text{for } x \geq L \\ 0 & \text{for } 0 < x < L \end{cases}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$$

$$-\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi$$

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0$$

$$\frac{2mE}{\hbar^2} = K^2$$

$$K = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0$$

$$\psi = A\sin kx + B\cos kx \quad \ldots(1)$$

Boundary conditions:

$\psi(x) = 0$

$\psi(x=0) = 0$

$A\sin(k\times 0) + B\cos(k\times 0) = 0$

$\sin 0 = 0$, $\cos 0 = 1$

$B = 0$

eqn (1)

$$\psi = A\sin kx \quad \ldots(2)$$

$\psi(x=L) = 0$

$A\sin(kL) + B\cos kx = 0$

$A\sin kL = 0$

$\sin kL = 0$

$kL = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$

Not allowed

$kL = n\pi$

$k_n = \frac{n\pi}{L}$ $\quad n = 1, 2, 3, \ldots$

Finally,

$$\psi = A\sin\left[\left(\frac{n\pi}{L}\right)x\right]$$

$n = 1, 2, 3, \ldots$

$\psi = A\sin kx + B\cos kx$

$\frac{d\psi}{dx} = kA\cos kx - kB\sin kx$

$\frac{d^2\psi}{dx^2} = -k^2 A\sin kx - k^2 B\cos kx$

$\frac{d^2\psi}{dx^2} = -k^2(A\sin kx + B\cos kx)$

$\frac{d^2\psi}{dx^2} = -k^2\psi$

$\frac{d^2\psi}{dx^2} + k^2\psi = 0$

$$\psi_n = A\sin\left(\frac{n\pi}{L}x\right)$$

↓

Find?

$$\int_{x=-\infty}^{+\infty} \psi^*\psi\, dx = 1$$

$$\int_{x=0}^{x=L} \psi^*\psi\, dx = 1$$

$$\psi(x,t) = \psi(x)e^{-iEt/\hbar}$$

↓

Satisfy Born statistical Interpretation

$$\Rightarrow \int A\sin\left(\frac{n\pi}{L}x\right)e^{iEt/\hbar} \times A\sin\left(\frac{n\pi}{L}x\right)e^{-iEt/\hbar}\, dx = 1$$

$$\Rightarrow A^2\int \sin^2\left(\frac{n\pi}{L}x\right)dx = 1$$

$$\sin 2\theta = 2\sin^2\theta - 1$$

$$\frac{1}{2}(1+\sin 2\theta) = \sin^2\theta$$

$$\Rightarrow A^2\int \frac{1}{2}\left\{1+\sin\left(\frac{2n\pi}{L}x\right)\right\}dx = 1$$

$$\Rightarrow \frac{A^2}{2}\int dx + \frac{A^2}{2}\int \sin\left(\frac{2\pi nx}{L}\right)dx = 1$$

$$\Rightarrow \frac{A^2}{2}[x]_0^L - \frac{A^2}{2}\times\left(\frac{L}{2\pi n}\right)\left[\cos\left(\frac{2\pi nx}{L}\right)\right]_0^L = 1$$

$$\Rightarrow \frac{A^2}{2}\times L - \frac{A^2L}{4n\pi}\left[\underbrace{\cos\left(\frac{2n\pi L}{L}\right)}_{1} - \underbrace{\cos\left(\frac{2n\pi 0}{L}\right)}_{1}\right] = 0$$

$$\Rightarrow \frac{A^2 L}{2} - \frac{A^2 L}{4\pi n}(1-1) = 1$$

$$\Rightarrow \frac{A^2 L}{2} = 1$$

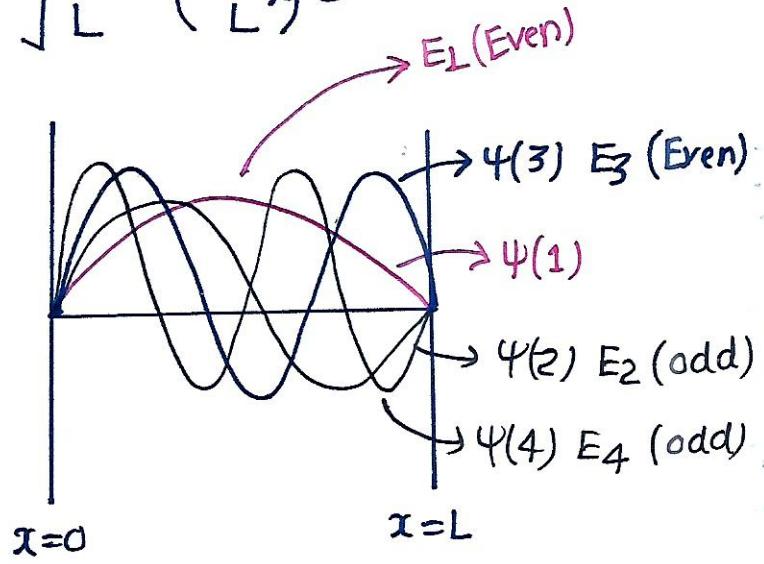
$$\Rightarrow A^2 = \frac{2}{L}$$

$$\Rightarrow \boxed{A = \sqrt{\frac{2}{L}}}$$

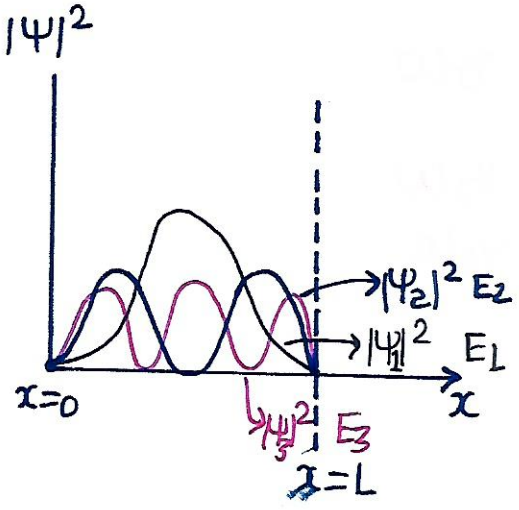
Eigen function sol^n

$$\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} x\right)$$

$$\Psi(x,t) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi n x}{L}\right) e^{-iEt/\hbar}$$



$$|\psi|^2 = P$$



Energy Levels :

$$K = \sqrt{\frac{2mE}{\hbar^2}}$$

$$K = \frac{n\pi}{L} \quad , n = 1, 2, 3, \ldots$$

$$\frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{L^2}$$

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

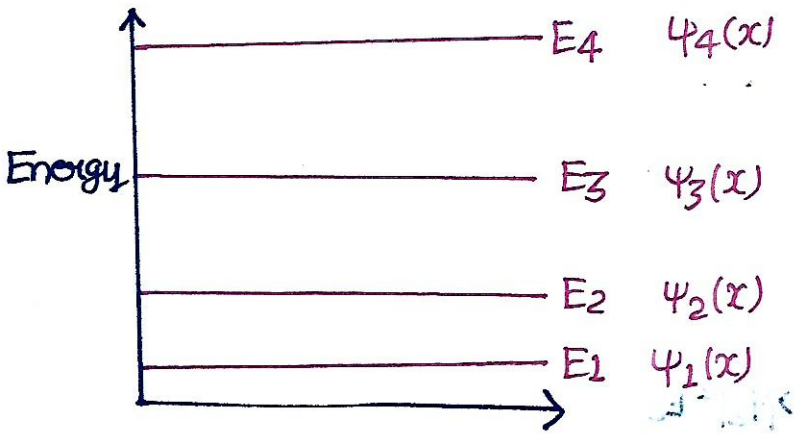
Discrete Energy Level

$$E_1 = \frac{\pi^2\hbar^2}{2mL^2}$$

$$E_2 = \frac{4\pi^2\hbar^2}{2mL^2}$$

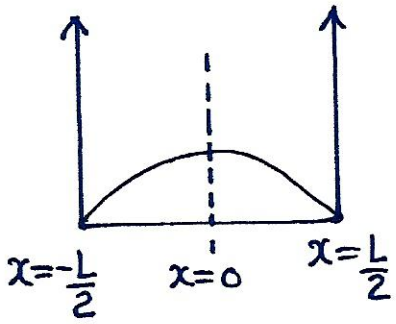
$$E_3 = \frac{9\pi^2\hbar^2}{2mL^2}$$

$$E_4 = \frac{16\pi^2\hbar^2}{2mL^2}$$



Properties of Eigen Functions:

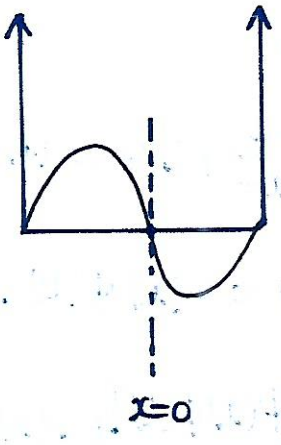
1.



$\psi(-x)=\psi(x)$

- It is even

Ground State



$\psi(-x)=-\psi(x)$

- It is odd.

Excited State

2. Node points increase.

$n=1,2,3,4,\ldots$

0 1 2 3, ...

3. $\int \psi_1 \psi_2 \, dx = 0 \quad (1 \neq 2)$

$\int \psi_i \psi_j \, dx = 0 \quad (i \neq j)$

⇓

Orthogonality

4. $\Psi(x,t) = \psi(x) e^{-iEt/\hbar}$

$\Psi(x,t) = \sum_{n=1}^{\infty} \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) e^{-i\frac{n^2\pi\hbar^2}{2mL^2}\cdot\frac{t}{\hbar}} \cdot C_1$

C_1 ⇓

What is the probability?

- Reducing the Time-dependent Schrodinger Equation to its time independent form-

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Time dependent SE :

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\psi(x,t) = \psi(x)\,\phi(t) \ldots\ldots (1)$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}[\psi(x)\phi(t)] + V(x)\psi(x)\phi(t) = i\hbar\frac{\partial}{\partial t}[\psi(x)\phi(t)]$$

$$-\frac{\hbar^2}{2m}\phi(t)\frac{\partial^2\psi(x)}{\partial x^2} + V(x)\psi(x)\phi(t) = i\hbar\,\psi(x)\frac{\partial\phi(t)}{\partial t}$$

$$\phi(t)\left[-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi(x)\right] = \psi(x)\left[i\hbar\frac{\partial\phi(t)}{\partial t}\right]$$

$$\frac{1}{\psi(x)}\left[-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi(x)\right] = \frac{1}{\phi(t)}\left[i\hbar\frac{d\phi(t)}{dt}\right]$$

Separation constant : G

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = G\psi \ldots\ldots (2)$$

$$i\hbar\frac{d\phi}{dt} = G\phi(t) \ldots\ldots (3)$$

$$i\hbar\frac{\partial\phi}{\partial t} = G\phi$$

$$\frac{d\phi}{dt} = -\frac{i}{\hbar}G\phi$$

$$\frac{d\phi}{dt} = \frac{-iG}{\hbar}\phi(t)$$

$$\frac{-iG}{\hbar} = \alpha$$

$$\phi(t) = e^{\alpha t}$$

$$\phi(t) = e^{\frac{-iG}{\hbar}t}$$

$$\frac{dy}{dt} = \alpha y$$

$$y = e^{\alpha t}$$

$$\frac{dy}{dt} = \alpha e^{\alpha t}$$

$$\frac{dy}{dt} = \alpha y$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{-i\theta} = \cos\theta - i\sin\theta$$

$$e^{-\frac{iGt}{\hbar}} = \cos\frac{G}{\hbar}t - i\sin\frac{G}{\hbar}t$$

$$\left(\because \hbar = \frac{h}{2\pi}\right)$$

$$e^{-\frac{iGt}{\hbar}} = \cos\left(\frac{2\pi G}{h}t\right) - i\sin\left(\frac{2\pi G}{h}t\right)$$

$$e^{-\frac{iGt}{\hbar}} = \cos(2\pi\nu t) - i\sin(2\pi\nu t)$$

where $\rightarrow \nu = \frac{G}{h}$

Frequency $\Rightarrow$

$E = h\nu$ [Plank-Einstein Postulate]

$G = h\nu$

$G = E$

From eq^n(3)

$$\phi(t) = e^{-\frac{iE}{\hbar}t}$$

$$\boxed{\frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = E\psi}$$

This is time independent Schrodinger equation.

$\psi \rightarrow$ Eigen Function

$$\Psi(x,t) = \psi(x)\phi(t)$$

$$= \psi(x)e^{-\frac{iE}{\hbar}t}$$

• Ehrenfest Theorem-

$$\frac{d}{dt}\langle p\rangle = \left\langle -\frac{\partial V}{\partial x}\right\rangle$$

↑ Momentum ↑ Negative gradient of potential

$$\langle p\rangle = \int_{-\infty}^{+\infty} \psi^* \hat{p}\,\psi\, dx \quad ; \quad \hat{p} = -i\hbar\frac{\partial}{\partial x}$$

↑ Operator for momentum

$$= \int_{-\infty}^{\infty} \psi^*\left(-i\hbar\frac{\partial\psi}{\partial x}\right)dx = \int_{-\infty}^{\infty} \psi^*\left(-i\hbar\frac{\partial\psi}{\partial x}\right)dx$$

$$\langle p\rangle = -i\hbar\int \psi^*\frac{\partial\psi}{\partial x}\,dx$$

$$\frac{d}{dt}\langle p\rangle = -i\hbar\frac{d}{dt}\int \psi^*\frac{\partial\psi}{\partial x}\,dx$$

$$\frac{d}{dt}\langle p\rangle = -i\hbar\frac{d}{dt}\int \frac{\partial}{\partial t}\left(\psi^*\frac{\partial\psi}{\partial x}\right)dx$$

$$\frac{d}{dt}\langle p\rangle = -i\hbar\int\left[\frac{\partial\psi^*}{\partial t}\frac{\partial\psi}{\partial x} + \psi^*\frac{\partial}{\partial x}\frac{\partial\psi}{\partial t}\right]dx \quad \ldots\ldots(1)$$

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\frac{\partial\psi}{\partial t} = \frac{1}{i\hbar}\left[\frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi\right]$$

$$\frac{\partial\psi}{\partial t} = \frac{-i}{\hbar}\left[\frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi\right]$$

$$\frac{\partial\psi}{\partial t} = \frac{i\hbar}{2m}\frac{\partial^2\psi}{\partial x^2} - \frac{i}{\hbar}V\psi$$

$$\frac{\partial \psi^*}{\partial t} = \frac{-i\hbar}{2m}\frac{\partial^2 \psi^*}{\partial x^2} - \frac{i}{\hbar} V\psi^*$$

$$\frac{d}{dt}\langle p \rangle = -i\hbar \int \left[\left(\frac{i\hbar}{2m}\frac{\partial^2 \psi^*}{\partial x^2} + \frac{i}{\hbar} V\psi^* \right) \frac{\partial \psi}{\partial x} + \psi^* \frac{\partial}{\partial x}\left(\frac{i\hbar}{2m}\frac{\partial^2 \psi}{\partial x^2} - \frac{i}{\hbar} V\psi \right) \right] dx$$

$$\frac{d}{dt}\langle p \rangle = (-i\hbar)\left(\frac{-i\hbar}{2m}\right) \int \left[\frac{\partial^2 \psi^*}{\partial x^2}\frac{\partial \psi}{\partial x} - \psi^* \frac{\partial}{\partial x}\left(\frac{\partial^2 \psi}{\partial x^2} \right) \right] dx$$

$$+ (-i\hbar)\left(\frac{i}{\hbar}\right) \int \left[V\psi^* \frac{\partial \psi}{\partial x} - \psi^* \frac{\partial}{\partial x}(V\psi) \right] dx$$

$$\frac{d}{dt}\langle p \rangle = \left(\frac{-\hbar^2}{2m}\right) \int \left[\frac{\partial^2 \psi^*}{\partial x^2}\frac{\partial \psi}{\partial x} - \psi^* \frac{\partial}{\partial x}\left(\frac{\partial^2 \psi}{\partial x^2} \right) \right] dx + \left\langle -\frac{dV}{dx} \right\rangle$$

$$\frac{d}{dt}\langle p \rangle = \left(\frac{-\hbar^2}{2m}\right) \left| \frac{\partial \psi^*}{\partial x}\frac{\partial \psi}{\partial x} \right|_{-\infty}^{\infty} + \left\langle -\frac{dV}{dx} \right\rangle$$

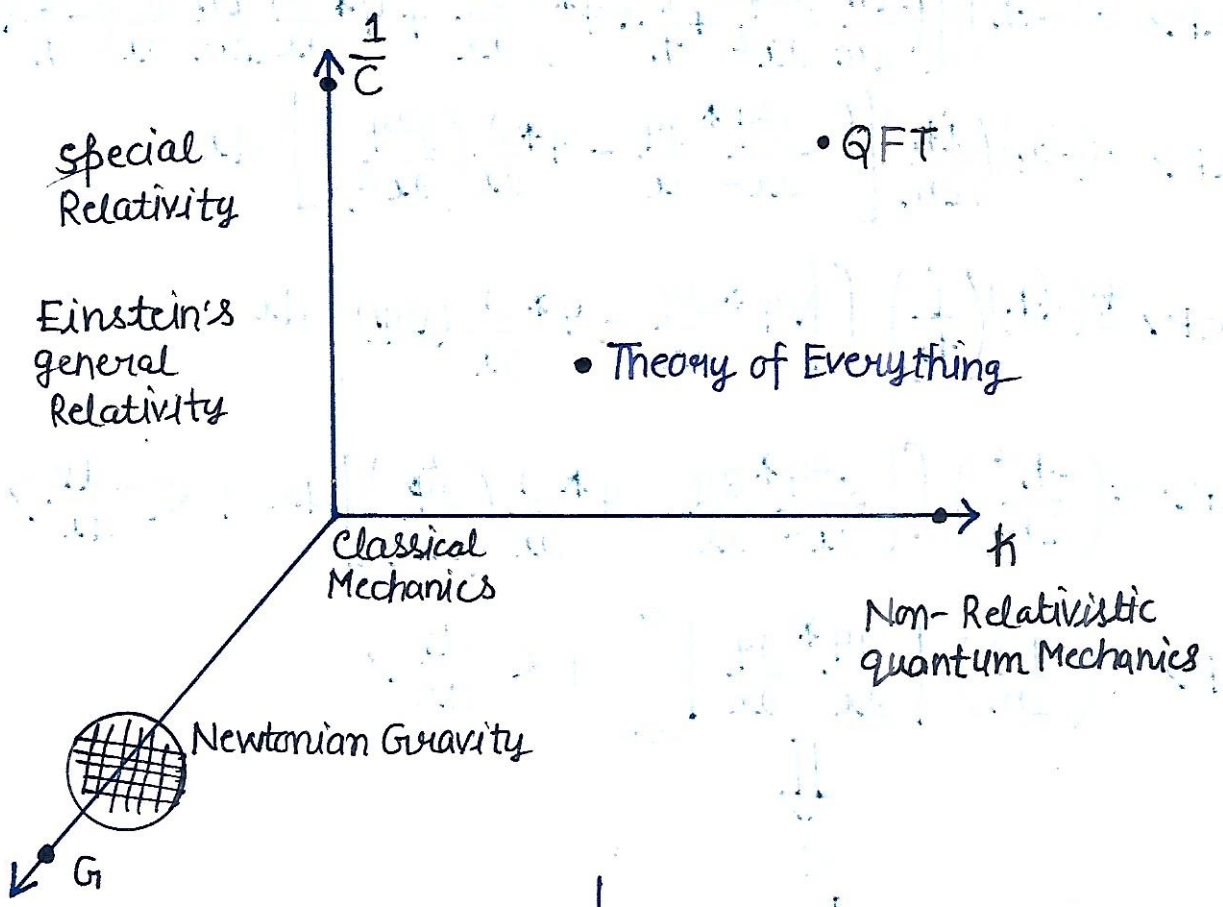
$$\Downarrow$$

$$0$$

$$\boxed{\frac{d}{dt}\langle p \rangle = \left\langle -\frac{dV}{dx} \right\rangle}$$

• Comments on studying Quantum Mechanics:-

- Feynman Lectures on YouTube.



Mathematics is common Language.

- Blackbody Radiation
- The Photoelectric effect
- Atomic spectra
- Bell Accident

Hertz Effect

$$E = \frac{hc}{\lambda} = \hbar\omega$$

COURSERA

$h = 2\pi\hbar$

$\lambda = \frac{h}{P}$ $\qquad P = mv$

$\lambda = \frac{h}{mv}$

- Wave & Particle duality
- Fourier transform

$$f(x) = \int dK A_K e^{iKx}$$

• Deriving Wave Equation-

$$\psi = Ce^{i(Kx-\omega t)}$$

$P = \hbar K$

$E = \hbar\omega$

$$\psi(x,t) = Ce^{\frac{i}{\hbar}(Px-Et)}$$

$K.E. = \frac{1}{2}mv^2$

$P = mv$

$K.E = \frac{P^2}{2m}$

$$\boxed{P = -i\hbar\nabla}$$

$$\left[i\hbar\frac{\partial}{\partial t} + \frac{\hbar^2\nabla^2}{2m}\right]\psi(\vec{x},t) = 0$$

#Qubit-

- A qubit is a unit of the amount of information.
- Superposition of $|0\rangle$ and $|1\rangle$.

• No cloaning Theorem-

Quantum Objects
↑
Probabilistic

Classical Objects
↑
Deterministic